Cubic couplings in higher-spin gauge theory

A review of old and new results

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Plan

General Ideas in Gauge Theory

★ **Gauge principle** [H. Weyl (1929)]: elevate local symmetry to an *a priori* fundamental principle of universal validity – an *expression of the simplicity of Nature at its deepest level* [S. Weinberg].

↪ **Standard Model**: 1-forms $\in$ compact Lie algebra $\mathfrak{h}$. Finite-dimensional unitarizable representations called $\mathfrak{h}$-types. Incorporates *Yang-Mills theory + matter*: spin-1 gauge fields plus 0-forms in $\mathfrak{h}$-types.
Non-dynamical spacetime

↪ **Gravity** : Non-compact $\mathfrak{g} \supset \mathfrak{m} \cong \mathfrak{so}(n-1,1)$. Infinite-dimensional unitarizable representations: $\mathfrak{m}$-type valued functions on a dynamical spacetime looking locally like $\mathfrak{g}/\mathfrak{m}$. Incorporates *Gravity + Yang-Mills + matter*.

Fully nonlinear higher-spin gauges theories

- Gauge Principle: Higher-spin gauge theories $\leftrightsquigarrow$ infinite-dimensional nonabelian gauge algebras containing the spacetime isometry algebra as maximal finite-dimensional subalgebra. Gravity is a subsector.
- Unification of fundamental interactions. Fields with *arbitrarily high spin* most probably necessary for consistency, as in string theory.
- Vasiliev's unfolding: *Geometric* approach to field theory. Manifest differomorphism covariance. Gauge invariance is a consequence of Cartan's integrability.
- AdS/CFT duality between Vasiliev's theory and free CFT [Sundborg, Sezgin–Sundell, Klebanov–Polyakov]. Relations with statistical physics, integrable models, strings etc.

THE PRINCIPLE OF UNFOLDING [VASILIEV, 1988 –]

- The concepts of spacetime, dynamics and observables are *derived* from infinite-dimensional FDA'a.
- Unfolded dynamics is an inclusion of local d.o.f. into field theories described *on-shell* by flatness conditions on generalized curvatures, and generically *infinitely many* local zero-form observables in the presence of a cosmological constant.
- Spin-2 couplings arise (albeit together with exotic higher-derivative couplings) in the limit in which the $\mathfrak{so}(2,n-1)$-valued part of the higher-spin connection one-form is treated exactly while its remaining spin $s>2$ components become weak fields together with all curvature zero-forms

Some questions

- HS algebras were derived from the Fradkin–Vasiliev nonabelian cubic vertices in AdS_4 background (1987).

  $\hookrightarrow$ How *unique* are those vertices ? Is AdS background mandatory ?

- Independently, higher-spin nonabelian cubic vertices were also obtained in flat background following various approaches (1983 —).

  $\hookrightarrow$ Any relation with the FV vertices in AdS_4 ?

- String theory first appeared as a model reproducing the Regge trajectories of *massive* hadronic resonances with increasing spin.

  Veneziano amplitudes: extremely soft UV behaviour thanks to the exchange of infinitely many massive HS states.

  $\hookrightarrow$ String (field) theory $\overset{?}{\leftrightsquigarrow}$ broken phase of higher-spin *gauge* theory ?

Interactions in flat space ($\Lambda = 0$)

In the 80's, the quest for HS interactions started, around flat spacetime.

- In the *light-cone gauge*, higher-spin $s - s' - s''$ cubic vertices were found in 4D by Bengtsson–Bengtsson–Brink, (1983). [Since 1993, these results have been considerably generalized by R. Metsaev.]
- *Manifestly covariant* cubic vertices of Berends–Burgers–van Dam (1984):

$\downarrow_{s_1}$ $\xrightarrow{s_2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
0	$\times$	$\times$	$\times$	$\times$	$\times$		
1	$\times$	$\times$	$\times$	$\times$	$\times$		
2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	
3	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
n	$\times$						

Table: $s_1 - s_2 - s_2$ *covariant vertices obtained by BBvD (1984).*

Gravitational HS coupling in flat space

- However, inconsistencies appeared in the HS gravitational coupling in flat background [Aragone–Deser (1979), Berends–van Holten–de Wit–van Nieuwenhuizen (1979), Aragone–La Roche (1982)].
- The standard 2-derivative Lorentz coupling, also called gravitational minimal coupling

$$\begin{aligned} \partial &\rightarrow \partial + \kappa\,\Gamma(h) \\ \kappa\, h_{\mu\nu} &:= g_{\mu\nu} - \eta_{\mu\nu} \end{aligned}$$

turned out to be *inconsistent* !

