

AdS₃ Vacua and RG Flows in Three Dimensional Gauged Supergravities

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ABSTRACT: We study AdS_3 supersymmetric vacua in $N = 4$ and $N = 8$, three dimensional gauged supergravities, with scalar manifolds $(\frac{SO(4,4)}{SO(4) \times SO(4)})^2$ and $\frac{SO(8,8)}{SO(8) \times SO(8)}$, non-semisimple Chern-Simons gaugings $SO(4) \ltimes \mathbf{R}^6$ and $(SO(4) \ltimes \mathbf{R}^6)^2$, respectively. These are in turn equivalent to $SO(4)$ and $SO(4) \times SO(4)$ Yang-Mills theories coupled to supergravity. For the $N = 4$ case, we study renormalization group flows between UV and IR AdS_3 vacua with the same amount of supersymmetry: in one case, with (3,1) supersymmetry, we can find an analytic solution whereas in another, with (2,0) supersymmetry, we give a numerical solution. In both cases, the flows turn out to be v.e.v. flows, i.e. they are driven by the expectation value of a relevant operator in the dual $SCFT_2$. These provide examples of v.e.v. flows between two AdS_3 vacua within a gauged supergravity framework.

KEYWORDS: AdS/CFT correspondence, gauged supergravity.

1. Introduction

Three dimensional gauged supergravities turn out to possess a very rich structure, and one reason to be interested in them, apart from their intrinsic geometrical elegance, is that they offer a convenient arena to discuss various aspects of AdS_3/CFT_2 correspondence, much in the same way the study of various backgrounds of five-dimensional gauged supergravity has been useful in uncovering interesting phenomena in the dual four dimensional Yang-Mills theory[1, 2, 3].

The construction of three dimensional, N -extended, gauged supergravities has been worked out systematically for any $N \leq 16$ in [4] extending previous results on $N = 8, 16$ obtained in [5, 6, 7]. When gauging isometries of the scalar manifold of the original, ungauged supergravity theory, one introduces gauge fields which have Chern-Simons kinetic terms and therefore do not represent propagating degrees of freedom. On the other hand, when reducing a higher dimensional supergravity theory down to three dimensions, which is the instance we are interested in, one generically obtains gauge-fields with Yang-Mills like kinetic terms. The apparent puzzle was solved in [4] and [8] and has to do with the duality between gauge fields and scalars in three dimensional space-time: more precisely, it has been shown there that, if the gauge group is not semisimple, but contains nilpotent shift symmetries, i.e. it is of the form $G \ltimes \mathbf{R}^{\dim G}$, then one can integrate out half of the $2 \dim G$ Chern-Simons gauge fields to produce a Yang-Mills action for the remaining ones. At the same time, $\dim G$ scalars can be set to zero by using the shift symmetries. In other words, one trades scalars with vectors and, of course, the number of physical degrees of freedom is unchanged. This mechanism has been employed, for example, in [9] for $N = 8$, where it has been shown that a gauging by $SO(4) \ltimes \mathbf{R}^6$ indeed reproduces, at the $N = 8$ point in the scalar manifold, the Kaluza-Klein spectrum of the six-dimensional (2,0) supergravity on $AdS_3 \times S^3$ [10]. The latter is the background one obtains by taking the near horizon geometry of a D1-D5 system of type IIB theory on $K3$ or T^4 , corresponding to a CFT_2 with (4,4) supersymmetry.

In this paper, we analyze two examples of gauged supergravities with non-semisimple gauging, with $N = 4$ and $N = 8$ supersymmetry, whose scalar manifolds take the forms of $(\frac{SO(4,4)}{SO(4) \times SO(4)})^2$ and $\frac{SO(8,8)}{SO(8) \times SO(8)}$, respectively. As for the gauging, we will consider gauge groups $SO(4) \ltimes \mathbf{R}^6$ and $(SO(4) \ltimes \mathbf{R}^6)^2$, respectively. These turn out to be subgroups of the isometry groups which can be gauged consistently with supersymmetry, as will be shown.

We will study supersymmetric AdS_3 vacua in both of these theories, with various amount of preserved supersymmetries. In the $N = 4$ case, we will be able to study the flow between different vacua with different cosmological constants but the same

amount of supersymmetry. Quite remarkably, we will be able to find an analytic flow solution between vacua with (3,1) supersymmetry involving two active scalar fields. For the case of flow between (2,0) vacua which involves three active scalars, we will discuss a numerical flow solution. The flows turn out to be v.e.v. flows driven by vacuum expectation values of some operators in the UV. Examples of v.e.v. flows are known in four dimensional super-conformal field theories, in particular in $N = 2$ SCFT, where they have been studied using Seiberg-Witten solution in connection with the Argyres-Douglas fixed points[11, 12, 13]. To the best of our knowledge, these are the first examples of v.e.v. flows between two AdS vacua in a gauged supergravity context.

From the higher dimensional perspective, the case with $N = 8$ supersymmetry (or better its maximally symmetric vacuum) is related to the brane configuration in type IIB theory whose near horizon geometry is $AdS_3 \times S^3 \times S^3 \times S^1$ [14], dual to a CFT_2 with “large” (4,4) superconformal algebra[15, 16]. For the $N = 4$ case, which has a (4,0) vacuum, the ten dimensional interpretation is far less clear. It could be related to some warped or orbifolded versions of the previous case. It would be interesting to establish this.

The paper is organized as follows. In section 2, we review the features of three dimensional gauged supergravity in the case where the target manifold is a symmetric space. In section 3, we specialize at the $N = 4$ theory and describe the vacua we found. In section 4, we discuss the analytic flow solutions between (3,1) vacua, and numerical flow solution between (2,0) vacua. In section 5, we move to the $N = 8$ case and describe the vacua we obtained. The algebraic manipulations and the numerical solution of the BPS differential equations have been performed with the help of *Mathematica*. In section 6, we make some conclusions.

2. Three Dimensional Gauged Supergravity

In this section, we review the basic features of 3 dimensional, N-extended, gauged supergravity, following the N-covariant formulation given in reference [4]. We will restrict our discussion to the case where the scalar manifold is a symmetric space G/H , although for $N < 5$ there are more general possibilities. Before gauging, the propagating bosonic sector of the theory is described by a non-linear sigma model whose target manifold is G/H , where H is a maximal compact subgroup of G . Thus there are scalar fields $\phi^i(x)$, $i = 1, \dots, \dim G/H$, which are coordinates of G/H . The subgroup H of G contains the R-symmetry group $SO(N)$. Gauging proceeds by introducing Chern-Simons gauge fields A_μ^M in the adjoint representation of a subgroup G' of the isometry group G , whose embedding in G is specified by a gauge invariant, symmetric embed-

ding tensor $\Theta_{\mathcal{MN}}$, with indices running over the Lie algebra of the gauged subgroup. Supersymmetry severely restricts the allowed gauged subgroup, and correspondingly the tensor Θ , as we will see in the following. For the reasons explained in the introduction, we will be interested in non-semisimple gaugings, where the gauged subgroup is a semi-direct product of a semisimple factor G_0 and an abelian factor $T = \mathbf{R}^{\dim G_0}$, $G' = G_0 \ltimes T$, with the latter transforming in the adjoint representation of G_0 .

Let us now introduce the basic data which allow us to construct the gauged supergravity theory in the symmetric space case: recall that by G/H we mean the manifold of right cosets, where H elements $h(x)$ act by right multiplication on the G -valued matrix $L(\phi^i(x))$. The generators of G decompose into $\{t^{\mathcal{M}}\} = \{X^{IJ}, X^\alpha, Y^A\}$. X^{IJ} generate $SO(N)$, and X^α generate a group H' commuting with $SO(N)$. Y^A are the non-compact generators of G . The isometry group is defined by the left action of G elements on the coset G/H . The geometry of G/H is encoded in the Lie algebra valued one-forms $L^{-1}\partial_i L$ and in $L^{-1}t^{\mathcal{M}}L$, through the following expansions over Lie algebra generators:

$$\begin{aligned} L^{-1}\partial_i L &= \frac{1}{2}Q_i^{IJ}X^{IJ} + Q_i^\alpha X^\alpha + e_i^A Y^A, \\ L^{-1}t^{\mathcal{M}}L &= \frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}X^{IJ} + \mathcal{V}_\alpha^{\mathcal{M}}X^\alpha + \mathcal{V}_A^{\mathcal{M}}Y^A. \end{aligned} \quad (2.1)$$

The e_i^A are vielbeins which determine the invariant metric $g_{ij} = e_i^A e_j^B \delta_{AB}$ of G/H . The Q 's are composite H -connections, and the \mathcal{V} 's give the Killing vectors, $\mathcal{V}^{\mathcal{M}i} = g^{ij}e_j^A \mathcal{V}_A^{\mathcal{M}}$. Pulling back on space-time and covariantizing with respect to the gauge action of G' from the right, we define:

$$L^{-1}D_\mu L = \frac{1}{2}Q_\mu^{IJ}X^{IJ} + Q_\mu^\alpha X^\alpha + e_\mu^A Y^A. \quad (2.2)$$

Here $D_\mu L = (\partial_\mu + \Theta_{\mathcal{MN}}A_\mu^{\mathcal{M}}t^{\mathcal{N}})L$ is a space-time covariant derivative and it is understood that the gauge coupling constant is contained in Θ . Thus the full gauge symmetry of the theory is $L(x) \rightarrow g'(x)L(x)h(x)$, where $g' \in G'$. The e_μ^A 's give the covariant kinetic term for scalars,

$$\mathcal{L}_{\text{kin}} = \frac{1}{4}\sqrt{g}g^{\mu\nu}e_\mu^A e_\nu^B \delta_{AB}. \quad (2.3)$$

The Lagrangian for gauge fields is of Chern-Simons type:

$$\mathcal{L}_{CS} = \frac{1}{4}\epsilon^{\mu\nu\rho}A_\mu^{\mathcal{M}}\Theta_{\mathcal{MN}}(\partial_\nu A_\rho^{\mathcal{N}} + \frac{1}{3}f_{\mathcal{Q}}^{\mathcal{NP}}\Theta_{\mathcal{PL}}A_\nu^{\mathcal{L}}A_\rho^{\mathcal{Q}}), \quad (2.4)$$

where $f_{\mathcal{Q}}^{\mathcal{NP}}$ are the structure constants of G' .

As it has been shown in [4] and in more detail in [8], in the non-semisimple case

where $G' = G_0 \ltimes T$, the Chern-Simons action for G' gauge fields is equivalent to a Yang-Mills plus Chern-Simons action for gauge fields transforming under the semisimple part G_0 . The point is that gauge invariance implies that the indices of $\Theta_{\mathcal{MN}}$ cannot be both along the G_0 direction and this allows to integrate the gauge fields carrying G_0 indices, producing a Yang-Mills action for gauge fields carrying T indices, which transform in the usual way under G_0 . At the same time, one can use the shift gauge symmetry to remove $\dim G_0$ scalars from the action.

A class of tensors that will play important role in our analysis are the two-form $SO(N)$ generators, f_{ij}^{IJ} , which originate from the existence of $N - 1$ hermitean almost complex structures f_j^{Pi} , $P = 1, \dots, N - 1$, on the scalar manifold. The existence of the latter is implied by the existence of N supersymmetries. They are vector valued one-forms obeying a Clifford algebra relation and therefore are essentially γ -matrices of $SO(N)$. With their commutators one constructs $SO(N)$ generators f_{ij}^{IJ} , which in our case can be expressed as:

$$f_{ij}^{IJ} = -\Gamma_{AB}^{IJ} e_i^A e_j^B, \quad (2.5)$$

with Γ_{AB}^{IJ} properly normalized generators in the spinor representation of $SO(N)$. Let us now specialize at the $N = 4$ and $N = 8$ cases. In the latter case, one proves that the allowed symmetric spaces are of the form $\frac{SO(8,k)}{SO(8) \times SO(k)}$, and in fact we will restrict our analysis to $k = 8$. For $N = 4$, the scalar manifold can actually be locally the product of two quaternionic manifolds, and even restricting to the symmetric space cases, this allows a finite number of different possibilities, but we will restrict the analysis to the quaternionic symmetric space $\frac{SO(4,4)}{SO(4) \times SO(4)}$.

