Therefore, at distance not more than 1 from each coordinate axis in the space of exponents there is the exponent of an existing monomial. Taking a monomial near to each of the axes (which is possible since the second differential is $\equiv 0$), we obtain a mapping $i \to j$ of the set of coordinate axes to itself. Thus, we have to classify mappings of a finite set to itself. For $n = 3$ this is not hard to do. A set with three elements has seven endomorphisms (apart from a re-naming of points), and this is what gives the seven classes of the table.

The values of the homogeneous exponents α_i and the multiplicity μ are calculated according to the formulae in §4.

11.2. PROPOSITION. 1) *A non-degenerate quasihomogeneous function of class* III *exists if and only if the least common multiple* $[b, c]$ *of* b *and* c *is divisible by* $a - 1$.

2) *A non-degenerate quasihomogeneous function of class* VI *exists if and only if* $(b - 1)c$ *is divisible by the product of* $a - 1$ *and the greatest common divisor* (b, c) *of* b *and* c.

3) *Non-degenerate quasihomogeneous functions of the remaining five classes exist for arbitrary* a, b, c.

PROOF. For the proof of 3) it is enough to add up the monomials shown in the table. To prove 1) and 2), we note that a quasihomogeneous function of class III or VI is degenerate if it contains none of the monomials $y^p z^q$ $(p \geqslant 0,\ q \geqslant 0)$. For the zero level set consists in that case of two components (one of which is the plane $x = 0$). This means that the critical point is not isolated (the set of critical points contains the line of intersection of the components), and the function is degenerate.

Conversely, it is easy to check that the quasihomogeneous functions

$$\text{III: } x^a + xy^b + xz^c + \varepsilon y^p z^q \quad \text{and VI: } x^a y + y^b x + z^c x + \varepsilon y^p z^q$$

are non-degenerate for almost all ε.

It remains to prove that a diagonal monomial $y^p z^q$ exists under exactly the above divisibility conditions.

In case III the generalized degree of $y^p z^q$ is $(pc + qb)(a - 1)/abc$. The monomial $y^p z^q$ is diagonal if and only if the degree is 1, that is, $(pc + qb)(a - 1) = (a - 1)bc + bc$. Thus, bc is divisible by $a - 1$, and the quotient (which is $pc + qb + bc$) is divisible by (b, c). In other words, bc is divisible by the product of $a - 1$ and (b, c), that is, $[b, c]$ is divisible by $a - 1$.

Conversely, suppose that $[b, c]$ is a multiple of $a - 1$. Then the number $\frac{abc}{a-1} = bc + \frac{bc}{a-1}$ is an integer and divisible by bc. But every number greater than bc and a multiple of (b, c) can be written in the form[1] $pc + qb$ $(p \geqslant 0,\ q \geqslant 0)$.

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[1] For there are not less than 2 integral points on the line $\{p, q\colon pc + qb = bc\}$ in the quadrant $p \geqslant 0$, $q \geqslant 0$. The distance between consecutive integral points on every parallel line $pc + qb = m > bc$ is the same, so that there is an integral point on the segment of the line within the square (even when $m \geqslant (b - 1)(c - 1)$).

Thus, $abc/(a-1) = pc + qb$, and the monomial $y^p z^q$ is the diagonal.

In case VI the diagonality condition takes the form

$$(a-1)(pc+qb) = (a-1)bc + (b-1)c.$$

Thus, $(b-1)c$ is divisible by $(a-1)(b, c)$. Conversely, if $(b-1)c$ is divisible by $(a-1)(b, c)$, then $bc+\frac{(b-1)c}{a-1}$ can be written in the form $pc + qb$, where $p \geqslant 0$, $q \geqslant 0$, as required.

An example of a non-degenerate function of class III is $x^7 + xy^3 + xz^4 + \varepsilon x^2y^2$; and one of class VI is $x^5y + xy^3 + xz^4 + \varepsilon x^2y^2$.

We recall that the inner modality μ_0 of a quasihomogeneous function is the number of monomials in the local ring lying on the diagonal or above.

