AdS_3 Vacua and RG Flows in Three Dimensional Gauged Supergravity

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- $SO(4) \ltimes T_6$ gauged $N = 4$ 3D supergravity

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- Conclusions

AdS/CFT correspondence: String theory on AdS spaces $\Leftrightarrow$ CFT on the boundary of AdS

$$Z_{\text{CFT}}[\phi_0] = Z_{\text{string}} \xrightarrow{\text{small curvature}} Z_{\text{SUGRA}} = e^{-S_{\text{SUGRA}}[\phi_0]} \qquad (1)$$

Generalization to gauge/gravity correspondence:
(super) gravity theory in the bulk $\Leftrightarrow$ (super) QFT on the boundary

Original $\text{AdS}_5/\text{CFT}_4$: strong coupling "real" world 4D CFT

2D CFT structure is well-known $\Rightarrow$ $\text{AdS}_3/\text{CFT}_2$ correspondence is interesting.

Original version: near horizon of D-brane configurations $\Rightarrow$ $AdS \times M$

Gauged supergravity viewpoint: D-brane $\Rightarrow$ domain wall

Solutions to lower dimensional SUGRA uplifted $\Rightarrow$ D-branes.

Asymptotically AdS: background interpolating between vacua

$$\Downarrow$$

RG Flow between conformal fixed points of QFT

C-theorem:

Theorem

In an RG flow from a UV to IR fixed points, the central charge is monotonically decrease along the flow.

Remarkably, the holographic RG flow (from gravity dual) can be proved to obey this theorem. (*D.Z. Freedman, S. Gubser, N. Warner and K. Pilch, 1999*)

Holographic RG flows in $\mathrm{CFT}_2 \Rightarrow$ 3D gauged supergravity theories.

Some examples of these 3D flows have been studied in *M. Berg and H. Samtleben, 2001* and *N. S. Deger, 2002*.

Ungauged supergravity (*Bernard de Wit, Ivan Herger and Henning Samtleben, 2003*):

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{2}i\epsilon^{\mu\nu\rho}(e^a_\mu R_{\nu\rho a} + \bar{\psi}^I_\mu D_\nu \psi^I_\rho) - \frac{1}{2}eg_{ij}(g^{\mu\nu}\partial_\mu\phi^i g^{\mu\nu}\partial_\nu\phi^j \\ &\quad +\frac{1}{N}\bar{\chi}^{iI}\not{D}\chi^{jI}) + \frac{1}{4}eg_{ij}\bar{\chi}^{iI}\gamma^\mu\gamma^\nu\psi^I_\mu(\partial_\nu\phi^j + \hat{\partial}_\nu\phi^j) \\ &\quad -\frac{1}{24N^2}eR_{ijkl}\bar{\chi}^{iI}\gamma_a\chi^{jI}\bar{\chi}^{kJ}\gamma^a\chi^{lJ} + \frac{1}{48N^2}e(3(g_{ij}\bar{\chi}^{iI}\chi^{jI})^2 \\ &\quad -2(N-2)(g_{ij}\bar{\chi}^{iI}\gamma^a\chi^{jJ})^2)\end{aligned} \tag{2}$$

where

$$\begin{aligned}D_\mu\psi^I_\nu &= (\partial_\mu + \frac{1}{2}\omega^a{}_\mu\gamma_a)\psi^I_\nu + \partial_\mu\phi^i Q^{IJ}_i\psi^J_\nu, \\ D_\mu\chi^{iI} &= (\partial_\mu + \frac{1}{2}\omega^a{}_\mu\gamma_a)\chi^{iI} + \partial_\mu\phi^j(\Gamma^i_{jk}\chi^{kI} + Q^{IJ}_j\chi^{iJ}).\end{aligned} \tag{3}$$

After gauging: g(fermionic mass-like terms) $+\, g^2$(scalar potential)
Gauged Lagrangian: (*Bernard de Wit, Ivan Herger and Henning Samtleben, 2003*)