$\hookrightarrow$ beginning of negative prejudices concerning HS gauge theory, strengthened by Weinberg's *low-energy theorems*, purely S-matrix.

Some ways out

- Fradkin–Vasiliev solved the problem by going to AdS_4. There is no S-matrix in AdS . Starting point of important progresses $\rightsquigarrow$ full nonlinear field equations for all HS fields around AdS_n [M. Vasiliev (2003)].
- On top of the vertices found in the 80's, there exist *some other* non-Abelian vertices which contain more derivatives. These vertices (at least $2-s-s$ and $1-s-s$) in flat space turn out to be the flat limit of the FV top vertices [N.B., S. Leclercq and P. Sundell].
- Weinberg's argument in flat space only concerns the low-energy regime $\rightsquigarrow$ vertices with the minimal number of derivatives. As stressed by Weinberg himself in his book: *Massless higher-spin particles may exist, but they cannot have couplings that survive in the limit of low energy.*

Reformulation of Noether's method

- The problem of consistently deforming a free theory, like Fronsdal's theory, into a full, *interacting* theory, can be reformulated within the BRST–BV antifield formalism [Barnich–Henneaux (1993)].
- To the free Fronsdal action $S^{Fr}[\varphi^i]$ (gauge invariant under $\delta\varphi^i = R^{(0)i}{}_\alpha\,\varepsilon^\alpha$) one associates the local functional $\overset{(0)}{W}[\Phi,\Phi^*]$ that depends on the fields $\{\varphi^i\}$, the ghosts $\{C^\alpha\}$ and their antifields $\Phi^* = \{\varphi^*_i, C^*_\alpha\}$.
  The local functional $\overset{(0)}{W}$ satisfies the master equation $\boxed{(\overset{(0)}{W},\overset{(0)}{W})_{A.B.} = 0}$.
- Then $W := \int w = \overset{(0)}{W} + g\,\overset{(1)}{W} + g^2\,\overset{(2)}{W} + \mathscr{O}(g^3)$.
  Consistent deformation: if $(W,W)_{A.B.} = 0$ to all orders in g.

First-order deformations

- The first order equation is $(\overset{(0)}{W},\overset{(1)}{W}) = \mathbf{s}\,\overset{(1)}{W} = 0\,.$ In the case of a local deformation, $\overset{(1)}{W}$ must be the integral of a *local* n-form $a := \overset{(1)}{w}$, and the equation $\mathbf{s}\,\overset{(1)}{W} = 0$ becomes an $\mathbf{s}$-cocycle relation modulo d:

$$\boxed{\mathbf{s}\,a + \mathrm{d}\,b = 0} \quad .$$

  Since $\mathbf{s}$-exact and d-exact terms in the cocycle $\overset{(1)}{w}$ correspond to trivial deformations, the first order inequivalent deformations are given by the cohomology classes of $H^{0,n}(\mathbf{s}|\mathrm{d})\,.$

- The cubic first order deformation can be expanded in the antifield number $a = a_0 + a_1 + a_2\,.$

Non-Abelian deformations

- The component a_2 contains the information about the first-order deformation of the (*a priori* open) gauge algebra. The non-Abelian deformations are thus characterized by a non-vanishing (and nontrivial) a_2 component.

  [A cubic a_2 is linear in the antifield number 2 antifields and quadratic in the ghosts, it does not depend on the fields. Consequently, the gauge algebra closes off-shell at first-order in the deformation, for cubic vertices.]

- The component a_1 contains the information about the first-order deformation of the gauge transformations.
- The component a_0 is the cubic vertex.

Bounds on derivatives for non-Abelian vertices: Covariant approach

Theorem ([X. Bekaert, N.B., S. Leclercq])

Given a cubic configuration of fields with spins $s \leqslant s' \leqslant s''$, the possible Poincaré invariants $a_2 = C^* U^{(i)} U^{(j)}\, d^n x$ are contractions of an undifferentiated antifield number-2 antighost and of two ghost tensors, involving i and j derivatives. The spins and the numbers of derivatives have to satisfy the following properties:

- $0 \leqslant |i - j| < s + s' - s''$
- $s + s' + s'' + i + j$ *is odd*
- *In the case of a spin-s antifield:* $i + j \leqslant 2s' - 2$
  In the case of a spin s' or s'' antifield: $i + j \leqslant 2s - 2$

Bounds on derivatives for cubic vertices: Light-cone gauge approach

In the light-cone gauge approach, Metsaev (2005) obtained a complete classification of cubic vertices for totally symmetric gauge fields in flat spacetime of dimension $n \geqslant 4$:

Theorem

Given a cubic configuration of totally symmetric fields with spins $s \leqslant s' \leqslant s''$, there exist cubic vertices involving k derivatives, where

$$s'' + s' - s \leqslant k \leqslant s'' + s' + s \quad .$$

The cases saturating the bounds are defined in $n \geqslant 4$ whereas the other intermediate cases appear in $n \geqslant 5$.