11.3. THEOREM. *The inner unimodal quasihomogeneous functions (that is, those with $\mu_0 = 1$) of corank 3 in three variables are exhausted (up to equivalence) by the following list:*

Type	Normal form	$\alpha_1, \quad \alpha_2, \quad \alpha_3$
P_8	$x^2z+y^3+ay^2z+z^3, \quad 4a^3+27\neq 0$	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$
Q_{10}	$x^3+y^4+yz^2$	$\frac{1}{3}, \frac{1}{4}, \frac{3}{8}$
Q_{11}	$x^3+y^2z+xz^3$	$\frac{1}{3}, \frac{7}{18}, \frac{2}{9}$
Q_{12}	$x^3+y^5+yz^2$	$\frac{1}{3}, \frac{1}{5}, \frac{2}{5}$
S_{11}	$x^4+y^2z+xz^2$	$\frac{1}{4}, \frac{5}{16}, \frac{3}{8}$
S_{12}	$x^2y+y^2z+xz^3$	$\frac{4}{13}, \frac{5}{13}, \frac{3}{13}$
U_{12}	$x^3+y^3+z^4$	$\frac{1}{3}, \frac{1}{3}, \frac{1}{4}$

Every non-degenerate quasihomogeneous function with quasihomogeneous exponents shown in the table can be reduced to the indicated normal form

PROOF. By §4, the last and last but one of the generalized degrees of the basis monomials are $d_{\max}$ and $d_{\max} - \alpha_{\min}$. The condition $\mu_0 = 1$ can therefore be rewritten in the form $d_{\max} \geqslant 1$, $d_{\max} - \alpha_{\min} < 1$. By §4, $d_{\max} = n - 2\Sigma\alpha_s$. Thus, for $n = 3$ the condition $\mu_0 = 1$ takes the form

$$\alpha_1 + \alpha_2 + \alpha_3 \leqslant 1, \quad 2\alpha_1 + 2\alpha_2 + 2\alpha_3 > 2 - \alpha_s, \qquad s = 1, 2, 3.$$

Now we must sort out the seven classes I—VII of Proposition 11.1, and choose an α in each case for which the given inequalities hold. We discard the case when monomials of degree 2 occur, since we are interested in functions of corank 3. Substituting the values of α, from Table 11.1, we get the following integral points corresponding to singularities of corank 3:

Class	(a, b, c) and notation for the function
I	$(3, 3, 3) = P_8$; $(3, 3, 4)$, $(3, 4, 3)$, $(4, 3, 3) = U_{12}$
II	$(3, 3, 2) = P_8$; $(4, 3, 2) = U_{12}$; $(3, 4, 2) = Q_{10}$; $(3, 5, 2) = Q_{12}$
III	$(3, 2, 2)$, $(2, 3, 2)$, $(2, 2, 3) = P_8$
IV	$(3, 2, 2) = P_8$; $(4, 2, 2) = U_{12}$; $(3, 3, 2)$, $(3, 2, 3) = Q_{12}$
V	$(3, 2, 2) = P_8$; $(3, 2, 3) = Q_{11}$; $(4, 2, 2) = S_{11}$
VI	$(2, 2, 2) = P_8$
VII	$(2, 2, 3)$, $(2, 3, 2)$, $(3, 2, 2) = S_{12}$; $(2, 2, 2) = P_8$

On the boundary of the domain of permissible α (that is, for $2\alpha_1 + 2\alpha_2 + 2\alpha_3 \geqslant 2 - \alpha_s$), there are some further integral points with integral a, b, c, namely:

Class	(a, b, c) and notation for the function
II	$(3, 3, 3) = U_{14}$; $(4, 4, 2) = V_{15}$; $(3, 6, 2) = Q_{14}$
III	$(3, 2, 3)$, $(3, 3, 2) = U_{14}$; $(4, 2, 3)$, $(4, 3, 2) = V_{15}$
V	$(3, 2, 4) = Q_{14}$; $(4, 2, 3) = V_{15}$; $(5, 2, 2) = S_{14}$; $(3, 3, 2) = U_{14}$
VI	$(2, 3, 2) = S_{14}$; $(2, 2, 3) = U_{14}$; $(3, 3, 2) = V_{15}$
I, IV, VII	—

For classes III and VI we have shown here only the integral points satisfying the divisibility relations of Proposition 11.2.

We show now that non-degenerate quasihomogeneous functions of the types listed really exist, and exhibit normal forms for them.

By Table 11.1, the homogeneous exponents α_s correspond to integral points (a, b, c). Sometimes triples (a, b, c) in different classes give rise to the same exponents. Such triples have been denoted in the preceding tables by the same letters (for instance, (4,3,3) in class I and (4, 2, 2) in class IV are denoted by U_{12}, and both give $\alpha_1 = 1/4$, $\alpha_2 = \alpha_3 = 1/3$).