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \frac{1}{4} i g \epsilon^{\mu\nu\rho} A_\mu^{\mathcal{M}} \Theta_{\mathcal{M}\mathcal{N}} (\partial_\nu A_\rho^{\mathcal{N}} - \frac{1}{3} g \hat{f}_{\mathcal{P}\mathcal{Q}}{}^{\mathcal{N}} A_\nu^{\mathcal{P}} A_\rho^{\mathcal{Q}}) \\ &\quad + eg \{ \frac{1}{2} A_1^{IJ} \bar{\psi}_\mu^I \gamma^{\mu\nu} \psi_\nu^J + A_{2j}^{IJ} \bar{\psi}_\mu^I \gamma^\mu \chi^{jJ} + \frac{1}{2} A_{3ij}^{IJ} \bar{\chi}^{iI} \bar{\chi}^{jJ} \} \\ &\quad + \frac{4eg^2}{N} (A_1^{IJ} A_1^{IJ} - \frac{1}{2} N g^{ij} A_{2i}^{IJ} A_{2i}^{IJ}) \qquad (4)\end{aligned}$$

with “derivative” $\rightarrow$ “derivative” $+ g\Theta_{\mathcal{M}\mathcal{N}} A_\mu^{\mathcal{M}} X^{\mathcal{N}i}$

We are interested in the $N = 4$ theory with symmetric scalar target spaces $SO(4,4)/SO(4) \times SO(4)$.

Coset space G/H can be described by $L(\phi^i(x))$, transforming as $L(x) \to gL(x)h(x)$, with the decompositions

$$\begin{aligned} L^{-1}\partial_i L &= \frac{1}{2}Q_i^{IJ}X^{IJ} + Q_i^{\alpha}X^{\alpha} + e_i^A Y^A, \\ L^{-1}t^{\mathcal{M}}L &= \frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}X^{IJ} + \mathcal{V}^{\mathcal{M}}_{\ \alpha}X^{\alpha} + \mathcal{V}^{\mathcal{M}}_{\ A}Y^A. \end{aligned} \tag{5}$$

In the gauged theory, we have

$$L^{-1}D_\mu L = L^{-1}D(\partial_\mu + \Theta_{\mathcal{M}\mathcal{N}}A_\mu^{\mathcal{M}}t^{\mathcal{N}})L = \frac{1}{2}Q_\mu^{IJ}X^{IJ} + Q_\mu^{\alpha}X^{\alpha} + e_\mu^A Y^A. \tag{6}$$

The target space of the $N = 4$ theory is a product of two quaternionic manifolds, $\frac{SO(4,4)}{SO(4)\times SO(4)} \times \frac{SO(4,4)}{SO(4)\times SO(4)}$ in our case.

The gauge group is $SO(4) \ltimes T_6$ with generators

$$t^{\mathcal{A}} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}, \qquad t^{\mathcal{B}} = \begin{pmatrix} b & b \\ -b & -b \end{pmatrix}. \qquad (7)$$

We parametrize the coset by

$$L_i = \frac{1}{2} \begin{pmatrix} X_i + e_i^t & Y_i + e_i^t \\ -X_i - e_i^t & e_i^t - Y_i \end{pmatrix}, \, i = 1, 2, \qquad (8)$$

where $X_i = E_i + B_i e_i^t$, $Y_i = -E_i + B_i e_i^t$ and $E_i = e_i^{-1}$.

The embedding tensor: $T_{\mathrm{gauge}\,\mathcal{M}} = \Theta_{\mathcal{M}\mathcal{N}} t^{\mathcal{N}}$

Components: $\Theta_{\mathcal{A}\mathcal{B}}$ and $\Theta_{\mathcal{B}\mathcal{B}}$ subject to some constraints imposed by closure of gauge algebra

$SO(4) \sim SU(2) \times SU(2) \Rightarrow 4\ SU(2)$ factors for the two gaugings, $\mathcal{A}\mathcal{B}$ and $\mathcal{B}\mathcal{B}$.

$$\Downarrow$$

Four coupling constants: g_{1s}, g_{1a}, g_{2s}, and g_{2a}

T-tensor:

$$T_{\mathcal{A}\mathcal{B}} = \mathcal{V}^{\mathcal{M}}_{\ \mathcal{A}} \Theta_{\mathcal{M}\mathcal{N}} \mathcal{V}^{\mathcal{N}}_{\ \mathcal{B}} \qquad (9)$$

