Quantization, geometry and background (in)dependence in high-spin gauge theory

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Warning: Revisionist attempt to collect various ideas connecting "high-spin gauge theory" to "quasi-topological field theory" — some details are based on work in collaboration with N. Boulanger, C. Iazeolla and E. Sezgin, other parts stolen from literature.

Generalized curvatures

Unfolded dynamics concerns the formulation of dynamical systems (with finitely or infinitely many degrees of freedom) as external differential systems generated by generalized curvature constraints

$$R^\alpha \ := \ dX^\alpha + Q^\alpha(X) \ \approx \ 0 \ ,$$

- fields X^α : locally defined differential forms on base manifold $\mathcal{S}$.
- structure functions Q^α : sums of exterior products of X's (that can be non-polynomial in zero-forms) obeying the compatibility condition

$$Q^\alpha \partial_\alpha Q^\beta \ \equiv \ 0 \quad \Rightarrow \quad dR^\alpha + R^\beta \partial_\beta Q^\alpha \ \equiv \ 0 \ ,$$

which is assumed to hold independently of the topology of $\mathcal{S}$.

Classical Q-manifolds

The basic dynamical variable is a sigma-model map f of vanishing intrinsic degree:

$$f \ : \ \ \mathcal{M} := T[1]\mathcal{S} \ \to \ \mathcal{N} \ , \qquad f^* \ : \ \Omega^{[n|p]}(\mathcal{N}) \ \to \ \Omega^{[p]}\mathcal{S}) \ ,$$

where $\mathcal{M}$ and $\mathcal{N}$ are graded commutative Q-manifolds, *i.e.* $\mathbb{N}$-graded differentiable manifolds with nilpotent vector fields of degree 1, *viz.*

- $\mathcal{M} := T[1]\mathcal{S}$ with local coordinates (σ^M, θ^M) of degrees $(0,1)$ and Q-structure $d = \theta^M \frac{\partial}{\partial \sigma^M}$.
- $\mathcal{N}$ with local coordinates $\{X^\alpha\}$ of degrees $|X^\alpha| = p_\alpha \in \mathbb{N}$, and

$$Q \ := \ Q^\alpha \partial_\alpha \ , \quad \mathcal{L}_Q Q \ = \ [Q,Q]_{S.B.} \ = \ 2(Q^\alpha \partial_\alpha Q^\beta)\partial_\beta \ \equiv \ 0 \ .$$

The equations of motion now read

$$df^* + f^* Q \approx 0 \ .$$

One also assigns $\{X^\alpha\}$ dual graded abelian modules $\mathfrak{R} = \bigoplus_{p=0}^\infty \mathfrak{R}_{[p]}$.

Cartan gauge transformations

- The constraint surface $\{R^\alpha \approx 0\}$ is left invariant under

$$\delta_\epsilon(f^*X^\alpha) \;:=\; d\epsilon^\alpha - \epsilon^\beta f^*(\partial_\beta Q^\alpha)$$

$$\Rightarrow\; \delta R^\alpha \;\equiv\; (-1)^\beta \epsilon^\beta R^\gamma f^*(\partial^2_{\gamma\beta} Q^\alpha)\;,$$

  with unconstrained, locally defined gauge parameters ϵ^α dual to $f^*_{[-1]}\mathfrak{R}$ where $f^*_{[-1]}$ is a non-dynamical sigma-model map of intrinsic degree -1 ($\rightsquigarrow$ dynamical ghosts in BRST-BV-AKSZ action).
- Soft, open, $\mathbb{N}$-graded algebra $\mathfrak{a} := (\mathfrak{R}[-1], \mathfrak{R})$, *viz.*