With the data introduced above, namely the embedding tensor Θ and the \mathcal{V} 's, we define the T-tensors:

$$\begin{aligned} T^{IJ,KL} &\equiv \mathcal{V}^{MIJ} \Theta_{\mathcal{MN}} \mathcal{V}^{NKL}, & T^{IJi} &\equiv \mathcal{V}^{MIJ} \Theta_{\mathcal{MN}} \mathcal{V}^{Ni}, \\ T^{ij} &\equiv \mathcal{V}^{Mi} \Theta_{\mathcal{MN}} \mathcal{V}^{Nj}, & T^i_\alpha &\equiv \mathcal{V}^\mathcal{M}_\alpha \Theta_{\mathcal{MN}} \mathcal{V}^{Ni}, \\ T_{\alpha\beta} &\equiv \mathcal{V}^\mathcal{M}_\alpha \Theta_{\mathcal{MN}} \mathcal{V}^\mathcal{M}_\beta, & T^{IJ}_\alpha &\equiv \mathcal{V}^{MIJ} \Theta_{\mathcal{MN}} \mathcal{V}^\mathcal{N}_\alpha. \end{aligned} \quad (2.6)$$

The fundamental consistency constraint on the gauging, implied by supersymmetry, can be expressed through the following identity:

$$T^{IJ,KL} = T^{[IJ,KL]} - \frac{4}{N-2} \delta^{I[K} T^{L]M,MJ} - \frac{2}{(N-1)(N-2)} \delta^{I[K} \delta^{L]J} T^{MN,MN}, \quad (2.7)$$

or equivalently,

$$\mathbb{P}_\boxplus T^{IJ,KL} = 0 \quad (2.8)$$

which means that the representation \boxplus of $SO(N)$ is projected out. The scalar potential of the theory can be expressed in terms of the tensors:

$$\begin{aligned} A_1^{IJ} &= -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \\ A_{2j}^{IJ} &= \frac{2}{N}T^{IJ}_j + \frac{4}{N(N-2)}f_j^{M(I}T^{J)M}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f_j^{KL}T^{KL}_m. \end{aligned} \quad (2.9)$$

Supersymmetry implies a quadratic identity involving A_1 and A_2 :

$$2A_1^{IK}A_1^{KJ} - NA_2^{IKi}A_{2i}^{JK} = \frac{\delta^{IJ}}{N}(2A_1^{KL}A_1^{KL} - NA_2^{KLi}A_{2i}^{KL}), \quad (2.10)$$

which offers a non-trivial check on the consistency of the construction. The scalar potential is given by:

$$V = -\frac{4}{N}\sqrt{g}(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}). \quad (2.11)$$

Since Θ 's are linear in the gauge couplings, V depends quadratically on them. The other piece of information we will need is given by the supersymmetry variations of the matter fermions χ and the gravitinos ψ_μ^I . For the former, in order to use the $SO(N)$ covariant notations, we extend the fermion fields χ^i to an overcomplete set χ^{iI} defined by, [4],

$$\chi^{iI} = (\chi^i, f_j^{Pi}\chi^j). \quad (2.12)$$

The Lagrangian and supersymmetry transformation rules can be expressed in a form that no longer depends explicitly on the almost complex structures. The fields χ^{iI} have to satisfy the projection constraint

$$\chi^{iI} = \mathbb{P}_{Jj}^{Ii}\chi^{jJ} \equiv \frac{1}{N}(\delta^{IJ}\delta_j^i - f_j^{Ij})\chi^{jJ}. \quad (2.13)$$

Omitting terms which are of higher order in the fermionic fields, the supersymmetry transformations which are relevant for us are given by:

$$\begin{aligned} \delta\psi_\mu^I &= \mathcal{D}_\mu\epsilon^I + A_1^{IJ}\gamma_\mu\epsilon^J, \\ \delta\chi^{iI} &= \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i_j\mathcal{D}\phi^j\epsilon^J - NA_2^{JIi}\epsilon^J, \end{aligned} \quad (2.14)$$

where

$$\begin{aligned} \mathcal{D}_\mu\epsilon^I &= (\partial_\mu + \frac{1}{2}\omega_\mu^a\gamma_a)\epsilon^I + \partial_\mu\phi^iQ_i^{IJ}\epsilon^J + \Theta_{MN}A_\mu^M\mathcal{V}^{NIJ}\epsilon^J, \\ \mathcal{D}_\mu\phi^i &= \partial_\mu\phi^i + A_\mu^M\mathcal{V}^{Ni}\Theta_{MN}, \end{aligned} \quad (2.15)$$

and ω_μ^a is the 3-dimensional spin connection constructed with the dreibein e_μ^a . As shown in [4], assuming a maximally symmetric space-time (in particular AdS_3), the supersymmetric critical points of the potential are given by the two equivalent conditions on spinors ϵ^I :

$$A_{2i}^{JI} \epsilon^J = 0$$

and

$$A_1^{IK} A_1^{KJ} \epsilon^J = -\frac{V_0}{4} \epsilon^I = \frac{1}{N} (A_1^{IJ} A_1^{IJ} - \frac{1}{2} N g^{ij} A_{2i}^{IJ} A_{2i}^{IJ}) \epsilon^I, \quad (2.16)$$

where V_0 is the potential at the critical point. The equivalence of the two statements follows from the quadratic identity (2.10) involving A_1 and A_2 . This result says that the preserved supersymmetries correspond to the eigenvalues of A_1^{IJ} which equal $\pm \sqrt{\frac{-V_0}{4}}$, since in our normalization $-V_0 = R^{-2}$, where R is the radius of AdS_3 . More in detail, let us choose AdS_3 coordinates r, x_0, x_1 , and metric $ds^2 = dr^2 + e^{2r/R} (-dx_0^2 + dx_1^2)$. From the previous remarks, it follows that for each eigenvector v_\pm^I of A_1^{IJ} , with eigenvalue $\pm \sqrt{\frac{-V_0}{4}}$, if we form the spinor $\epsilon_\pm^I = \epsilon_\pm \otimes v_\pm^I$, then the BPS condition for the gravitino variation (2.14) becomes identical to the Killing spinor equation for ϵ_\pm on AdS_3 i.e. $\mathcal{D}_\mu \epsilon_\pm = \pm \frac{1}{2R} \gamma_\mu \epsilon_\pm$. Using the explicit expression for the spin connection for the above metric, one can see that one solution to this equation is an x_0, x_1 -independent spinor obeying $\gamma^r \epsilon_\pm = \pm \epsilon_\pm$, where γ^r is the flat gamma matrix. This corresponds to a left (right) Poincare' supersymmetry in the boundary CFT. The other solution gives rise to the superconformal charge in the boundary CFT, has a non-trivial x_0, x_1 dependence and is constructed with a constant spinor obeying the opposite γ^r projection condition.

Therefore, it is convenient to classify the critical points by presenting their preserved supersymmetries in the form of (N_+, N_-) corresponding to the N_+ and N_- positive and negative eigenvalues of A_1^{IJ} whose modulus equals $\sqrt{\frac{-V_0}{4}}$. These coincide with the number of left-(right-) moving Poincare' supersymmetries of the dual $SCFT_2$. Of course the total number of supersymmetries is doubled by the inclusion of the superconformal ones.

To summarize, the procedure of finding supersymmetric vacua is the following. From (2.16), we look for the Killing spinors ϵ^I which are annihilated by some of the A_{2i}^{JI} . At the same time, ϵ^I must also be the eigenvector of A_1^{IJ} . Clearly, maximal supersymmetric vacua are annihilated by all of the components of A_{2i}^{JI} , and ϵ^I is an eigenvector of A_1^{IJ} for all directions I . The ϵ^I characterizing partially supersymmetric vacua will be an eigenvector of A_1^{IJ} for certain directions labeled by some values of I , and will be annihilated only by the A_{2i}^{JI} in the corresponding directions. We also find many supersymmetric vacua with $V_0 = 0$, and there might be non-supersymmetric AdS_3 vacua as well. However, in this work, we will not discuss them.

3. Vacua of the $N = 4$ Theory

The target space in our case is the product of two quaternionic manifolds, that we take to be $SO(4, 4)/SO(4) \times SO(4)$. A convenient (redundant) parametrization of cosets is given by the following $SO(4, 4)$ group element

$$L_i = \frac{1}{2} \begin{pmatrix} X_i + e_i^t & Y_i + e_i^t \\ -X_i - e_i^t & e_i^t - Y_i \end{pmatrix}, \quad (3.1)$$

where $i = 1, 2$ refers to the two spaces. e_i is a 4×4 matrix in $GL(4, \mathbf{R})$, $X_i = E_i + B_i e_i^t$, $Y_i = -E_i + B_i e_i^t$. B_i is an antisymmetric 4×4 matrix, and $E_i = e_i^{-1}$. The inverse of L_i is

$$L_i^{-1} = \frac{1}{2} \begin{pmatrix} X_i^t + e_i & X_i^t - e_i \\ -Y_i^t - e_i & e_i - Y_i^t \end{pmatrix}. \quad (3.2)$$

One can eliminate 6 of the 22 parameters in L by using the right action of the diagonal $SO(4)$ action, for example by bringing e_i into an upper triangular form. The following Lie algebra elements,

$$t^A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \quad t^B = \begin{pmatrix} b & b \\ -b & -b \end{pmatrix} \quad (3.3)$$

where all entries are 4×4 antisymmetric blocks, together with an identical copy for the second space, will be gauged. In other words, the semisimple part of the gauge group will be the diagonal $SO(4)_D$ in the $(SO(4))^4$ of the product $(\frac{SO(4,4)}{SO(4) \times SO(4)})^2$, corresponding to generators t^A . On the other hand, the nilpotent generators, t^B , generate diagonal shift symmetries $B_{1,2} \rightarrow B_{1,2} + 2b$. Also, it is clear that the \mathcal{B} -generators transform in the adjoint representation with respect to the diagonal $SO(4)$. For a and b , we can take a basis of antisymmetric matrices given by $J^{IJ} = \varepsilon^{IJ} - \varepsilon^{JI}$, with $(\varepsilon^{IJ})_{KL} = \delta_{IK}\delta_{JL}$. Similarly, we can use the following basis for the 16 non-compact generators of $SO(4, 4)$:

$$Y^{ab} = \begin{pmatrix} 0 & \varepsilon^{ab} \\ (\varepsilon^t)^{ab} & 0 \end{pmatrix}. \quad (3.4)$$

Since in the present case both the R symmetry group and the gauge group are $SO(4)$, it is convenient to split the corresponding Lie algebras generators into self-dual and anti-self-dual components J_+ and J_- respectively:

$$J_+^{IJ} = J^{IJ} + \frac{1}{2}\epsilon^{IJKL}J^{KL} \quad \text{and} \quad J_-^{IJ} = J^{IJ} - \frac{1}{2}\epsilon^{IJKL}J^{KL} \quad (3.5)$$

which are $SU(2)_+$ and $SU(2)_-$ generators in the $SO(4) = SU(2)_+ \oplus SU(2)_-$ Lie algebra decomposition. We will adopt this decomposition both for \mathcal{A} - and \mathcal{B} -type generators.