Supersymmetry $\Rightarrow$ Constraint on T $\Rightarrow g_{2a} = -g_{2s}$

Supersymmetry transformations:

$$\begin{aligned} \delta\psi^I_\mu &= \mathcal{D}_\mu \epsilon^I + A_1^{IJ}\gamma_\mu \epsilon^J, \\ \delta\chi^{iI} &= \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i{}_j \not{\mathcal{D}}\phi^j \epsilon^J - NA_2^{JIi}\epsilon^J, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathcal{D}_\mu \epsilon^I &= (\partial_\mu + \frac{1}{2}\omega^a{}_\mu \gamma_a)\epsilon^I + \partial_\mu \phi^i Q_i^{IJ}\epsilon^J + \Theta_{\mathcal{M}\mathcal{N}} A_\mu^{\mathcal{M}} \mathcal{V}^{\mathcal{N}IJ}\epsilon^J, \\ \mathcal{D}_\mu \phi^i &= \partial_\mu \phi^i + A_\mu^{\mathcal{M}} \mathcal{V}^{\mathcal{N}i}\Theta_{\mathcal{M}\mathcal{N}} \end{aligned} \tag{11}$$

Ansatz $B_1 = B_2 = 0$,

$$\begin{aligned} e_1 &= (a_1(r), a_2(r), a_3(r), a_4(r)) \qquad \text{and} \\ e_2 &= (a_1(r), a_2(r), a_3(r), a_4(r)). \end{aligned} \tag{12}$$

Also, define $g_{1s} = g_p + g_n$ and $g_{1a} = g_p - g_n$.

We use non compact generators of $SO(4,4)$ of the form

$$Y^{ab} = \begin{pmatrix} 0 & \varepsilon^{ab} \\ (\varepsilon^t)^{ab} & 0 \end{pmatrix}. \quad (13)$$

$SO(4)$ generators:

$$J_+^{IJ} = J^{IJ} + \frac{1}{2}\epsilon^{IJKL}J^{KL} \qquad \text{and} \qquad J_-^{IJ} = J^{IJ} - \frac{1}{2}\epsilon^{IJKL}J^{KL} \quad (14)$$

where $J^{IJ} = \varepsilon^{IJ} - \varepsilon^{JI}$, with $(\varepsilon^{IJ})_{KL} = \delta_{IK}\delta_{JL}$. Finally, the tensor f^{IJ} is given by

$$f_{\pm\ ab,cd}^{IJ} = \mathrm{Tr}((\varepsilon^t)^{ab}J_\pm^{IJ}\varepsilon^{cd}). \quad (15)$$

Now, we can find critical points of the potential and flow equations coming from $\delta\chi^{Ii} = 0$.

Number of supersymmetries: $\mathcal{N} = (n_L, n_R) \Leftrightarrow$ number of $\pm$ eigenvalues of A_1 tensor

This is equivalent to the number of supersymmetries in the dual 2D field theories.

Results:

- $N = 4$ vacua
- Flows between (3,1) and (2,0)
- Vacua of $N = 8$, $SO(8,8)/SO(8) \times SO(8)$ scalar manifold and $(SO(4) \ltimes T_6)^2$ gauging

Only (3,1) flows and the vacua involved will be discussed.

- I. $\mathcal{N} = (3,1)$

$$\begin{aligned} e_1 &= \sqrt{\frac{-2(g_n+g_p)}{g_{2s}}}\mathbb{I}_{4\times 4} \\ e_2 &= \sqrt{\frac{-2(g_n+g_p)}{g_{2s}}}(-1,1,1,1) \\ A_1 &= \frac{32(g_n+g_p)^2}{g_{2s}} \text{ and } V_0 = \frac{-4096(g_n+g_p)^4}{g_{2s}^2}. \quad (16) \end{aligned}$$

- II. $\mathcal{N} = (3,1)$

$$\begin{aligned} e_1 &= \sqrt{\frac{2(g_p-g_n)}{g_{2s}}}(1,-1,-1,-1) \\ e_2 &= -\sqrt{\frac{2(g_p-g_n)}{g_{2s}}}\mathbb{I}_{4\times 4} \quad (17) \end{aligned}$$

$$A_1 = \frac{-32(g_n - g_p)^2}{g_{2s}} \text{ and } V_0 = \frac{-4096(g_n - g_p)^4}{g_{2s}^2}. \tag{18}$$