When we know the homogeneous exponents, we can find all diagonal monomials. For example, for the values of α_s of type U_{12} just indicated, the equation of the diagonal Γ has the form $3p + 4q + 4r = 12$. The diagonal monomial $x^p y^q z^r$ corresponds to a non-negative solution of this equation in integers. There are five such solutions:

$$(4,0,0),\ (0,3,0),\ (0,2,1),\ (0,1,2),\ (0,0,3).$$

When we carry out the analogous computations for all the cases listed above, we come to the following table (in which the cases reducing to a permutation of the coordinate axes are omitted):

Type	A_1, A_2, A_3; N	Exponents of diagonal monomial $x^p y^q z^r$
$\mathrm{I}_{333} = P_8$	1, 1, 1; 3	(300), (210), (201), (120), (102), (111), (021), (012), (030), (003)
$\mathrm{II}_{342} = Q_{10}$	8, 6, 9; 24	(300), (040), (012)
$\mathrm{V}_{323} = Q_{11}$	6, 7, 4; 18	(300), (021), (103)
$\mathrm{II}_{352} = Q_{12}$	5, 3, 6; 15	(300), (050), (031), (012)
$\mathrm{V}_{422} = S_{11}$	4, 5, 6; 16	(400), (021), (102)
$\mathrm{VII}_{223} = S_{12}$	4, 5, 3; 13	(210), (021), (103)
$\mathrm{I}_{433} = U_{12}$	3, 4, 4; 12	(400), (030), (021), (012), (003)

Here $\alpha_s = A_s/N$, so that $A_1 p + A_2 q + A_3 r = N$.

The boundary points of the domain of permissible α give four more types of quasihomogeneity:

Type	$A_1,\ A_2,\ A_3;\ N$	Exponents of diagonal monomials $x^p y^q z^r$
$\mathrm{II}_{362} = Q_{14}$	4, 2, 5; 12	(300), (220), (140), (060), (012)
$\mathrm{VI}_{232} = S_{14}$	4, 2, 3; 10	(050), (130), (210), (102), (022)
$\mathrm{II}_{333} = U_{14}$	3, 3, 2; 9	(300), (210), (120), (030), (103), (013)
$\mathrm{II}_{442} = V_{15}$	2, 2, 3; 8	(400), (310), (220), (130), (040), (102), (012)

We now choose normal forms for the functions with these 11 types of quasihomogeneity.

CASES Q_{10}, Q_{11}, S_{11} and S_{12}. All of these have three diagonal monomials each. The coefficients must be different from 0 (otherwise the function is degenerate). After magnification of coordinates the function reduces to a sum of diagonal monomials, that is,

$$Q_{10} = x^3 + y^4 + yz^2, \quad Q_{11} = x^3 + y^2z + xz^3,$$
$$S_{11} = x^4 + y^2z + xz^2, \quad S_{12} = x^2y + y^2z + xz^3.$$

CASE Q_{12}. The general linear combination of the diagonal monomials in the table is

$$Ax^3 + By^5 + Cy^3z + Dyz^2.$$

For a non-degenerate function, $AD \neq 0$. Replacing z by $z + \lambda y^2$ we reduce the function to a form in which $C = 0$. After this, magnification of the coordinates x, y, z converts the coefficients A, B, C to 1, and we obtain

$$Q_{12} = x^3 + y^5 + yz^2.$$

CASE U_{12}. According to the table, the general form of a quasihomogeneous function of type U_{12} is $Ax^4 + By^3 + Cy^2z + Dyz^2 + Ez^3$. For a non-degenerate function $A \neq 0$, and the cubic form in y and z is non-degenerate. A non-degenerate binary cubic form can be reduced by a linear change of variables to $y^3 + z^3$. The coefficient A can be made unity by magnifying the x-axis. So we obtain

$$U_{12} = x^4 + y^3 + z^3.$$

CASE P_8. It is well known that a non-degenerate ternary cubic form can be reduced to the Weierstrass normal form

$$x^2z - 4y^3 + g_2yz^2 + q_2z^3, \quad g_2^3 - 27g_3^2 \neq 0.$$

Replacing y by $y + \lambda z$, we reduce the form to $x^2z - 4y^3 + Ay^2z + Bz^3$. By non-degeneracy, $B \neq 0$. By a magnification of the x- and z-coordinates we can reduce the form to

$$P_8 = x^2z + y^3 + ay^2z + z^3.$$

This form is non-degenerate if $4a^3 + 27 \neq 0$.

