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Instantonic field theories and new observables

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Outline

Motivation

- The development of String theory has raised a question: How to formulate field theories on target manifolds without global linear structure?
- The general path integral is not rigorously defined for curved space of fields.
- Idea: postulate the value of the path integral looking as infinite-dimensional δ-function, it localizes to the finite-dimensional space of generalized instantons.

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"Instantons beyond topological theory II " arXiv:0803.3302
A.Losev and SS "New observables in topological instantonic field theories", arXiv: 0911.2928
"Polyvector fields in instantonic field theories and A-I-B mirror", to appear

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Connection to “Physics”

2D σ model

$$\int_\Sigma \left(\frac{1}{2}\lambda(g_{a\bar{b}}(\partial_{\bar{z}}X^a\partial_z X^{\bar{b}} + \partial_z X^a \partial_{\bar{z}} X^{\bar{b}}) + i\pi_a D_{\bar{z}}\psi^a + i\pi_{\bar{a}} D_z \psi^{\bar{a}} + \frac{1}{2}\lambda^{-1} R^{a\bar{b}}{}_{c\bar{d}}\pi_a\pi_{\bar{b}}\psi^c\psi^{\bar{d}}\right) d^2z, \quad (1)$$

Bogomol’ny trick

$$\lambda \int_\Sigma d^2z\, |\partial_{\bar{z}} X|^2 + \lambda \int_\Sigma \Phi^*(\omega_K) = \lambda \int_\Sigma g_{a\bar{b}}\partial_{\bar{z}}X^a \partial_z X^{\bar{b}} d^2z + \lambda \int_\Sigma \frac{i}{2} g_{a\bar{b}} dX^a \wedge dX^{\bar{b}}, \quad (2)$$

Remove the topological term and take the infinite volume limit $\lambda \to \infty$.
Then we get *exact localization on instantons* and suppression of anti-instantons. This is the example of “instantonic field theory”.

Rigorous mathematical formulation (FLN)

Let X be a finite-dimensional manifold, and v is a section of vector bundle V over X. Then

$$\int \mathrm{d}p_a \mathrm{d}\pi_a \mathrm{d}x^i \mathrm{d}\psi^i \; \exp\left(ip_a v^a(x) - i\pi_a \partial_j v^a \psi^j\right) F(x,\psi) = \int_{\text{zeroes of } v} \omega_F$$

where ω_F denotes the differential form on X corresponding to the function F on the ΠTX with even coordinates x^i and odd coordinates ψ^i. The variables p_a and π_a correspond to the even and odd coordinates on V

Generalization to field theory

- Take X to be a space of maps from world-sheet Σ to target X. Now we have a path integral in the l.h.s.
- Take a vector field on this space of maps such that it vanishes on finite-dimensional space.
- Use a r.h.s. as a definition of path integral in the l.h.s.

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Deformations (FLN)

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Let us deform v:

$$v_\epsilon = v_0 + \epsilon^\alpha v_\alpha,$$

where ϵ^α are deformation parameters.

$$\int \mathrm{d}p_a \mathrm{d}\pi_a \mathrm{d}x^i \mathrm{d}\psi^i \ \exp\left(ip_a v^a_\epsilon(x) - i\pi_a \partial_j v^a_\epsilon \psi^j + id\epsilon^\alpha v^a_\alpha \pi_a\right) F(x,\psi) = \int_{\text{zeroes of } v_\epsilon} \omega_F$$

This integral may be treated as a generating function for *deformation observables* π_{v_α} and $\mathcal{O}_{v_\alpha}$, given by

$$\pi_{v_\alpha} = \pi_a v^a_\alpha$$

and

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$$\boxed{\left\langle \mathcal{O}_F e^{\epsilon^\alpha \mathcal{O}_{v_\alpha} + id\epsilon^\alpha \pi_{v_\alpha}} \right\rangle = \int_{\text{zeroes of } v_\epsilon} \omega_F}$$

Consider infinite dimensional version of the above statement as the *definition* of the generating function for the correlators.
NB. Non-trivial due to compactification subtleties.

summary on formalism (FLN)

We get ∞-dimensional theory

The space of deformations of instanton equation $v_\epsilon = 0$ is ∞ - dimensional, so we get a full-fledged quantum field theory!