Correspondingly, the two-forms tensors f^{IJ} introduced in the previous section have, say, self-dual components on the first quaternionic space and anti-self-dual components on the second. In our formalism and in a flat basis, they can be expressed as:

$$f_{\pm}^{IJ}{}_{ab,cd} = \text{Tr}((\varepsilon^t)^{ab} J_{\pm}^{IJ} \varepsilon^{cd}). \quad (3.6)$$

At this stage, we can proceed to construct the supergravity theory with the gauging of $SO(4) \ltimes \mathbf{R}^6$ and in particular, verify its consistency, along the lines reviewed in the previous section. As explained there, the main ingredients are given by the tensors A_1 and A_2 , which determine the scalar potential and the supersymmetry variations of the fermionic fields. They are constructed through the T -tensors, which in turn are obtained by uplifting the embedding tensor $\Theta_{\mathcal{MN}}$ into G by using $\mathcal{V}_{\mathcal{P}}^{\mathcal{M}}$, with \mathcal{P} running over the generators of G corresponding to the R-symmetries $\mathcal{P} = IJ$, and the non-compact coset directions $\mathcal{P} = ab$ in the first and second space. We give in the Appendix A expressions for the relevant components of \mathcal{V} .

Gauge invariance restricts the Θ tensors to have components, $\Theta_{\mathcal{AB}}$ and $\Theta_{\mathcal{BB}}$, which are proportional to the $SO(4)$ Killing form, schematically $\delta_{\mathcal{AB}}$ and $\delta_{\mathcal{BB}}$, respectively. The proportionality constants are gauge couplings, and, of course, we should specify here to which of the four $SU(2)$'s the \mathcal{A} , \mathcal{B} indices belong. Therefore, a priori we expect four couplings g_{1s} , g_{1a} , g_{2s} , and g_{2a} . The a and s labels indicate the self-dual and anti-self-dual $SU(2)$, respectively, and 1 refers to the \mathcal{AB} couplings whereas 2 refers to the \mathcal{BB} ones.

With this notation and with the meaning of \mathcal{V} indices explained in the Appendix A, the T -tensors turn out to be:

$$\begin{aligned} T^{LJ,MK} &= g_{1s}(\mathcal{V}_{+a}^{LJ,PQ} \mathcal{V}_{+b}^{MK,PQ} + \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+a}^{MK,PQ}) + g_{1a}(\mathcal{V}_{-a}^{LJ,PQ} \mathcal{V}_{-b}^{MK,PQ} \\ &\quad + \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-a}^{MK,PQ}) + g_{2s} \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+b}^{MK,PQ} + g_{2a} \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-b}^{MK,PQ}, \\ T_{1ab}^{LJ} &= g_{1s}(\mathcal{V}_{+a}^{LJ,PQ} \mathcal{V}_{+1b}^{PQ} + \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+1a}^{PQ}) + g_{1a}(\mathcal{V}_{-a}^{LJ,PQ} \mathcal{V}_{-1b}^{PQ} \\ &\quad + \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-1a}^{PQ}) + g_{2s} \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+1b}^{PQ} + g_{2a} \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-1b}^{PQ}, \\ T_{2ab}^{LJ} &= g_{1s}(\mathcal{V}_{+a}^{LJ,PQ} \mathcal{V}_{+2b}^{PQ} + \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+2a}^{PQ}) + g_{1a}(\mathcal{V}_{-a}^{LJ,PQ} \mathcal{V}_{-2b}^{PQ} \\ &\quad + \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-2a}^{PQ}) + g_{2s} \mathcal{V}_{+b}^{LJ,PQ} \mathcal{V}_{+2b}^{PQ} + g_{2a} \mathcal{V}_{-b}^{LJ,PQ} \mathcal{V}_{-2b}^{PQ}. \end{aligned} \quad (3.7)$$

It turns out that the consistency requirement on $T^{IJ,KL}$, discussed in the previous section, requires $g_{2a} = -g_{2s}$. Moreover, we find it is convenient for the subsequent analysis to redefine the couplings from g_{1s} , g_{1a} to g_n , g_p as follows:

$$g_{1s} = g_p + g_n \quad \text{and} \quad g_{1a} = g_p - g_n. \quad (3.8)$$

Now, we study various vacua of this theory. We begin by choosing an ansatz for the coset L . We have two spaces. We set $B_1 = B_2 = 0$ and choose diagonal e_i 's:

$$e_1 = \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{pmatrix} \quad \text{and} \quad e_2 = \begin{pmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{pmatrix}. \quad (3.9)$$

Notice that the shift gauge symmetry would allow us to set one of the two B 's to zero and the left $SO(4)$ gauge symmetry can be used to diagonalize one of the two e 's, so the ansatz above is indeed a truncation of the full twenty-dimensional moduli space. We have checked the consistency of this truncation explicitly. That is, we have verified that the remaining fields appear at least quadratically in the action, and therefore setting them to zero solves their equations of motion. We then proceed to analyze the BPS conditions $\delta\psi_\mu^I = 0$ and $\delta\chi^{iI} = 0$ using (2.14), within this eight-dimensional subspace.

We give below the vacuum expectation values of e_1, e_2 , the A_1^{IJ} eigenvalue (A_1) satisfying $|A_1|^2 = -V_0/4$ and the corresponding preserved supersymmetries (N_+, N_-) for the AdS_3 vacuum solutions that are relevant to the flow solutions we will show in the next section. Other vacua are shown in Appendix B.

3.1 (3,1) vacua

- I.

$$\begin{aligned} e_1 &= \sqrt{\frac{-2(g_n + g_p)}{g_{2s}}} \mathbb{I}_{4 \times 4} \\ e_2 &= \sqrt{\frac{-2(g_n + g_p)}{g_{2s}}} (-1, 1, 1, 1) \\ A_1 &= \frac{32(g_n + g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = \frac{-4096(g_n + g_p)^4}{g_{2s}^2}. \end{aligned} \quad (3.10)$$

- II.

$$\begin{aligned} e_1 &= \sqrt{\frac{2(g_p - g_n)}{g_{2s}}} (1, -1, -1, -1) \\ e_2 &= -\sqrt{\frac{2(g_p - g_n)}{g_{2s}}} \mathbb{I}_{4 \times 4} \\ A_1 &= \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = \frac{-4096(g_n - g_p)^4}{g_{2s}^2}. \end{aligned} \quad (3.11)$$

• III.

$$\begin{aligned}
e_1 &= \sqrt{\frac{g_n(g_p^2 - g_n^2)}{g_{2s}g_n^2}} \left(\frac{g_n}{g_p}, -1, -1, -1 \right) \\
e_2 &= -\sqrt{\frac{g_n(g_p^2 - g_n^2)}{g_{2s}g_n^2}} \left(\frac{g_n}{g_p}, 1, 1, 1 \right) \\
A_1 &= \frac{-8(g_n^2 - g_p^2)^2}{g_{2s}g_n g_p} \quad \text{and} \quad V_0 = \frac{-256(g_n^2 - g_p^2)^4}{g_{2s}^2 g_n^2 g_p^2}.
\end{aligned} \tag{3.12}$$

3.2 (2,0) vacua

• IV.

$$e_1 = (-a_1, a_1, a_2, a_2) \quad e_2 = (b_1, b_1, b_2, b_2) \tag{3.13}$$

$$\begin{aligned}
a_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_p - g_n + \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\
a_2 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_n - g_p + \sqrt{5g_p^2 + 2g_p g_n + g_n^2})}} \\
b_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(3g_n + g_p + \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\
b_2 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(\sqrt{g_n^2 + 2g_n g_p + 5g_p^2} + g_n + 3g_p)}}
\end{aligned} \tag{3.14}$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = -\frac{4096(g_n - g_p)^4}{g_{2s}^2}. \tag{3.15}$$

• V.

$$e_1 = (a_1, a_2, a_3, a_3) \quad e_2 = (b_1, b_1, b_2, b_2) \tag{3.16}$$

$$\begin{aligned}
a_1 &= -\frac{1+t}{1-t+\sqrt{1+t^2}} \sqrt{\frac{2g_p(1-t+\sqrt{1+t^2})}{g_{2s}t(1+t)\sqrt{1+t^2}}} \times \\
&\quad \sqrt{(t-1) \left\{ t^3 - t^2 + t - 1 + (t-t^2-1)\sqrt{1+t^2} \right\}} \\
a_2 &= \sqrt{\frac{2tg_p(t-1)^2(1+t)\sqrt{1+t^2}}{g_{2s}(1-t+\sqrt{1+t^2})}} \times \\
&\quad \frac{1}{\sqrt{(t-1-t^2)(t-1)\sqrt{1+t^2}-t^2+(1-t+t^2)^2}} \\
a_3 &= \sqrt{\frac{2g_p(1-t^2)}{g_{2s}(t-1+\sqrt{1+t^2})}} \\
b_1 &= \sqrt{\frac{2g_p(1-t^2)}{g_{2s}(1+t+\sqrt{1+t^2})}} \\
b_2 &= \sqrt{\frac{2g_p(1-t^2)}{g_{2s}(1+t+\sqrt{1+t^2})}} \\
A_1 &= \frac{-8(g_n^2 - g_p^2)^2}{g_{2s}g_n g_p} \quad \text{and} \quad V_0 = -\frac{256(g_n^2 - g_p^2)^4}{(g_{2s}g_n g_p)^2}, \tag{3.17}
\end{aligned}$$

where we have introduced $t = \frac{g_n}{g_p}$.

Out of all vacua, there are only three possibilities in connecting two vacua. That means we will have only three RG flows in the dual field theories. All these three flows are the flows between I and III, II and III, and between IV and V. The last flow is the only possible flow among V and other (2,0) points. This is because we cannot find any values of g_n , g_p and g_{2s} so that both e_1 and e_2 of the two end points of the flow are real apart from the IV and V pair. There are three possibilities in order to make IV and V real at the same time. These are given by

$$\begin{aligned}
&t < -1, g_p < 0, g_{2s} < 0 \\
\text{or} \quad &t < 1, g_p > 0, g_{2s} > 0 \\
\text{or} \quad &t > 1, g_p > 0, g_{2s} < 0. \tag{3.18}
\end{aligned}$$

For definiteness, we choose the last range and further choose $t = 2$, $g_p = 1$ and $g_{2s} = -1$ in our numerical solution. For all the critical points given above, we have checked that there exist at least one possible set of g_p , g_n and g_{2s} such that all the square roots in any critical points are real, although any two different critical points may not be made real with the same values of g_p , g_n and g_{2s} .

There might be more possibilities apart from these three flows. However, we could not find any interpolating solutions both analytically and numerically apart from those three mentioned above. Remarkably, we find only the flows between critical points which have the same supersymmetries. In the next section, we will give these solutions explicitly.

4. Supersymmetric Flow Solutions

In this section, we study flows between some pairs of AdS vacua found in the previous section. We assume the standard form for the 3D metric:

$$ds^2 = e^{2A(r)}(-dt^2 + dx^2) + dr^2. \quad (4.1)$$

This becomes the AdS_3 metric for $A(r) = r/R$, where R is the AdS_3 radius. This is related to the vacuum energy V_0 as $R^2 = -1/V_0$, since in our normalization Einstein's equations read $R_{\mu\nu} = -2V_0 g_{\mu\nu}$. Also, we recall that the eigenvalue A_1 introduced in the previous section satisfies $4A_1^2 = -V_0$. We will look for solutions of the BPS equations interpolating between AdS vacua from the UV region ($r \rightarrow +\infty$) to the IR region ($r \rightarrow -\infty$), where the scalar fields reach the vev's determined in the previous section. The central charge of the CFT's at an AdS_3 vacuum is proportional to R , and therefore proportional to $1/A'(r)$. In fact, the latter quantity can be used to define, up to a positive proportionality constant, a C -function, $C(r)$, on the full flow interpolating between the UV and IR fixed points and can be proved to be monotonic, $A''(r) \leq 0$ [1]. This nicely agrees with the c-theorem in conformal field theories. The result in [1] depends on the validity of the weaker energy condition, which is met in all the flows involving only scalars and the metric. This is the case for our flows as we will see below. Notice that, since $A(r)$ is related to A_1 through a first order differential equation given by the gravitino variation (2.14), this also implies that A_1 should not change sign along the flow because this would imply an unphysical infinity for $C(r)$ at some value of r . Examples of RG flows in 3D gauged supergravity have been studied in [17, 18].