This completes the proof of Theorem 11.3.

11.4. PROPOSITION. *Non-degenerate quasihomogeneous functions with quasihomogeneous exponents Q_{14}, S_{14}, U_{14} and V_{15} (see above) can be reduced by a quasihomogeneous diffeomorphism to the following normal forms:*

Type	Normal form	Non-degeneracy condition
Q_{14}	$yz^2+x^3+ax^2y^2+y^6$	$4a^3+27 \neq 0$
S_{14}	$xz^2+x^2y+axy^3+y^5$	$a^2-4 \neq 0$
U_{14}	$x^3+y^3+xz^3+ayz^3$	$a^3-1 \neq 0$
V_{14}	$xz^2+ayz^2+x^4+bx^2y^2+y^4$	$(b^2-4)(a^4+a^2b+1) \neq 0$

PROOF. CASE Q_{14}. From the table of diagonal monomials we find

$$Q_{14} = A_1x^3 + A_2x^2y^2 + A_3xy^4 + A_4y^6 + A_5yz^2.$$

The non-degeneracy condition gives $A_1 \neq 0$. Replacement of x by $x + \lambda y^2$ removes A_3. In the expression so obtained, $A_4A_5 \neq 0$ (non-degeneracy). By magnifying coordinates we make $A_1 = A_4 = A_5 = 1$. When this is done, the non-degeneracy conditions takes the form $4A_2^3 + 27 \neq 0$.

CASE S_{14}. From the table of diagonal monomials,

$$S_{14} = A_1xz^2 + A_2x^2y + A_3xy^3 + A_4y^5 + A_5y^2z^2.$$

It follows from non-degeneracy that $A_1 \neq 0$. Replacement of x by $x + \lambda y^2$ removes A_5. After this, non-degeneracy gives $A_1A_2A_4 \neq 0$. We make $A_1 = A_2 = A_4 = 1$ by magnifiying coordinates. The non-degeneracy condition then takes the form $A_3^2 \neq 4$.

CASE U_{14}. From the table of diagonal monomials,

$$U_{14} = A_1x^3 + A_2x^2y + A_3xy^2 + A_4y^3 + A_5xz^3 + A_6yz^3.$$

The cubic form $A_1x^3 + \ldots + A_4y^3$ is non-degenerate, and by a linear change of the variables x, y it can be reduced to the form $x^3 + y^3$. It follows from non-degeneracy that one of the coefficients A_5, A_6 is not zero, say A_5. We make $A_5 = 1$ by magnifying the z-coordinate. The non-degeneracy condition then takes the form $A_6^3 \neq 1$.

CASE V_{15}. From the table of diagonal monomials,

$$V_{15} = A_1z^2x + A_2z^2y + A_3x^4 + A_4x^3y + A_5x^2y^2 + A_6xy^3 + A_7y^4.$$

By non-degeneracy, the binary 4-form $A_3x^4 + \ldots + A_7y^4$ is itself non-degenerate. By a linear change of x and y we can annihilate A_4 and A_6 (see 8.3). By non-degeneracy, one of A_1, A_2 is not 0, say A_1. By a magnification of coordinates we achieve that $A_1 = A_3 = A_7 = 1$. The non-degeneracy condition then assumes the form $A_5^2 \neq 4$, $A_2^4 + A_5A_2^2 + 1 \neq 0$. This proves Proposition 11.4.