Two types of observbales

- evaluation observables are functions of coordinates : evaluated on generalized instanton solution
- deformation observables are functions of momenta : deform the vector field v $=>$ deform the instanton solution

Formalism is geometric

- All basic observables are geometric objects – forms and vector fields, so the formalism is completely coordinate-independent.
- If we introduce coordinates for calculations, then gluing between patches is a-priory guaranteed.

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Examples (FLN)

1D: SUSY Quantum mechanics

$$\text{space} = Maps([0,1] \to X)$$

$$\text{vector field on Maps:} \quad v = \frac{d}{dt}X(t) - V_0(X(t))$$

localization on integral curves $\frac{d}{dt}X(t) = V_0(X(t))$
Hamiltonian: $H = \mathcal{L}_{V_0}$.
If gradient vector field, $V_0^i = g^{ij}\partial_j f$ this is called *Morse theory*.

2D: Gromov-Witten theory

$$\text{space} = Maps(\Sigma \to X)$$

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localization on holomorphic maps $\bar{\partial}X(z) = 0$ This is called $\beta\gamma - bc$ *system*
Topological subsector is a *Gromov-Witten theory*

4D: $N = 2$ SYM and Donaldson theory

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4D: N = 2 SYM and Donaldson theory

space = gauge equivalence classes of connections
localization on self-dual connections
Called $N = 2$ *twisted Super Yang-Mills and* topological subsector is *Donaldson theory*

Simplest example

$$\boxed{\left\langle \mathcal{O}_F e^{\epsilon^\alpha \mathcal{O}_{v_\alpha} + i\, d\epsilon^\alpha \pi_{v_\alpha}} \right\rangle = \int_{\text{zeroes of } v_\epsilon} \omega_F}$$

Simplest example of non-topological correlator in QM

- Simplest setting:

$$\text{space} = X(\cdot) = Maps([0,1] \to \mathbb{S}^1)$$

$$\text{vector field on Maps:} \quad v = \frac{d}{dt}$$

localization on constant maps $\frac{d}{dt}X = 0$

- Compute the correlation function $\langle X(t_x)\, p(t_p)\, \psi(1) \rangle = \frac{\partial}{\partial \epsilon} \langle X(t_x)\; e^{i\epsilon p(t_p)}\, \psi(1) \rangle$
- Deformed instanton equation: $\frac{d}{dt}X(t) = \epsilon\delta(t - t_p)$. Moduli space of solutions: constant maps with jump $X(t) = c + \epsilon\; \theta(t - t_p)$
- $$\langle X(t_x)\; e^{i\epsilon p(t_p)}\, \psi(1) \rangle = \int (c + \epsilon\; \theta(t_x - t_p))dc$$

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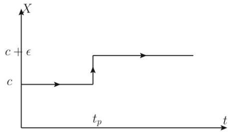
$$X(t_x)p(t_p) = -i\theta(t_X - t_p)$$

$$=> \; [p, X] = i$$



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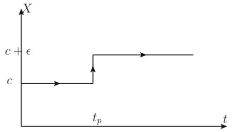

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Subtleties of ∞-dimensional case

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- Want the space of zeroes of v_ϵ (i.e. moduli space) to be compact.
  Requires *compactification*, e.g. *Kontsevich space of stable maps*.
- The deformation is expected to be smooth, for example

$$\bar{\partial} X(z) = \epsilon V(X) \omega(z - z_0)$$

- Local deformation observables $\mathcal{O}_V(z_0)$ are in closure of *compactification of space of deformations*.
- *Smooth correlation functions are always finite!* Divergencies and ambiguities may arise from taking a limit of local observables, hence we have a full control over divergencies.

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Briefly some results

Morse theory, $\beta\gamma - bc$ system and $\mathcal{N} = 2$ SYM

- The framework leads to rigorous non-perturbative formulation and calculations for these theories
- These theories are *logarithmic* (conformal) field theories. Frenkel, Losev, Nekrasov
  This is *an essentially non-perturbative phenomenon due to non-trivial topology of the target:* $H_2(X) \neq 0$

New topological observables (Losev, SS)

- We have found new topological observables in supersymmetric quantum mechanics that are non-trivial in BRST cohomologies and do not commute with evaluation-observables.
- These observables correspond to arbitrary jumps at instant t along fiber of any fibration of the target X.
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Morse theory, $\beta\gamma - bc$ system and $\mathcal{N} = 2$ SYM