4.1 The Flow Between $(3, 1)$ Vacua

In this subsection, we study a supersymmetric flow between two of the AdS_3 vacua with the same, $(3, 1)$, amount of supersymmetries but with different cosmological constants, found in the previous section.

We start by giving an ansatz for the scalars with non-trivial r -dependence,

$$e_1 = \begin{pmatrix} b(r) & 0 & 0 & 0 \\ 0 & a(r) & 0 & 0 \\ 0 & 0 & a(r) & 0 \\ 0 & 0 & 0 & a(r) \end{pmatrix}, \quad e_2 = \begin{pmatrix} -b(r) & 0 & 0 & 0 \\ 0 & a(r) & 0 & 0 \\ 0 & 0 & a(r) & 0 \\ 0 & 0 & 0 & a(r) \end{pmatrix}. \quad (4.2)$$

Since now we are going to allow the scalars to have r dependence, we need to worry about possible contributions of the intrinsic connection Q_μ^{IJ} and the gauge fields $A_\mu^{\mathcal{M}}$ to the BPS equations (2.14). In addition, of course, the Yang-Mills equations of motion may be non-trivial. Indeed, r -dependent scalars may a priori source the gauge fields in case they give rise to a non-trivial gauge current $J_\mu^{\mathcal{M}}$. From the kinetic term (2.3), we have

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & \frac{1}{2} \sqrt{g} [\text{Tr}(L^{-1} \partial_\mu L L^{-1} \partial^\mu L) + 2 \Theta_{\mathcal{MN}} A^{\mathcal{M}\mu} \text{Tr}(L^{-1} t^\mathcal{N} \partial_\mu L) \\ & + \Theta_{\mathcal{MN}} \Theta_{\mathcal{KL}} A^{\mathcal{M}\mu} A_\mu^\mathcal{K} \text{Tr}(L^{-1} t^\mathcal{N} t^\mathcal{L} L)]. \end{aligned} \quad (4.3)$$

From (4.3), we see that the gauge fields couple to the scalar fields via a current

$$J_\mu^\mathcal{N} = \sqrt{g} \text{Tr}(L^{-1} t^\mathcal{N} \partial_\mu L). \quad (4.4)$$

For diagonal e_1 and e_2 , the current is zero, so we can consistently satisfy the equation of motion for the gauge fields by setting $A_\mu^{\mathcal{M}} = 0$. As promised, our flows involve only scalars and the metric. So, the holographically proved c-theorem mentioned before is guaranteed in our flow ansatz. Furthermore, all of the composite connections Q 's are also zero in this diagonal ansatz. The BPS equations can be obtained by using (2.14). The $\delta\chi^{iI} = 0$ conditions give

$$\frac{db}{dr} = 24g_n a b^2 + 16g_p b^3 - 8a^3(g_n - g_{2s} b^2) \quad (4.5)$$

$$\frac{da}{dr} = 16g_p a^3 + 8g_n a^2 b + \frac{8a^4(g_n + g_{2s} b^2)}{b}. \quad (4.6)$$

This ansatz preserves (3,1) supersymmetry, so we have (3,1) supersymmetry throughout the flow. We proceed by taking one of the scalars as an independent variable. Changing the variables to $b(r) = z$ and $a(r) = a(z)$, we can write (4.5) and (4.6) as a single equation

$$\frac{da}{dz} = \frac{a^2(g_n z^2 + 2g_p z a + (g_n + g_{2s} z^2) a^2)}{2g_p z^4 + 3g_n z^3 a + (g_{2s} z^3 - g_n z) a^3}. \quad (4.7)$$

We solve this by writing $a(z) = z f(z)$. Then, (4.7) becomes

$$z \frac{df}{dz} = - \frac{2f(g_p + g_n f)(f^2 - 1)}{(g_n - g_{2s} z^2) f^3 - 2g_p f}. \quad (4.8)$$

This equation can be solved for z as a function of f . We find

$$z = \pm \sqrt{\frac{g_n(f^2 - 1)}{g_{2s}f^2 + (g_n^2f^3 + g_ng_pf^2)c_1}}. \quad (4.9)$$

We then obtain

$$b = \pm \sqrt{\frac{g_n(f^2 - 1)}{g_{2s}f^2 + (g_n^2f^3 + g_ng_pf^2)c_1}}, \quad (4.10)$$

$$a = fb, \quad (4.11)$$

and (4.5) and (4.6) lead to the same equation for f

$$\frac{df}{dr} = \frac{16g_n(g_p + g_nf)(f^2 - 1)^2}{f(g_{2s} + (g_ng_p + g_n^2f)c_1)}. \quad (4.12)$$

We can solve for r in term of f and find

$$r = c_2 + \frac{1}{64g_n} \left[\frac{2(-fg_{2s}g_n + g_{2s}g_p + g_n(g_p^2 - g_n^2)c_1)}{(f^2 - 1)(g_n^2 - g_p^2)} - \frac{g_{2s}g_n \ln(1 - f)}{(g_n + g_p)^2} \right. \\ \left. + \frac{g_{2s}g_n \ln(1 + f)}{(g_n - g_p)^2} - \frac{4g_{2s}g_n^2g_p \ln(fg_n + g_p)}{(g_n^2 - g_p^2)^2} \right]. \quad (4.13)$$

The constant c_2 is irrelevant and can be set to zero by shifting the coordinate r . So, from now on, we will use $c_2 = 0$ and choose a definite sign, + sign, for z .

We now move to the gravitino variation $\delta\psi_\mu^I$. The BPS condition gives an equation for the warp factor $A(r)$:

$$\frac{dA}{dr} = -\frac{1}{f^2(g_{2s} + (g_ng_p - g_n^2f)c_1)^2} [8g_n(f^2 - 1)(3f^2(c_1g_n(g_n^2 + g_p^2) + g_{2s}g_p) \\ - 2g_nf^3(2c_1g_ng_p + g_{2s}) - 2g_nf(2c_1g_ng_p + g_{2s}) + c_1g_n^3f^4 \\ + g_p(c_1g_ng_p + g_{2s}))]. \quad (4.14)$$

Changing the variable from r to f , we find

$$\frac{dA}{df} = \frac{1}{2(fg_n + g_p)} \left[\frac{g_p + f(3fg_p + g_n(3 + f^2))}{f(f^2 - 1)} - \frac{g_{2s}g_n}{g_{2s} + g_n(fg_n + g_p)c_1} \right]. \quad (4.15)$$

This can be solved and give

$$A = c_3 + \frac{1}{2} \ln f - \ln(1 - f^2) + \frac{1}{2} \ln(g_p + fg_n) + \frac{1}{2} \ln(g_{2s} + g_n(g_p + g_nf)c_1). \quad (4.16)$$

The constant c_3 can be set to zero by rescaling coordinates x^0 and x^1 . We require that A_1 must not change sign along the flow, so these are the only two possible flows namely the flow between I and III critical points and between II and III points. We choose the value of c_1 in such a way that the solution goes to one critical point at one end and to the other critical point at the other end. In order to identify the UV point with $r = \infty$ and the IR point with $r = -\infty$, we choose $g_{2s} < 0$ in the followings.

In the flow between I and III critical points, we chose $c_1 = -\frac{g_{2s}}{g_n(g_n+g_p)}$, $g_n g_p < 0$ and obtain

$$\begin{aligned}
b &= \sqrt{-\frac{(g_n + g_p)(1 + f)}{g_{2s}f^2}} \\
a &= \sqrt{-\frac{(g_n + g_p)(1 + f)}{g_{2s}}} \\
r &= \frac{1}{64} \left[-\frac{2g_{2s}}{(1 + f)(g_n^2 - g_p^2)} - \frac{g_{2s} \ln(1 - f)}{(g_n + g_p)^2} \right. \\
&\quad \left. + \frac{g_{2s} \ln(1 + f)}{(g_n - g_p)^2} - \frac{4g_{2s}g_n g_p \ln(fg_n + g_p)}{(g_n^2 - g_p^2)^2} \right] \\
A &= \frac{1}{2} \ln f - \frac{1}{2} \ln(1 - f) - \ln(1 + f) + \frac{1}{2} \ln(g_p + fg_n)
\end{aligned} \tag{4.17}$$

where we have absorbed all the constants in c_3 for the last equation. We see that $A \rightarrow \infty$ at $f = 1$ and $A \rightarrow -\infty$ at $f = -\frac{g_p}{g_n}$. In the dual CFT, the I point corresponds to the UV fixed point while the III point corresponds to the IR point. The ratio of the central charges is given by

$$\frac{c_{UV}}{c_{IR}} = -\frac{(g_n - g_p)^2}{4g_n g_p}. \tag{4.18}$$

It is easy to show that this is always greater than 1 as it should.

The flow between II and III are given by $c_1 = \frac{g_{2s}}{g_n(g_n - g_p)}$, and $g_n g_p > 0$. We find that

$$\begin{aligned}
b &= \sqrt{\frac{(g_n - g_p)(f - 1)}{g_{2s} f^2}} \\
a &= \sqrt{\frac{(g_n - g_p)(f - 1)}{g_{2s}}} \\
r &= \frac{1}{64} \left[\frac{2g_{2s}}{(1 - f)(g_n^2 - g_p^2)} - \frac{g_{2s} \ln(1 - f)}{(g_n + g_p)^2} \right. \\
&\quad \left. + \frac{g_{2s} \ln(1 + f)}{(g_n - g_p)^2} - \frac{4g_{2s} g_n g_p \ln(f g_n + g_p)}{(g_n^2 - g_p^2)^2} \right] \\
A &= \frac{1}{2} \ln f - \ln(1 - f) - \frac{1}{2} \ln(1 + f) + \frac{1}{2} \ln(g_p + f g_n). \tag{4.19}
\end{aligned}$$

In this case, we see that $A \rightarrow \infty$ at $f = -1$ and $A \rightarrow -\infty$ at $f = -\frac{g_p}{g_n}$. In the dual CFT, the II point corresponds to the UV fixed point while the III point corresponds to the IR point. The ratio of the central charges is given by

$$\frac{c_{UV}}{c_{IR}} = \frac{(g_n + g_p)^2}{4g_n g_p}. \tag{4.20}$$

Again, this agrees with the c-theorem.

We next compute the scalar mass spectrum for the eight scalars. We parametrize the eight scalars as follow:

$$\begin{aligned}
a_1(r) &= a_{10} e^{s_1(r)} & a_2(r) &= a_{20} e^{s_2(r)} \\
a_3(r) &= a_{30} e^{s_3(r)} & a_4(r) &= a_{40} e^{s_4(r)} \\
b_1(r) &= a_{50} e^{s_5(r)} & b_2(r) &= a_{60} e^{s_6(r)} \\
b_3(r) &= a_{70} e^{s_7(r)} & b_4(r) &= a_{80} e^{s_8(r)} \tag{4.21}
\end{aligned}$$

where all the s_i , $i = 1, \dots, 8$ are canonically normalized scalars. From the scalar mass matrix M^2 , we can find the conformal dimensions (Δ) of the operators in the dual CFT by using the relation

$$\Delta(\Delta - 2) = m^2 R^2. \tag{4.22}$$

We find the following mass matrices.