Bottom-to-Top: Cubic Vertices $\Lambda = 0$

- **YM/Gravity**: remarkable in that spin-1/spin-2 fields have cubic 1-derivative/2-derivative self-interactions that survive at the full level provided structure functions obey Jacobi identities (internal/diffeo.)
- **Light-cone methods** [Bengtsson^2; Brink; Linden; Fradkin; Metsaev]: cubic symmetric-rank-$s - s' - s''$ vertices with $m = s + s' + s'' - 2p$ derivatives $p = 0, 1, \ldots, \min(s, s', s'')$
- **Minimal non-Abelian vertices**: e.g. 3-derivative BBvD $3 - 3 - 3$ vertex. Suffers incurable quartic obstruction of the Jacobi identity. Remains obstructed also in the presence of spin 2,4,5 fields [X. Bekaert, N.B., S. Leclercq]. However, perhaps no problem for even spin gauge fields [cf. recent work by Manvelyan et al.]

Bottom-to-Top: Cubic Vertices $\Lambda = 0$

- **Maximal nonabelian vertices**: Non-Abelian interactions for $n \geqslant 5$ (*e.g.* 5-derivative $3-3-3$ vertex). These vertices automatically satisfy the Jacobi identity for the gauge algebra.
- **Born-Infeld/Chern–Simon/Bell-Robinson-like** vertices. Correspond to Abelian gauge algebra. They may become relevant at order g^2 and require a non-Abelian algebra at that order.
- From S-matrix open-string tree amplitudes, seems that all these vertices are used [Sagnotti–Tarona]. Broken gauge invariance by Stückelberg mechanism. See also recent work by D. Polyakov.

Fradkin–Vasiliev mechanism: $\Lambda \neq 0$

- **Minimal coupling**: $\nabla = \partial + \Gamma(g)$, $[\nabla, \nabla] \sim \Lambda g g + W(g)$
- **Fradkin-Vasiliev** $2 - s - s$ **vertex**: Einstein-Hilbert + Λ-term + covariantized Fronsdal action + polynomial albeit exotic $\Lambda^{-m/2}\nabla^m$-expansion terminating at the top-vertex; *e.g.* spin-3:

$$S^{\Lambda}_{\rm FV}[g,\phi] = \frac{1}{(\ell_p)^2}\int \left((\nabla\phi)^2 + \Lambda\phi^2 + W\phi^2 + \frac{1}{\Lambda}W\{\nabla^2\phi^2\}\right)$$

- **Naive flat limit**: the FV vertex $\rightarrow$ the top-vertex $a_0^{\Lambda=0} + d(\cdot)$ in nonuniform multiple-scaling limit [Λ, ℓ_p, ϕ_s, $W(g) \rightarrow 0$ in units of $\tilde{\ell}$ (rescaled Planck length) nonuniformly in s]. [N.B., S. Leclercq and P. Sundell]

Local vs Exotic ∂^m-Interactions

- **Separated length scales**: $\ell_p \ll \ell \ll \lambda^{-1}$ where ℓ_p is Planck scale (overall normalization of action); $\ell \sim \| \phi \| / \| \partial\phi \|$ (physical wave lengths); and $\lambda^2 \sim \Lambda$ (infra-red cutoff)
- **Mild non-locality**: weakly coupled $V_m \sim (\ell_p/\ell)^{m-2} \ll 1$ (broken HS symmetry, tensionful sigma model)
- **Exotic non-locality** (Fradkin, Vasiliev): strongly coupled $V_m \sim (\ell\lambda)^{-m+2} \gg 1$ (unbroken HS symmetry, topological phase-space sigma models)

Conclusions

- Existence of higher-spin gauge theories, perturbation around AdS_n: Full non-linear and consistent field equations [Vasiliev, 2003].
- Uniqueness of Fradkin–Vasiliev non-Abelian $2-s-s$ vertex.

  Quasi-minimal coupling = minimal coupling + finite derivative expansion terminating at top vertex. Flat limit of Fradkin–Vasiliev cubic vertex $\rightsquigarrow$ non-Abelian and non-minimal flat space cubic vertex [N.B., S. Leclercq and P. Sundell].

  Appear in string theory [D. Polyakov, Sagnotti–Tarona].