- III. $\mathcal{N} = (3,1)$

$$\begin{aligned} e_1 &= \sqrt{\frac{g_n(g_p^2 - g_n^2)}{g_{2s}g_n^2}}\left(\frac{g_n}{g_p}, -1, -1, -1\right) \\ e_2 &= -\sqrt{\frac{g_n(g_p^2 - g_n^2)}{g_{2s}g_n^2}}\left(\frac{g_n}{g_p}, 1, 1, 1\right) \\ A_1 &= \frac{-8(g_n^2 - g_p^2)^2}{g_{2s}g_ng_p} \text{ and } V_0 = \frac{-256(g_n^2 - g_p^2)^4}{g_{2s}^2g_n^2g_p^2}. \end{aligned} \tag{19}$$

Supersymmetry variations(10)

The ansatz for the metric is

$$ds^2 = e^{2A(r)}(-dt^2 + dx^2) + dr^2. \tag{20}$$

Preserving 2D Poincare symmetries.

AdS$_3$: $A(r) = \frac{r}{L}$

Scalar current is zero for diagonal e_i

$$\Downarrow$$

$$A_\mu^{\mathcal{M}} = 0$$

The full truncation is consistent.

Taking $a_1(r) = -b_1(r) = b(r)$ and $a_2(r) = a_3(r) = a_4(r) = b_2(r) = b_3(r) = b_4(r) = a(r)$, we find

$$\begin{aligned} \frac{db}{dr} &= 24g_n ab^2 + 16g_p b^3 - 8a^3(g_n - g_{2s}b^2) && (21) \\ \frac{da}{dr} &= 16g_p a^3 + 8g_n a^2 b + \frac{8a^4(g_n + g_{2s}b^2)}{b}. && (22) \end{aligned}$$

and $\delta\psi^I_\mu = 0$ gives

$$\begin{aligned} \frac{dA}{dr} &= -\frac{1}{f^2(g_{2s} + (g_n g_p - g_n^2 f)c_1)^2}[8g_n(f^2 - 1)(3f^2(c_1 g_n(g_n^2 + g_p^2) \\ &\quad + g_{2s}g_p) - 2g_n f^3(2c_1 g_n g_p + g_{2s}) - 2g_n f(2c_1 g_n g_p + g_{2s}) \\ &\quad + c_1 g_n^3 f^4 + g_p(c_1 g_n g_p + g_{2s}))]. && (23) \end{aligned}$$

Solutions:

$$
\begin{aligned}
b &= \pm\sqrt{\frac{g_n(f^2-1)}{g_{2s}f^2+(g_n^2f^3+g_ng_pf^2)c_1}},\ a = fb \\
r &= c_2+\frac{1}{64g_n}\left[\frac{2(-fg_{2s}g_n+g_{2s}g_p+g_n(g_p^2-g_n^2)c_1)}{(f^2-1)(g_n^2-g_p^2)}\right. \\
&\quad -\frac{g_{2s}g_n\ln(1-f)}{(g_n+g_p)^2}+\frac{g_{2s}g_n\ln(1+f)}{(g_n-g_p)^2} \\
&\quad \left.-\frac{4g_{2s}g_n^2g_p\ln(fg_n+g_p)}{(g_n^2-g_p^2)^2}\right] \text{ and} \\
A &= c_3+\frac{1}{2}\ln f-\ln(1-f^2)+\frac{1}{2}\ln(g_p+fg_n) \\
&\quad +\frac{1}{2}\ln(g_{2s}+g_n(g_p+g_nf)c_1). \qquad (24)
\end{aligned}
$$

Choosing $c_1 = -\frac{g_{2s}}{g_n(g_n+g_p)}$, $g_n g_p < 0$,

$$
\begin{aligned}
b &= \sqrt{-\frac{(g_n+g_p)(1+f)}{g_{2s}f^2}} \\
a &= \sqrt{-\frac{(g_n+g_p)(1+f)}{g_{2s}}} \\
r &= \frac{1}{64}\left[-\frac{2g_{2s}}{(1+f)(g_n^2-g_p^2)}-\frac{g_{2s}\ln(1-f)}{(g_n+g_p)^2}\right. \\
&\quad \left.+\frac{g_{2s}\ln(1+f)}{(g_n-g_p)^2}-\frac{4g_{2s}g_ng_p\ln(fg_n+g_p)}{(g_n^2-g_p^2)^2}\right] \\
A &= \frac{1}{2}\ln f-\frac{1}{2}\ln(1-f)-\ln(1+f)+\frac{1}{2}\ln(g_p+fg_n)
\end{aligned}
\tag{25}
$$