11.5. PROPOSITION. *The following sets of monomials are bases for the local rings of the functions in Theorem* 11.3:

Type	Normal form	Basis monomials and their weights
P_8	$x^2z+y^3+ay^2z+z^3$ $4a^3+27\neq 0 \quad N=3$	$1,\ x,\ y,\ z,\ xy,\ yz,\ z^2,\ yz^2$ $0,\ 1,\ 1,\ 1,\ 2,\ 2,\ 2,\ 3$
Q_{10}	$x^3+y^4+yz^2$ $N=24$	$1,\ y,\ x,\ z,\ y^2,\ xy,\ yz,\ y^3,\ xy^2,\ xy^3$ $0,\ 6,\ 8,\ 9,\ 12,\ 14,\ 17,\ 18,\ 20,\ 26$
Q_{11}	$x^3+y^2z+xz^3$ $N=18$	$1,\ z,\ x,\ y,\ z^2,\ xz,\ z^3,\ xy,\ xz^2,\ z^4,\ z^5$ $0,\ 4,\ 6,\ 7,\ 8,\ 10,\ 12,\ 13,\ 14,\ 16,\ 20$
Q_{12}	$x^3+y^5+yz^2$ $N=15$	$1,\ y,\ x,\ z,\ y^2,\ xy,\ y^3,\ xz,\ xy^2,\ y^4,\ xy^3,\ xy^4$ $0,\ 3,\ 5,\ 6,\ 6,\ 8,\ 9,\ 11,\ 11,\ 12,\ 14,\ 17$
S_{11}	$x^4+y^2z+xz^2$ $N=16$	$1,\ x,\ y,\ z,\ x^2,\ xy,\ xz,\ x^3,\ x^2y,\ x^2z,\ x^3z$ $0,\ 4,\ 5,\ 6,\ 8,\ 9,\ 10,\ 12,\ 13,\ 14,\ 18$
S_{12}	$x^2y+y^2z+xz^3$ $N=13$	$1,\ z,\ x,\ y,\ z^2,\ xz,\ yz,\ xy,\ xz^2,\ yz^2,\ z^4,\ z^5$ $0,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 15$
U_{12}	$x^3+y^3+z^4$ $N=12$	$1,\ z,\ x,\ y,\ z^2,\ xz,\ yz,\ xy,\ xz^2,\ yz^2,\ xyz,\ xyz^2$ $0,\ 3,\ 4,\ 4,\ 6,\ 7,\ 7,\ 8,\ 10,\ 10,\ 11,\ 14$

Here the number below the monomial denotes its weight: $x^py^qz^r$ has weight $i = A_1p + A_2q + A_3r$, where the $\alpha_s = A_s/N$ are the homogeneous exponents (the values of A_s and N are shown in pages 44/5).

The proof is based on the formula in §4 for the number μ_i of monomials of weight i. The choice of μ_i monomials out of all solutions of the equation $i = A_1p + A_2q + A_3r$ proceeds by means of the "crossword solution" as described in 9.6.

Similar calculations for the functions $Q_{14} - V_{15}$ lead to the following result:

11.6. PROPOSITION. *The following sets of monomials are bases for the local rings of the functions in Proposition* 11.4:

Type	Monomials below the diagonal	On	Above
Q_{14}	$1,\ y,\ x,\ y^2,\ z,\ y^3,\ xy,\ y^4,\ xy^2,\ xz,\ y^5,\ xy^3$ $0,\ 2,\ 4,\ 4,\ 5,\ 6,\ 6,\ 8,\ 8,\ 9,\ 10,\ 10$	y^6 12	y^7 14
S_{14}	$1,\ y,\ z,\ x,\ y^2,\ yz,\ y^3,\ xy,\ y^2z,\ xy^2,\ y^4,\ y^3z$ $0,\ 2,\ 3,\ 4,\ 4,\ 5,\ 6,\ 6,\ 7,\ 8,\ 8,\ 9$	xy^3 10	xy^4 12
U_{14}	$1,\ z,\ x,\ y,\ z^2,\ xz,\ yz,\ xy,\ z^3,\ yz^2,\ z^4,\ xyz$ $0,\ 2,\ 3,\ 3,\ 4,\ 5,\ 5,\ 6,\ 6,\ 7,\ 8,\ 8$	yz^3 9	yz^4 11
V_{15}	$1,\ x,\ y,\ z,\ x^2,\ xy,\ y^2,\ yz,\ x^2y,\ xy^2,\ z^2,\ y^2z$ $0,\ 2,\ 2,\ 3,\ 4,\ 4,\ 4,\ 5,\ 6,\ 6,\ 6,\ 7$	$yz^2,\ x^2y^2$ 8, 8	y^2z^2 10

Thus, in all the cases listed we have succeeded in giving a single system of monomials furnishing a basis of the local ring for all values of the parameters for which the function is non-degenerate. The possibility of such a choice of normal forms and bases in the cases P_8, Q_{14}, S_{14}, U_{14}, V_{15} has not been clear up to now.