- The framework leads to rigorous non-perturbative formulation and calculations for these theories
- These theories are *logarithmic* (conformal) field theories.
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  This is *an essentially non-perturbative phenomenon due to non-trivial topology of the target:* $H_2(X) \neq 0$

New topological observables (Losev, SS)

- We have found new topological observables in supersymmetric quantum mechanics that are non-trivial in BRST cohomologies and do not commute with evaluation-observables.
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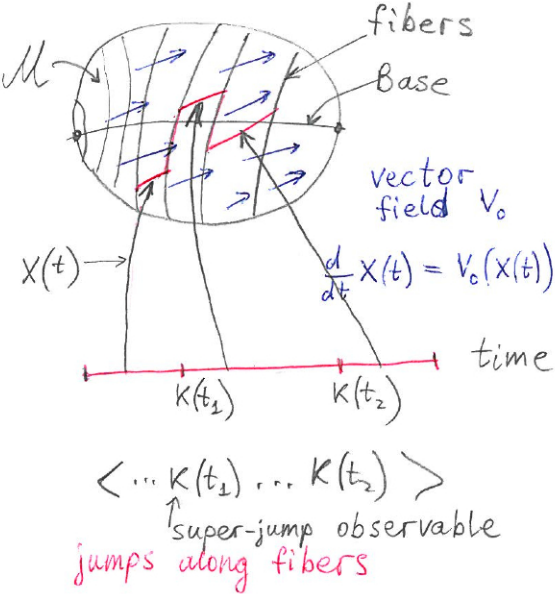
Arbitrary jumps along fibers: non-trivial extension of moduli space



Arbitrary jumps: possible definitions

Jump along fibers of fibration

Let the target X be a fibration with projection pr.

$$K_{fib}\,\omega = pr^* pr_* \omega = pr^* \int_{fiber} \omega$$

Jump along cycle in $\mathrm{Diff}\,X$

Let C be a cycle in the group of diffeomorphisms of the target X.

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Arbitrary jumps: applications

Deforming the theory with arbitrary jump

$$Q \to Q + \tau K_{fib}$$

Geometrically: any number of jumps at any time are allowed.
The theory computes the cohomology of the base as τ-equivariant cohomology of the fibration.

Loop rotation for QM on the loop space

The arbitrary jump creates observables, integrated over ϕ:
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Summary and outlook

We try to develop a formalism leading to mathematically rigorous geometric formulation of a rich class of field theories.
It is based on localization on deformed instanton moduli spaces.

Outlook: to do list

- Adding a bivector field deformation observables of the type $p\bar{p}$ to get A-model from $\beta\gamma - bc$ system
- Soft breaking of supersymmetry to get a geometric understanding of bosonic theories
- Mirror symmetry beyond topological sector
- Understanding of anti-instanton corrections

E.Frenkel, A.Losev, N.Nekrasov , "Instantons beyond topological theory I, II", hep-th/0610149, hep-th/0702137, arXiv:0803.3302
A.Losev and SS "New observables in topological instantonic field theories", arXiv: 0911.2928
A.Losev and SS "Polyvector fields in instantonic field theories and A-I-B mirror", to appear

Deformations of $\beta\gamma - bc$ system (Losev, SS, to appear soon)

$$S = i \int p\bar{\partial}X - \pi\bar{\partial}\psi + (c.c.) \quad (3)$$

To get the A-model

$$S = i \int p\bar{\partial}X - \pi\bar{\partial}\psi + g^{ij}(X)p_i p_j + ferm. + (c.c.) \quad (4)$$

we need to understand the observables of the type

$$g^{ij}(X)p_i p_j$$

We propose that these correspond to fusing two vector field deformation observables:

$$g^{ij}(X)p_i p_j \to (pv_1 + \pi v_1'\psi)(pv_2 + \pi v_2'\psi)$$

No notion of normal ordering on curved targets!

Towards bosonic system

$$S = i \int p\bar{\partial}X - \pi\bar{\partial}\psi + (c.c.) \tag{5}$$

To get the bosonic theory

$$S = i \int p\bar{\partial}X - \pi\bar{\partial}\psi + m\pi\psi \quad m \to \infty \tag{6}$$

So, we should expand in m and consider correlation functions of many $\pi\psi$ insertions in a geometric theory.