- $f = 1$:

$$M^2 = \frac{2048(g_n + g_p)^4}{g_{2s}^2} \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}. \quad (4.23)$$

The eigenvalues of $M^2 R^2$ are $(3, -1, -1, -1, 0, 0, 0, 0)$ corresponding to $\Delta = (3, 1, 2)$. All the eight eigenvectors are given by

$$\begin{aligned} v_1 &= (1, 1, 1, 1, 1, 1, 1, 1) & v_2 &= (-1, 0, 0, 1, -1, 0, 0, 1) \\ v_3 &= (-1, 0, 1, 0, -1, 0, 1, 0) & v_4 &= (-1, 1, 0, 0, -1, 1, 0, 0) \\ v_5 &= (0, 0, 0, -1, 0, 0, 0, 1) & v_6 &= (0, 0, -1, 0, 0, 0, 1, 0) \\ v_7 &= (0, -1, 0, 0, 0, 1, 0, 0) & v_8 &= (-1, 0, 0, 0, 1, 0, 0, 0). \end{aligned} \quad (4.24)$$

Our flow corresponds to the combination $v_2 + v_3 + v_4$ which has eigenvalue -1, $\Delta = 1$. This is consistent with the fact that the flow is driven by a relevant operator.

- $f = -1$:

$$M^2 = \frac{2048(g_n - g_p)^4}{g_{2s}^2} \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}. \quad (4.25)$$

The eigenvalues of $M^2 R^2$ are $(3, -1, -1, -1, 0, 0, 0, 0)$ corresponding to $\Delta = (3, 1, 2)$. All the eight eigenvectors are given by

$$\begin{aligned} u_1 &= (1, 1, 1, 1, 1, 1, 1, 1) & u_2 &= (-1, 0, 0, 1, -1, 0, 0, 1) \\ u_3 &= (-1, 0, 1, 0, -1, 0, 1, 0) & u_4 &= (-1, 1, 0, 0, -1, 1, 0, 0) \\ u_5 &= (0, 0, 0, -1, 0, 0, 0, 1) & u_6 &= (0, 0, -1, 0, 0, 0, 1, 0) \\ u_7 &= (0, -1, 0, 0, 0, 1, 0, 0) & u_8 &= (-1, 0, 0, 0, 1, 0, 0, 0). \end{aligned} \quad (4.26)$$

As in the previous case, the flow ansatz is the combination $u_2 + u_3 + u_4$ which has eigenvalue -1, $\Delta = 1$ and corresponds to a relevant operator.

- $f = -\frac{g_p}{g_n}$:

$$M^2 = \frac{256(g_n^2 - g_p^2)^4}{(g_{2s}g_n g_p)^2} \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & 0 & 0 & -\frac{1}{2} & 1 & 1 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 1 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 1 & 1 & -\frac{1}{2} \\ \frac{3}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 1 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 1 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & 0 & 0 & 0 & \frac{3}{2} \end{pmatrix}. \quad (4.27)$$

The eigenvalues of $M^2 R^2$ are $(3, 3, 3, 3, 0, 0, 0, 0)$ corresponding to $\Delta = (3, 2)$. All the eight eigenvectors are given by

$$\begin{aligned} w_1 &= (0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{3}, 0, 0, 0, 1) & w_2 &= (0, \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0, 1, 0) \\ w_3 &= (0, -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}, 0, 1, 0, 0) & w_4 &= (1, 0, 0, 0, 1, 0, 0, 0) \\ w_5 &= (0, -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, 0, 0, 0, 1) & w_6 &= (0, -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}, 0, 0, 1, 0) \\ w_7 &= (0, \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}, 0, 1, 0, 0) & w_8 &= (-1, 0, 0, 0, 1, 0, 0, 0). \end{aligned} \quad (4.28)$$

Our flow corresponds to the combination $w_1 + w_2 + w_3 - 3w_4$ which has eigenvalue 3, $\Delta = 3$. This is consistent with the fact that at the IR, the operator must be irrelevant.

We also compute the mass spectrum for the full scalar manifold. Using gauge transformation, we are left with twenty scalars. At the UV points $f = \pm 1$, six of the extra twelve scalars have $M^2 R^2 = -\frac{1}{4}$, and the other six are massless. At the IR point $f = -\frac{g_p}{g_n}$, out of the extra twelve scalars, there are six massless scalars and six scalars with $M^2 R^2 = \frac{3}{4}$.

The behavior of the scalars at large r is given by the linearized equations

$$\begin{aligned} \frac{da}{dr} &= \frac{8a_0}{b_0} [2a(r)(2a_0^2(b_0^2 g_{2s} + g_n) + 3a_0 b_0 g_p + b_0^2 g_n) + b(r)(a_0^2(b_0^2 g_{2s} - g_n) + b_0^2 g_n) \\ &\quad + a_0^2 b_0^2 g_{2s} + a_0^2 g_n + 2a_0 b_0 g_p + b_0^2 g_n] \\ \frac{db}{dr} &= \frac{8}{b_0} [3a(r)(a_0^3(b_0^2 g_{2s} - g_n) + a_0 b_0^2 g_n) + 2b_0^2 b(r)(a_0^3 g_{2s} + 3a_0 g_n + 3b_0 g_p) \\ &\quad + a_0^3 b_0^2 g_{2s} - a_0^3 g_n + 3a_0 b_0^2 g_n + 2b_0^3 g_p] \end{aligned} \quad (4.29)$$

where a_0 and b_0 are the values of $a(r)$ and $b(r)$ at the critical point. For the UV ($r \rightarrow \infty$) point, $f = 1$ and $f = -1$, we find

$$a(r) \sim e^{-r/R}, \quad b(r) \sim e^{-r/R}. \quad (4.30)$$

For the IR point ($r \rightarrow -\infty$), we find

$$a(r) \sim e^{r/R}, \quad b(r) \sim e^{r/R}. \quad (4.31)$$

The general behavior of a scalar field near the UV fixed point is given by [19], [20]

$$\phi(x, r) = e^{-(2-\Delta)r} (1 + \dots) \hat{\phi}(x) + e^{-\Delta r} (1 + \dots) \check{\phi}(x), \quad (4.32)$$

where $\hat{\phi}(x)$ and $\check{\phi}(x)$ correspond to the source and the vacuum expectation value of the operator of dimension Δ , respectively [20], [21]. In (4.32), $1 < \Delta \leq 2$. For $\Delta = 1$ or $\Delta = \frac{d}{2}$ in d dimensional field theory, the behavior of the scalar is given by [20]

$$\phi(r, x) = e^{-r/R} \left(\frac{r}{R} \hat{\phi}(x) + \check{\phi}(x) \right) + \dots \quad (4.33)$$

We see that in our flow, the first term in (4.33) is absent, so there is no source. The flow is therefore of the so-called v.e.v. type, corresponding to the deformation of the UV theory by an expectation value of an operator of dimension one. Near the IR point, the scalar behaves as $e^{(\Delta-2)r/R}$ [22]. We then find that, in the IR, the corresponding operator is irrelevant with dimension 3.

4.2 The Flow Between (2, 0) Vacua

Now, we consider the flow between IV and V critical points.

We begin by giving the ansatz for e_1 and e_2 ,

$$\begin{aligned} e_1 &= \sqrt{\frac{2(g_p - g_n)}{g_{2s}}} \begin{pmatrix} x(r) & 0 & 0 & 0 \\ 0 & q(r) & 0 & 0 \\ 0 & 0 & z(r) & 0 \\ 0 & 0 & 0 & z(r) \end{pmatrix} \\ e_2 &= \sqrt{\frac{2(g_p - g_n)}{g_{2s}}} \begin{pmatrix} y(r) & 0 & 0 & 0 \\ 0 & y(r) & 0 & 0 \\ 0 & 0 & w(r) & 0 \\ 0 & 0 & 0 & w(r) \end{pmatrix}. \end{aligned} \quad (4.34)$$

Consistency condition for the BPS equations requires

$$x = -\frac{(g_n + g_p)y^2}{q(g_n + g_p - 2g_n y^2)} \quad (4.35)$$

$$w = \sqrt{\frac{g_n + g_p}{g_n + g_p + 2g_p z^2}} z. \quad (4.36)$$

The $\delta\chi^{iI}$ equations give

$$\frac{dz}{dr} = \frac{1}{g_{2s}(g_n + g_p)q^2y^2(g_n + g_p - 2g_ny^2)} \{8(g_n + g_p)z^3(2q^2y^2(2(g_n^3 - g_ng_p^2)y^4 + (2g_p^3 + 6g_ng_p^2 - 4g_n^3)y^2 + (g_n - 2g_p)(g_n + g_p)^2) + g_nq^4(g_n + g_p - 2g_ny^2)^2 + g_n(g_n + g_p)^2y^4)\} \quad (4.37)$$

$$\frac{dy}{dr} = \frac{8y(g_n + g_p - 2g_ny^2)}{g_{2s}(g_n + g_p)} \left\{ -\frac{2(g_n + g_p)y^2}{g_n + g_p} \left((g_n - g_p)^2z^2 - 2g_n^2 + 3g_ng_p - g_p^2 + \frac{(g_n - g_p)^2(g_n + g_p)z^2}{g_n + g_p + 2g_pz^2} \right) + \frac{g_p(g_p - g_n)(g_n + g_p)^2y^4}{q^2(g_n + g_p - 2g_ny^2)^2} + g_p(g_p - g_n)q^2 \right\} \quad (4.38)$$

$$\frac{dq}{dr} = -\frac{8(g_n - g_p)q(g_n + g_p - 2g_ny^2)}{g_{2s}(g_n + g_p)y^2} \left\{ \frac{(g_n + g_p)^2y^4}{q^2(g_n + g_p - 2g_ny^2)} \left(g_nz^2 - g_py^2 + \frac{g_n(g_n + g_p)z^2}{g_n + g_p + 2g_pz^2} \right) + q^2 \left(\frac{2(g_n + g_p)^2y^4(g_p - 2g_n + (g_n - g_p)z^2)}{q^2(g_n + g_p - 2g_ny^2)^2} + \frac{(g_n + g_p)z^2}{g_n + g_p + 2g_pz^2} \left(\frac{2(g_n - g_p)(g_n + g_p)^2y^4}{q^2(g_n + g_p - 2g_ny^2)} - g_n \right) - g_nz^2 + g_py^2 \right) + \frac{2g_p(g_n + g_p)q^2y^2}{g_n + g_p - 2g_ny^2} \right\}. \quad (4.39)$$

This flow ansatz preserves (2,0) supersymmetry along the entire flow. We now change the variables to z_1 , h , and p

$$y = \sqrt{\frac{g_n + g_p}{2g_n(1 + z_1)}} \quad (4.40)$$

$$z = \sqrt{\frac{g_n + g_p}{2g_ph}} \quad (4.41)$$

$$q = \sqrt{-\frac{(g_n + g_p)\sqrt{p^2 - 4}}{g_nz_1(p^2 - 4 + p\sqrt{p^2 - 4})}} \quad (4.42)$$

and rescale r to $r \frac{8(g_n^2 - g_p^2)}{g_{2s}g_ng_p}$. The final forms of (4.37), (4.38), and (4.39) are

$$\frac{dz_1}{dr} = \frac{(g_n^2 - g_p^2 - h(g_p^2p - 2g_n(g_n - 2g_p) + g_ph(4g_n - 2g_p + g_pp)))}{h(h + 1)} \quad (4.43)$$

$$\frac{dh}{dr} = \frac{g_n^2 - g_p^2 + z_1(g_n^2p(1 + z_1) - 2(g_p(g_p - 2g_n) + g_n(g_n - 2g_p)z_1))}{z_1(1 + z_1)} \quad (4.44)$$

$$\frac{dp}{dr} = -(p^2 - 4) \left[g_n^2 \left(\frac{1}{h} + \frac{1}{1 + h} \right) - \frac{g_p^2}{z_1} - \frac{g_p^2}{1 + z_1} \right]. \quad (4.45)$$