11.7. COROLLARY. *Every semiquasihomogeneous function with quasihomogeneous part of any of the types $P_8 - U_{12}$ (or with quasihomogeneous exponents as in Table 11.3) reduces under diffeomorphism to one of the following normal forms:*

Type	Normal form	s
P_8	$x^3z+y^3+ay^2z+z^3,\quad 4a^3+27\neq 0$	3
Q_{10}	$x^3+y^4+yz^2+axy^3$	4
Q_{11}	$x^3+y^2z+xz^3+az^5$	5
Q_{12}	$x^3+y^5+yz^2+axy^4$	5
S_{11}	$x^4+y^2z+xz^2+ax^3z$	4
S_{12}	$x^2y+y^2z+xz^3+az^5$	5
U_{12}	$x^3+y^3+z^4+axyz^2$	4

Bases for the local rings are the same as for the quasihomogeneous singularities shown in Table 11.5. The number s is the order of a sufficient s-jet.

11.8. COROLLARY. *Every semiquasihomogeneous function with quasihomogeneous exponents of the singularities Q_{14}, S_{14}, U_{14}, V_{15} can be* *reduced by a diffeomorphism to one of the following normal forms*:

Type	Normal form	Non-degeneracy condition	s
Q_{14}	$yz^2+x^3+ax^2y^2+y^6+by^7$	$4a^3+27\neq 0$	7
S_{14}	$xz^2+x^2y+axy^3+y^5+bxy^4$	$a^2-4\neq 0$	6
U_{14}	$x^3+y^3+xz^3+ayz^3+byz^4$	$a^3-1\neq 0$	5
V_{15}	$xz^2+ayz^2+x^4+bx^2y^2+y^4+cy^2z^2$	$b^2-4\neq 0$ $a^4+a^2b+1\neq 0$	5

Proofs of these corollaries are obtained from Theorem 7.2 and Tables 11.3, 11.4, 11.5, 11.6. The number s is computed on the basis of 9.10 (the proof of the equation $s(S_{11}) = 4$ uses the fact that x^5 lies in the ideal for arbitrary a).

11.9. COROLLARY. *Each of the singularities in the list* 11.7 *has inner modality* $\mu_0 = 1$ *and modality not less than* 1. *Singularities of the types* Q_{14}, S_{14}, U_{14} *have* $\mu_0 = 2$ *and modality not less than* 2; *the type* V_{15} *has* $\mu_0 = 3$ *and modality not less than* 3.

11.10. REMARK. In fact *the modalities of the singularities listed in* §§10 *and* 11 *are the same as the corresponding inner modalities* μ_0 (see [3]); however, we do not prove this here.

11.11. THEOREM. *Every quasihomogeneous function of inner modality* $\mu_0 = 1$ *is stably equivalent to a function of one of the* 17 *types in Theorems* 10.3 (part 2) *and* 11.3.

PROOF. The finiteness of μ follows from that of μ_0. By a theorem of Saito (1.3 of [13]), every quasihomogeneous function with finite μ is stably equivalent to a quasihomogeneous function (possibly in fewer variables) whose second differential is identically 0. The inner modality of the new function is the same as that of the original one. The new function is not a function of a single variable, or else $\mu_0 = 0$. If it is a function of two variables, then by Theorem 10.3(2), the function can be reduced to one of the 10 forms shown in this theorem. If it is a function of three variables, Theorem 11.3 says that it can be reduced to one of the seven forms shown in this theorem.

For corank greater than 3 there are no functions of inner modality 1. For Saito proved that the (generalized) degree of a basis monomial of highest degree in the local ring of any quasihomogeneous function of generalized degree 1 with zero second differential is not less than that of a non-degenerate cubic form in the same number of variables (Theorem 2.11 of [11]): $d_{\max} = n-2\sum\alpha_s \geqslant n-\frac{2}{3}n$. On the other hand, it follows from this inequality that $\Sigma\alpha_s \leqslant n/3$, which means that $\min\alpha_s$ is not more than

1/3. On the other hand, for $n \geqslant 4$ we have $d_{\max} = n - 2\sum \alpha_s \geqslant 4/3$. Thus, for $n \geqslant 4$ we have not only $d_{\max} \geqslant 1$, but $d_{\max} - \alpha_{\min} \geqslant 1$. Therefore, $\mu_0 \geqslant 2$ by Corollary 7.11. This proves the theorem.

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