With $g_{2s} < 0$:

I: $f = 1, \Leftrightarrow$ UV point where $A, r \to \infty$ and

III: $f = -\frac{g_p}{g_n}, \Leftrightarrow$ IR point where $A, r \to -\infty$

The ratio of the central charges, $c = \frac{3L}{2G}$, is given by

$$\frac{c_{UV}}{c_{IR}} = -\frac{(g_n - g_p)^2}{4g_n g_p} > 1. \tag{26}$$

We see the agreement with the c-theorem.

Choosing $c_1 = \frac{g_{2s}}{g_n(g_n - g_p)}$, $g_n g_p > 0$,

$$
\begin{aligned}
b &= \sqrt{\frac{(g_n - g_p)(f-1)}{g_{2s} f^2}} \\
a &= \sqrt{\frac{(g_n - g_p)(f-1)}{g_{2s}}} \\
r &= \frac{1}{64}\left[\frac{2g_{2s}}{(1-f)(g_n^2 - g_p^2)} - \frac{g_{2s}\ln(1-f)}{(g_n+g_p)^2}\right. \\
&\quad \left. + \frac{g_{2s}\ln(1+f)}{(g_n - g_p)^2} - \frac{4g_{2s}g_n g_p \ln(fg_n + g_p)}{(g_n^2 - g_p^2)^2}\right] \\
A &= \frac{1}{2}\ln f - \ln(1-f) - \frac{1}{2}\ln(1+f) + \frac{1}{2}\ln(g_p + fg_n). \quad (27)
\end{aligned}
$$

With $g_{2s} < 0$:

II: $f = -1$, $\Leftrightarrow$ UV point where A, $r \to \infty$ and

III: $f = -\frac{g_p}{g_n}$, $\Leftrightarrow$ IR point where A, $r \to -\infty$

The ratio of the central charges is given by

$$\frac{c_{UV}}{c_{IR}} = \frac{(g_n + g_p)^2}{4g_n g_p} > 1. \tag{28}$$

We see again the agreement with the c-theorem.

Recall mass dimension relation:

$$m^2 L^2 = \Delta(\Delta - 2) \tag{29}$$

The analysis of mass spectra indicates that the two flows are driven by a relevant operator of dimension $\Delta = 1$. For $\Delta = \frac{d}{2}$, the scalars behave near $r = \infty$ as (*Wolfgang Mück, 2002*)

$$\phi(r,x) = e^{-r/R}\left(\frac{r}{R}\hat{\phi}(x) + \check{\phi}(x)\right) + \dots . \quad (30)$$

From our solutions, we have

$$a(r) \sim e^{-r/R}, \qquad b(r) \sim e^{-r/R} \quad (31)$$

at the UV points. Both of the flows are called vev. flows driven by a vacuum expectation value of an operator of dimension 1. At the IR point, we have

$$a(r) \sim e^{r/R}, \qquad b(r) \sim e^{r/R} \quad (32)$$

meaning that the operator becomes irrelevant with dimension 3.

- We have found some supersymmetric AdS vacua of the $N = 4$ 3D gauged supergravity and studied the RG flow solutions between them. The flows are vev. flows driven by a vacuum expectation value of a relevant operator of dimension 1.

- We have found some supersymmetric AdS vacua of the $N = 4$ 3D gauged supergravity and studied the RG flow solutions between them. The flows are vev. flows driven by a vacuum expectation value of a relevant operator of dimension 1.
- $G_c \ltimes T_{\mathrm{dim}(G_c)}$ CS gauging $\Leftrightarrow$ G_c YM gauging (on-shell) (*H. Nicolai and H. Samtleben, 2003*)

  Dimensional reduction from higher dimensional theories gives YM gaugings. It is interesting to find a higher dimensional description to the model we have studied in the context of string or M theories.