We proceed by taking p as an independent variable and obtain

$$\frac{dz_1}{dp} = \frac{(g_p^2 - g_n^2 + (g_p^2 p + 4g_n g_p - 2g_n^2)h + g_p(4g_n + g_p(p - 2))h^2)z_1(1 + z_1)}{(p^2 - 4)(g_n^2(1 + 2h)z_1(1 + z_1) - g_p^2 h(1 + h)(1 + 2z_1))} \quad (4.46)$$

$$\frac{dh}{dp} = \frac{h(1 + h)(g_p^2 - g_n^2 + 2g_p(g_p - 2g_n)z_1 + 2g_n(g_n - 2g_p)z_1^2 - g_n^2 p z_1(1 + z_1))}{(p^2 - 4)(g_n^2(1 + 2h)z_1(1 + z_1) - g_p^2 h(1 + h)(1 + 2z_1))} \quad (4.47)$$

Recall that $g_n = t g_p$, we find that the two critical points are given by

• IV:

$$\begin{aligned} p &= -2, & h &= \frac{1}{4}(t - 1 + \sqrt{5 + 2t + t^2}), \\ \text{and} & & z_1 &= \frac{1 - t + \sqrt{1 + 2t + 5t^2}}{4t}, \end{aligned} \quad (4.48)$$

and

• V:

$$\begin{aligned} p &= 2 - \frac{2}{t} - 2t, & h &= \frac{1}{2}(t - 1 + \sqrt{1 + t^2}), \\ \text{and} & & z_1 &= \frac{1 - t + \sqrt{1 + t^2}}{2t}. \end{aligned} \quad (4.49)$$

We now give the numerical solution. Choosing $t = 2$, we find the numerical values for the critical points

$$\begin{aligned} \text{IV :} & \quad p = -2.000, & h &= 1.151, & z_1 &= 0.500 \\ \text{V :} & \quad p = -3.000, & h &= 1.618, & z_1 &= 0.309. \end{aligned} \quad (4.50)$$

The numerical solutions for the flow are given in Fig.1 and Fig.2.

The gravitino variation gives an equation for $A(p)$, with $t = 2$,

$$\begin{aligned} \frac{dA}{dp} &= -\frac{8g_p^2[(p^2 - 2)\sqrt{p^2 - 4} + p^3 - 4p]}{\sqrt{p^2 - 4}(p\sqrt{p^2 - 4} + p^2 - 4)^2} \times \\ & \quad [(p + 6)h(p)^2(2z_1(p) + 1) + ph(p)(1 - 2z_1(p)(4z_1(p) + 3)) \\ & \quad - 2z_1(p)(2pz_1(p) + 2p + 3) - 3]/[-4(2h(p) + 1)z_1(p)^2 \\ & \quad + 2((h(p) - 3)h(p) - 2)z_1(p) + h(p)(h(p) + 1)]. \end{aligned} \quad (4.51)$$

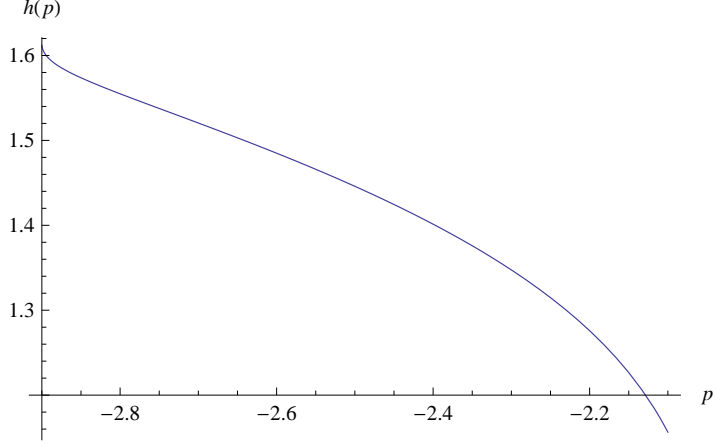


Figure 1: $h(p)$ solution.

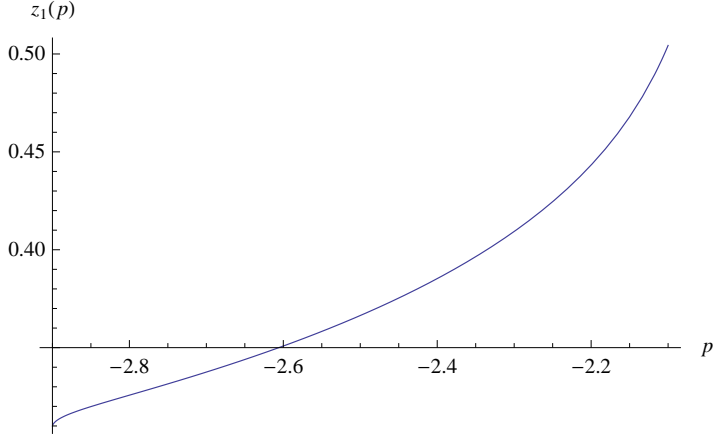


Figure 2: $z_1(p)$ solution.

Choosing $g_{2s} = -1$ and $g_p = 1$, we find the numerical solution for A as shown in Fig.3.

In this flow, the point IV is the UV fixed point, and V is the IR. The ratio of the central charges is

$$\frac{c_{UV}}{c_{IR}} = \frac{(g_n + g_p)^2}{4g_n g_p}. \quad (4.52)$$

This ratio is greater than 1 in consistent with the c-theorem. We also compute the scalar mass matrices at both critical points, but the form of the matrices is too complicated to be written here. We give only the numerical values of the eigenvalues in our choice of $g_p = 1$, $g_n = 2$ and $g_{2s} = -1$.

- IV: Eigenvalues of $M^2 R^2$ are $(3.70, -1.00, -1.00, -0.97, 0.36, 0.36, 0.00, 0.00)$

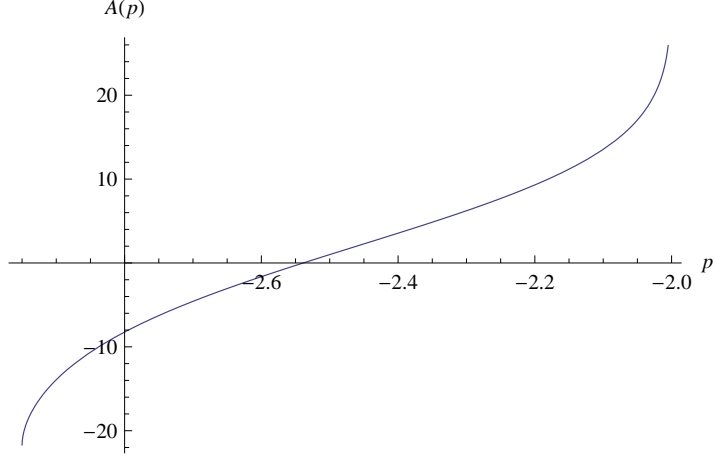


Figure 3: $A(p)$ solution.

with eigenvectors

$$\begin{aligned}
U_1 &= (-0.47, -0.47, -0.44, -0.44, -0.16, -0.16, -0.24, -0.24) \\
U_2 &= (0.33, -0.33, 0.44, -0.44, 0.00, 0.00, 0.44, -0.44) \\
U_3 &= (0.63, -0.63, -0.23, 0.23, 0.00, 0.00, -0.23, 0.23) \\
U_4 &= (0.47, 0.47, -0.44, -0.44, 0.16, 0.16, -0.24, -0.24) \\
U_5 &= (0.00, 0.00, -0.49, 0.49, -0.14, 0.14, 0.49, -0.49) \\
U_6 &= (0.00, 0.00, -0.10, 0.10, 0.69, -0.69, 0.10, -0.10) \\
U_7 &= (0.22, 0.22, -0.06, -0.06, -0.66, -0.66, 0.11, 0.11) \\
U_8 &= (-0.04, -0.04, -0.33, -0.33, 0.12, 0.12, 0.61, 0.61). \tag{4.53}
\end{aligned}$$

Our flow ansatz corresponds to U_4 with $\Delta = 1.168$ which is dual to a relevant operator. Note also that, our ansatz does not correspond to the one which saturates the bound $M^2 R^2 = -1$. This means the dual operator is not the most relevant one.

- V: Eigenvalues of $M^2 R^2$ are $(4.17, 3.33, 3.33, 3.33, 0.84, 0.84, 0.84, 0.00)$

with eigenvectors

$$\begin{aligned}
V_1 &= (-0.211, -0.894, -0.211, -0.211, -0.130, -0.130, -0.130, -0.130) \\
V_2 &= (0.201, -0.031, 0.063, -0.159, -0.609, 0.001, 0.090, 0.742) \\
V_3 &= (0.390, -0.398, 0.523, 0.432, 0.400, 0.011, -0.103, 0.241) \\
V_4 &= (-0.293, 0.159, 0.015, -0.258, 0.359, -0.004, -0.801, 0.227) \\
V_5 &= (0.255, 0.002, -0.712, 0.572, -0.039, 0.086, -0.300, 0.046) \\
V_6 &= (-0.146, 0.011, 0.387, 0.287, -0.526, 0.391, -0.384, -0.411) \\
V_7 &= (0.757, 0.004, -0.047, -0.499, 0.007, 0.156, -0.207, -0.328) \\
V_8 &= (0.130, 0.130, 0.130, 0.130, -0.211, -0.893, -0.211, -0.211). \tag{4.54}
\end{aligned}$$

Our flow ansatz corresponds to V_1 with $\Delta = 3.275$ which is dual to an irrelevant operator.

The behavior near $r \rightarrow \infty$ can be obtained as in the previous case. With $g_p = 1$, $g_n = 2$, and $g_{2s} = -1$, we find that

$$p(r) \sim e^{-2r/R}, \quad z_1(r), \quad h(r) \sim e^{-1.168r/R}. \tag{4.55}$$

At the IR point, we find

$$z_1(r), \quad h(r), \quad p(r) \sim e^{1.275r/R}. \tag{4.56}$$

From the dominant term near the UV fixed point, we see that the flow solution describes the deformation of the UV theory by a vacuum expectation value of an operator of dimension 1.168. We find that this flow is also a v.e.v. flow. The corresponding operator in the IR theory is an irrelevant operator of dimension 3.275.

5. Vacua of the $N = 8$ Theory

In this section, we study a gauging of an $N = 8$ theory. We restrict our discussion to the target space $\frac{SO(8,8)}{SO(8) \times SO(8)}$. We parametrize the coset elements L as in the $N = 4$ case, but now obviously e is an element of $GL(8, \mathbf{R})$ and B is an antisymmetric 8×8 matrix. The resulting L depends on 92 parameters, but, again using the right action of a diagonal $SO(8)$, one can bring e to an upper triangular form, thereby reducing the number of parameters to 64. As for the non compact generators, the Y^{ab} introduced before carry over in the obvious way to the present case, with $a, b = 1, \dots, 8$.

We are going to gauge the subgroup $(SO(4) \ltimes \mathbf{R}^6)^2$. Accordingly, we introduce gauge group generators:

$$t^{\mathcal{A}} = \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & a_2 \end{pmatrix}, \quad t^{\mathcal{B}} = \begin{pmatrix} b_1 & 0 & b_1 & 0 \\ 0 & b_2 & 0 & b_2 \\ -b_1 & 0 & -b_1 & 0 \\ 0 & -b_2 & 0 & -b_2 \end{pmatrix}. \quad (5.1)$$

Here all entries are 4×4 matrices, a_1 (a_2) are generators of the first (second) $SO(4)$, b_1 and b_2 are antisymmetric and correspond to independent shifts of B . More precisely, the upper and lower 4×4 diagonal blocks of B will be shifted by $2b_1$ and $2b_2$, respectively, and therefore could be set to zero. Generators carrying index 1 commute with those carrying index 2, and one checks the structure of the gauge group stated above. The f -tensors are constructed as follows: we choose a basis of symmetric, real $SO(8)$ γ -matrices with 8×8 off-diagonal blocks Γ^I , so that:

$$f_{ab,cd}^{IJ} = -\frac{1}{2} \text{Tr}(\varepsilon^{ba}[\Gamma^I, \Gamma^J]\varepsilon^{cd}). \quad (5.2)$$

As for the embedding tensor Θ , the structure discussed in the $N = 4$ case extends naturally to the present case, and now we expect a priori 8 couplings corresponding to the 8 $SU(2)$'s (including the \mathcal{B} generators). We then proceed first by computing the \mathcal{V} 's which are given in the Appendix A and then the T tensors which are given by:

$$\begin{aligned} T^{LJ,MK} &= g_{1s}(\mathcal{V}_{+a}^{LJ,PQ}\mathcal{V}_{+b}^{MK,PQ} + \mathcal{V}_{+b}^{LJ,PQ}\mathcal{V}_{+a}^{MK,PQ}) + g_{1a}(\mathcal{V}_{-a}^{LJ,PQ}\mathcal{V}_{-b}^{MK,PQ} \\ &\quad + \mathcal{V}_{-b}^{LJ,PQ}\mathcal{V}_{-a}^{MK,PQ}) + g_{2s}(\mathcal{V}_{+a}^{LJ,P'Q'}\mathcal{V}_{+b}^{MK,P'Q'} + \mathcal{V}_{+b}^{LJ,P'Q'}\mathcal{V}_{+a}^{MK,P'Q'}) \\ &\quad + g_{2a}(\mathcal{V}_{-a}^{LJ,P'Q'}\mathcal{V}_{-b}^{MK,P'Q'} + \mathcal{V}_{-b}^{LJ,P'Q'}\mathcal{V}_{-a}^{MK,P'Q'}) + h_{1s}\mathcal{V}_{+b}^{LJ,PQ}\mathcal{V}_{+b}^{MK,PQ} \\ &\quad + h_{1a}\mathcal{V}_{-b}^{LJ,PQ}\mathcal{V}_{-b}^{MK,PQ} + h_{2s}\mathcal{V}_{+b}^{LJ,P'Q'}\mathcal{V}_{+b}^{MK,P'Q'} + h_{2a}\mathcal{V}_{-b}^{LJ,P'Q'}\mathcal{V}_{-b}^{MK,P'Q'}, \\ T_{ab}^{LJ} &= g_{1s}(\mathcal{V}_{+a}^{LJ,PQ}\mathcal{V}_{+b}^{PQ} + \mathcal{V}_{+b}^{LJ,PQ}\mathcal{V}_{+a}^{PQ}) + g_{1a}(\mathcal{V}_{-a}^{LJ,PQ}\mathcal{V}_{-b}^{PQ} \\ &\quad + \mathcal{V}_{-b}^{LJ,PQ}\mathcal{V}_{-a}^{PQ}) + g_{2s}(\mathcal{V}_{+a}^{LJ,P'Q'}\mathcal{V}_{+b}^{P'Q'} + \mathcal{V}_{+b}^{LJ,P'Q'}\mathcal{V}_{+a}^{P'Q'}) \\ &\quad + g_{2a}(\mathcal{V}_{-a}^{LJ,P'Q'}\mathcal{V}_{-b}^{P'Q'} + \mathcal{V}_{-b}^{LJ,P'Q'}\mathcal{V}_{-a}^{P'Q'}) + h_{1s}\mathcal{V}_{+b}^{LJ,PQ}\mathcal{V}_{+b}^{PQ} \\ &\quad + h_{1a}\mathcal{V}_{-b}^{LJ,PQ}\mathcal{V}_{-b}^{PQ} + h_{2s}\mathcal{V}_{+b}^{LJ,P'Q'}\mathcal{V}_{+b}^{P'Q'} + h_{2a}\mathcal{V}_{-b}^{LJ,P'Q'}\mathcal{V}_{-b}^{P'Q'}, \end{aligned} \quad (5.3)$$

where $P, Q, \dots = 1, \dots, 4$ and $P', Q', \dots = 5, \dots, 8$. Here L, J, M, K are $SO(8)$ R-symmetry indices, and $a, b = 1, \dots, 8$ label the 64 non-compact generators in $SO(8, 8)$. $P, Q = 1, \dots, 4$ and $P', Q' = 5, \dots, 8$ label the first and second $SO(4)$, respectively. We have included also the 8 coupling constants, but actually, consistency imposes relations among them:

$$\begin{aligned} g_{1a} &= -g_{1s}, & g_{2a} &= -g_{2s} \\ h_{1a} &= -h_{1s}, & \text{and} & & h_{2a} &= -h_{2s}. \end{aligned} \quad (5.4)$$

Notice that if we set the type-2 couplings to zero i.e. $g_{2s} = g_{2a} = h_{2s} = h_{2a} = 0$, we decouple the second $SO(4)$ and therefore we recover a truncation of the single $SO(4)$ gauging studied in [9] as the supergravity dual of the D1-D5 system in IIB theory on $K3$ or T^4 . It can be obtained by reducing (2,0) six-dimensional supergravity on $AdS_3 \times S^3$.

A simple class of supersymmetric AdS vacua can be obtained as follows. We parameterize e and B as:

$$e = \begin{pmatrix} a_1 & 0 & 0 & 0 & e_{15} & e_{16} & e_{17} & e_{18} \\ 0 & a_2 & 0 & 0 & e_{25} & e_{26} & e_{27} & e_{28} \\ 0 & 0 & a_3 & 0 & e_{35} & e_{36} & e_{37} & e_{38} \\ 0 & 0 & 0 & a_4 & e_{45} & e_{46} & e_{47} & e_{48} \\ 0 & 0 & 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_8 \end{pmatrix} \quad (5.5)$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & b_{15} & b_{16} & b_{17} & b_{18} \\ 0 & 0 & 0 & 0 & b_{25} & b_{26} & b_{27} & b_{28} \\ 0 & 0 & 0 & 0 & b_{35} & b_{36} & b_{37} & b_{38} \\ 0 & 0 & 0 & 0 & b_{45} & b_{46} & b_{47} & b_{48} \\ -b_{15} & -b_{25} & -b_{35} & -b_{45} & 0 & 0 & 0 & 0 \\ -b_{16} & -b_{26} & -b_{36} & -b_{46} & 0 & 0 & 0 & 0 \\ -b_{17} & -b_{27} & -b_{37} & -b_{47} & 0 & 0 & 0 & 0 \\ -b_{18} & -b_{28} & -b_{38} & -b_{48} & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (5.6)$$

We have used the shift symmetry to set to zero the diagonal 4×4 blocks of B and the $SO(4) \times SO(4)$ left action to diagonalize the diagonal blocks of e . For diagonal $e = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B = 0$, we cannot find any interesting solutions apart from the trivial one with (4,4) supersymmetry. All the truncations below have been checked to be consistent, in the sense that there are no tadpoles for the remaining scalars.

We find a class of solutions by setting:

$$\begin{aligned}
a_2 &= a_3 = a_4 = a_1 \\
a_6 &= a_7 = a_8 = a_5 \\
b_{15} &= \frac{1}{4}(c_{15} - c_{26} - c_{37} + c_{48}) & b_{16} &= \frac{1}{4}(-c_{16} - c_{25} - c_{38} - c_{47}) \\
b_{17} &= \frac{1}{4}(c_{18} + c_{27} - c_{36} - c_{45}) & b_{18} &= \frac{1}{4}(c_{17} - c_{28} + c_{35} - c_{46}) \\
b_{25} &= \frac{1}{4}(-c_{16} - c_{25} + c_{38} + c_{47}) & b_{26} &= \frac{1}{4}(-c_{15} + c_{26} - c_{37} + c_{48}) \\
b_{27} &= \frac{1}{4}(c_{17} - c_{28} - c_{35} + c_{46}) & b_{28} &= \frac{1}{4}(-c_{18} - c_{27} - c_{36} - c_{45}) \\
b_{35} &= \frac{1}{4}(c_{18} - c_{27} + c_{36} - c_{45}) & b_{36} &= \frac{1}{4}(-c_{17} - c_{28} + c_{35} + c_{46}) \\
b_{37} &= \frac{1}{4}(-c_{15} - c_{26} - c_{37} - c_{48}) & b_{38} &= \frac{1}{4}(-c_{16} + c_{25} + c_{38} - c_{47}) \\
b_{45} &= \frac{1}{4}(-c_{17} - c_{28} - c_{35} - c_{46}) & b_{46} &= \frac{1}{4}(-c_{18} + c_{27} + c_{36} - c_{45}) \\
b_{47} &= \frac{1}{4}(-c_{16} + c_{25} - c_{38} + c_{47}) & b_{48} &= \frac{1}{4}(c_{15} + c_{26} - c_{37} - c_{48}), \tag{5.7}
\end{aligned}$$

and all other parameters are zero. We can choose

$$\begin{aligned}
c_{16} &= c_{17} = c_{18} = c_{25} = c_{27} = c_{28} = 0 \\
c_{35} &= c_{36} = c_{38} = c_{45} = c_{46} = c_{47} = 0. \tag{5.8}
\end{aligned}$$

Supersymmetric vacua require

$$g_{1s} = -a_1^2 h_{1s} \quad g_{2s} = -a_5^2 h_{2s} \quad h_{2s} = \frac{a_1^4}{a_5^4} h_{1s}. \tag{5.9}$$

- **(1,1) critical point**

This point is given by $c_{15} = 0$,

$$\begin{aligned}
A_1 &= \left(-\frac{16g_{1s}^2}{h_{1s}}, \frac{16g_{1s}^2}{h_{1s}}, -\frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{26}^2}, \frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{26}^2}, \right. \\
&\quad \left. -\frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{37}^2}, \frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{37}^2}, -\frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{48}^2}, \right. \\
&\quad \left. \frac{8g_{1s}^2}{h_{1s}}\sqrt{4 + a_1^2 a_5^2 c_{48}^2} \right) \tag{5.10}
\end{aligned}$$

and $V_0 = -\frac{1024g_{1s}^4}{h_{1s}^2}$.

- **(2,2) critical point**

This point is given by $c_{15} = 0$ and $c_{26} = 0$.

- **(3,3) critical point**

This point is given by $c_{15} = 0, c_{26} = 0$ and $c_{37} = 0$.

- **(4,4) critical point**

This point is given by $c_{15} = 0, c_{26} = 0, c_{37} = 0$ and $c_{48} = 0$.

All of them have the same cosmological constant. A_1 for the last three points is given by setting some of the appropriate values of c 's to zero in (5.10).

We also find other solutions with non zero parameters

$$\begin{aligned} a_2 &= a_3 = a_4 = a_1 & a_6 &= a_7 = a_8 = a_5 \\ e_{15} &= e_{26} = e_{37} = e_{48} = e \\ b_{16} &= -b_{25} & b_{38} &= -b_{47} \end{aligned} \tag{5.11}$$

subject to these relations $a_5^2 + e^2 = -\frac{g_{2s}}{h_{2s}}$, $a_1^2 = -\frac{g_{1s}}{h_{1s}}$ and $\frac{g_{1s}^2}{h_{1s}} = \frac{g_{2s}^2}{h_{2s}}$. Note that in this case, we also turn on some off-diagonal elements of e . The solutions are given by:

- **(2,2) critical point**

This solution has $A_1 = \frac{16g_{1s}^2}{h_{1s}}$ giving the same cosmological constant as in the previous case.

- **(3,2) critical point**

This can be obtained from the previous case by setting $b_{25} = b_{47}$ or $b_{25} = -b_{47}$.

So, there is no possible flow solution between all these critical points.

6. Conclusions

In this paper, we have studied three dimensional gauged supergravities and their AdS_3 supersymmetric vacua. We have discussed the $N = 4$ and $N = 8$ theories with $SO(4) \ltimes \mathbf{R}^6$ and $(SO(4) \ltimes \mathbf{R}^6)^2$ gaugings, respectively. Several supersymmetric AdS_3 vacua with different amount of supersymmetry have been found.

In the $N = 4$ theory, we have found analytic solutions interpolating between two (3,1) vacua. These solutions describe Renormalization Group flows between two fixed points of the dual boundary field theory. We have checked that the flows agree with the c-theorem, in particular the central charges of UV fixed points are strictly greater than those of the IR ones. We have also found a numerical solution describing the

flow between (2,0) vacua with similar qualitative features. In both cases, we found v.e.v. flows, i.e. flows driven by vacuum expectation values of relevant operators with dimensions $\Delta = 1$ and $\Delta = 1.168$, respectively, as opposed to the most common case where the flow is driven by a perturbing relevant operator.

In the $N = 8$ theory, we have found several vacua. However, they all have the same cosmological constant/central charge and the flow issue does not arise.

The gaugings considered here are of non semi-simple Chern-Simons type, giving rise to semi-simple Yang-Mills theories. In the $N = 8$ case, the (4, 4) point is related to the KK reduction of type IIB theory on $AdS_3 \times S^3 \times S^3 \times S^1$, and it would be interesting to identify the marginal deformations which take the theory to other less supersymmetric vacua, i.e. to generalize the discussion of [16], where the marginal deformation from (4, 4) to (3, 3) vacua has been worked out in detail, to the (k, k) vacua with $k < 3$. The $N = 4$ case seems to be related, via a \mathbb{Z}_2 projection, to the $N = 8$ theory, and it would be interesting to see how this is acting on the corresponding type IIB theory background. This would presumably help us in understanding the nature of the dual $SCFT_2$.

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A. Essential formulae

In this appendix, we give the expressions for the \mathcal{V} 's. Indices referring to each target space coordinates, i, j, k, \dots , will be traded by a pair of indices of the type a, b, c, \dots from 1 to 4. Antisymmetric pairs of capital letters I, J, K, \dots label $SO(4)$ adjoint indices.

$$\begin{aligned}
\mathcal{V}_{\pm a}^{LJ, MK} &= -\frac{1}{4} \text{Tr}[(e_1^t J_+^{LJ} X_1^t + X_1 J_+^{LJ} e_1) J_{\pm}^{MK} + (e_2^t J_-^{LJ} X_2^t + X_2 J_-^{LJ} e_2) J_{\pm}^{MK}], \\
\mathcal{V}_{\pm 1, 2a}^{MK} &= \text{Tr}[(e_{1,2}^t \varepsilon_{ab} X_{1,2}^t + Y_{1,2} \varepsilon_{ab} e_{1,2}) J_{\pm}^{MK}], \\
\mathcal{V}_{\pm b}^{LJ, MK} &= -\frac{1}{4} \text{Tr}[(e_1^t J_+^{LJ} e_1^t + e_2^t J_-^{LJ} e_2^t) J_{\pm}^{MK}], \\
\mathcal{V}_{\pm 1, 2b}^{MK} &= \text{Tr}(e_{1,2}^t \varepsilon_{ab} e_{1,2} J_{\pm}^{MK}).
\end{aligned} \tag{A.1}$$

The string of indices $\pm 1, 2a$ ($\pm 1, 2b$) indicates \mathcal{A} (\mathcal{B})-type gauging in the first (second) space with (anti-)self-dual $SU(2)$.

For completeness, we give below the analogous expressions for the $N = 8$ case:

$$\begin{aligned}
\mathcal{V}_{\pm a}^{LJ,MK} &= \frac{1}{4\sqrt{2}} \text{Tr}[\Gamma^{JL}(eJ_{\pm}^{MK}X + X^t J_{\pm}^{MK}e^t)], \\
\mathcal{V}_{\pm b}^{LJ,MK} &= \frac{1}{2\sqrt{2}} \text{Tr}[J_{\pm}^{JL}e\Gamma^{MK}e^t], \\
\mathcal{V}_{\pm a ab}^{MK} &= \frac{1}{\sqrt{2}} \text{Tr}[\varepsilon_{ab}(X^t J_{\pm}^{MK}e^t + eJ_{\pm}^{MK}Y)], \\
\mathcal{V}_{\pm b ab}^{MK} &= \frac{2}{\sqrt{2}} \text{Tr}[\varepsilon_{ab}eJ_{\pm}^{MK}e^t].
\end{aligned} \tag{A.2}$$

Here $\Gamma^{JL} = -[\Gamma^J, \Gamma^L]/2$ and all indices run from 1 to 8 and J_{\pm}^{MK} are the (anti-)self-dual $SU(2)$ generators in $SO(4) \times SO(4) \subset SO(8)$, corresponding to the first (second) $SO(4)$ for $M, K = 1, \dots, 4$ ($M, K = 5, \dots, 8$), respectively.

B. The other vacua of the $N = 4$ theory

In this appendix, we give all vacua we have found in $N = 4$ theory apart from those involved in the flows.

B.1 (4,0) vacua

- VI.

$$\begin{aligned}
e_1 &= \sqrt{\frac{-2(g_n + g_p)}{g_{2s}}} \mathbb{I}_{4 \times 4}, & e_2 &= \sqrt{\frac{2(g_p - g_n)}{g_{2s}}} \mathbb{I}_{4 \times 4}, \\
A_1 &= \frac{32g_n g_p}{g_{2s}}, & \text{and} & & V_0 &= -\frac{4096g_n^2 g_p^2}{g_{2s}^2}.
\end{aligned} \tag{B.1}$$

B.2 (3,0) vacuum

- VII.

$$\begin{aligned}
e_1 &= a \left(1, -\frac{g_m + g_p}{g_n + g_p + g_{2s}a^2}, 1, 1 \right), & e_2 &= \sqrt{\frac{(g_p^2 - g_n^2)}{g_p^2 - g_n^2 + g_{2s}g_p a}} a \mathbb{I}_{4 \times 4}, \\
a &= \sqrt{\frac{g_n^3 - g_n^2 g_p - g_n g_p^2 + g_p^3 + \sqrt{g_n^6 - g_n^4 g_p^2 - g_n^2 g_p^4 + g_p^6}}{g_n g_p g_{2s}}}, \\
A_1 &= -\frac{8(g_n^2 - g_p^2)^2}{g_{2s} g_n g_p}, & \text{and} & & V_0 &= \frac{-256(g_n^2 - g_p^2)^4}{(g_{2s}^2 g_n g_p)^2}
\end{aligned} \tag{B.2}$$

- VIII.

$$\begin{aligned}
e_1 &= a \left(1, -\frac{g_m + g_p}{g_n + g_p + g_{2s}a^2}, 1, 1 \right), & e_2 &= \sqrt{\frac{(g_p^2 - g_n^2)}{g_p^2 - g_n^2 + g_{2s}g_p a}} a \mathbb{I}_{4 \times 4}, \\
a &= \sqrt{\frac{g_n^3 - g_n^2 g_p - g_n g_p^2 + g_p^3 - \sqrt{g_n^6 - g_n^4 g_p^2 - g_n^2 g_p^4 + g_p^6}}{g_n g_p g_{2s}}} \\
A_1 &= -\frac{8(g_n^2 - g_p^2)^2}{g_{2s}g_n g_p}, & \text{and} & & V_0 &= \frac{-256(g_n^2 - g_p^2)^4}{(g_{2s}^2 g_n g_p)^2}
\end{aligned} \tag{B.3}$$

B.3 (2,0) vacua

- IX.

$$e_1 = -(a_1, a_1, b_1, b_2) \quad e_2 = (b_1, b_1, b_2, b_2) \tag{B.4}$$

$$\begin{aligned}
a_1 &= 2\sqrt{\frac{g_n^2 - g_p^2}{g_{2s}(g_n - g_p + \sqrt{5g_n^2 + 2g_n g_p + g_p^2})}} \\
a_2 &= 2\sqrt{\frac{g_n^2 - g_p^2}{g_{2s}(g_p - g_n + \sqrt{5g_p^2 + 2g_n g_p + g_n^2})}} \\
b_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(3g_n + g_p - \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\
b_2 &= 2\sqrt{\frac{g_n^2 - g_p^2}{g_{2s}(\sqrt{g_n^2 + 2g_n g_p + 5g_p^2} - g_n - 3g_p)}}
\end{aligned} \tag{B.5}$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = -\frac{4096(g_n - g_p)^4}{g_{2s}^2}. \tag{B.6}$$

- X.

$$e_1 = (-a_1, a_1, a_2, a_2) \quad e_2 = (b_1, b_1, b_2, b_2) \tag{B.7}$$

$$\begin{aligned}
a_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_p - g_n + \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\
a_2 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_n - g_p - \sqrt{5g_p^2 + 2g_p g_n + g_n^2})}} \\
b_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(3g_n + g_p + \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\
b_2 &= 2\sqrt{\frac{g_n^2 - g_p^2}{g_{2s}(\sqrt{g_n^2 + 2g_n g_p + 5g_p^2} - g_n - 3g_p)}}
\end{aligned} \tag{B.8}$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = -\frac{4096(g_n - g_p)^4}{g_{2s}^2}. \quad (\text{B.9})$$

• XI.

$$e_1 = (-a_1, a_1, a_2, a_2) \quad e_2 = (b_1, b_1, b_2, b_2) \quad (\text{B.10})$$

$$\begin{aligned} a_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_p - g_n - \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\ a_2 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(g_n - g_p + \sqrt{5g_p^2 + 2g_p g_n + g_n^2})}} \\ b_1 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(3g_n + g_p - \sqrt{5g_n^2 + 2g_p g_n + g_p^2})}} \\ b_2 &= 2\sqrt{\frac{g_p^2 - g_n^2}{g_{2s}(\sqrt{g_n^2 + 2g_n g_p + 5g_p^2} + g_n + 3g_p)}} \end{aligned} \quad (\text{B.11})$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = -\frac{4096(g_n - g_p)^4}{g_{2s}^2}. \quad (\text{B.12})$$

• XII.

$$e_1 = (-a_1, a_1, a_2, a_2) \quad e_2 = (b_1, b_1, b_2, b_2) \quad (\text{B.13})$$

$$\begin{aligned} a_1 &= \sqrt{\frac{g_p - g_n}{g_{2s}}} \sqrt{\frac{g_n - g_p + \sqrt{5g_n^2 + 2g_n g_p + g_p^2}}{g_n}} \\ a_2 &= \sqrt{\frac{g_p - g_n}{g_{2s}}} \sqrt{-\frac{g_n - g_p + \sqrt{g_n^2 + 2g_n g_p + 5g_p^2}}{g_p}} \\ b_1 &= \sqrt{\frac{g_p - g_n}{g_{2s}}} \sqrt{\frac{3g_n + g_p - \sqrt{5g_n^2 + 2g_n g_p + g_p^2}}{g_n}} \\ b_2 &= \sqrt{\frac{g_p^2 - g_n^2}{g_{2s}g_p}} \sqrt{-\frac{g_n - g_p + \sqrt{g_n^2 + 2g_n g_p + 5g_p^2}}{2g_p - \sqrt{g_n^2 + 2g_n g_p + 5g_p^2}}} \end{aligned} \quad (\text{B.14})$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \quad \text{and} \quad V_0 = -\frac{4096(g_n - g_p)^4}{g_{2s}^2}. \quad (\text{B.15})$$

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