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}](f^*X^\alpha) \;\equiv\; \delta_{\epsilon_{12}}(f^*X^\alpha) + (-1)^\gamma \epsilon_1^\beta \epsilon_2^\gamma R^\delta f^*(\partial^3_{\delta\gamma\beta} Q^\alpha)\;,$$

  where $\epsilon^\alpha_{12} = (-1)^{\beta+1}\epsilon_1^\beta \epsilon_2^\gamma f^*(\partial^2_{\gamma\beta} Q^\alpha)$.
- Globally defined formulation with "unbroken" Cartan gauge structuroid

$$\mathfrak{a} \;\hookrightarrow\; \mathcal{P} \;\rightarrow \mathcal{M}\;.$$

Twisted-adjoint representation

- Subalgebra $\mathfrak{a}_{[0]} := ((\mathfrak{R}[-1])_{[0]}, \mathfrak{R})$: Lie algebra represented softly and linearly on fields of fixed degrees

$$\delta_\epsilon X^\alpha = \delta_{p_\alpha,1} d\epsilon^\alpha_{[0]} + \rho(\epsilon_{[0]})^\alpha_\beta X^\beta + O(X^2)$$

  Linearized representation matrices ρ define ordinary Lie algebra $\mathfrak{g}$.

- Twisted-adjoint module

$$\mathfrak{T} := ((\mathfrak{R}[-1])_{[0]}, \mathfrak{R}_{[0]}) \ .$$

  In quantum field theory, $\mathfrak{T}$ consists of the Weyl tensors and all their on-shell derivatives, forming a unitarizable representation of the space-time isometry algebra, $\mathfrak{g}$.

Globally defined local symmetries

On-shell Q- and d-morphisms connected to the identity:

- infinitesimal target-space diffeos preserving Q :

  $\delta X^\alpha = -[Q, B]^\alpha$ for globally defined vector field $B = B^\alpha \partial_\alpha$ on $\mathcal{N}$ of degree $|B| = -1$ (since $\mathcal{L}_{[Q,B]} Q = [[Q, B], Q] \equiv 0$) corresponding to

  $$\epsilon^\alpha = f^* B^\alpha(X)$$

- infinitesimal base-manifold diffeos preserving d :

  $\delta(f^* X^\alpha) = \mathcal{L}_v f^* X^\alpha = [d, i_v] f^* X^\alpha$ for globally defined vector field v on Σ corresponding to

  $$\epsilon^\alpha \approx i_v f^* X^\alpha$$

On-shell, these symmetries are broken spontaneously by $\langle X^\alpha \rangle$ but not by observables, *i.e.* observables must break symmetries “softly”.

Degrees of freedom and phases

Weyl zero-form $\Phi \in f^*\mathfrak{R}_{[0]}$ deform otherwise "topological systems" into "dynamical systems" in various phases characterized by different types of gauge-invariant classical observables/degrees of freedom:

- Unbroken phase:
  - Generalized Casimir invariants $\mathcal{I}_N[\Phi] \leftrightarrow$ semi-classical amplitudes in correspondence/twistor spaces.
  - Decorated Wilson loops $Tr[P\{\prod_i \mathcal{I}_{N_i}[\Phi]|_{p_i} \exp \oint_L A\}]$ for possibly composite connections A that are on-shell flat, $dA + \frac{1}{2}[A, A] \approx 0$.
- Softly broken Cartan geometries with soldering one-form E:
  - Charges $\oint_S (H^I + K^I)$ for globally defined, equivariantly on-shell closed, possibly composite, p-form curvatures $H^I[\Phi, E]$, $dH^I + f^I(H) \approx 0$ (*c.f.* Sullivan algebras)
  - Minimal areas for composite spin-s metrics $G_{M_1 \dots M_s}[\Phi, E]$ and brane partition functions.

Naively

$$\# \text{ d.o.f. in unbroken phase} << \# \text{ d.o.f. broken phase}$$

Locally defined gauge functions and integration constants

Expand Q (assume no constant forms in positive degrees and Stückelberg masses):

$$Q^\alpha(X) \;=\; \sum_{n\geq 2} Q^\alpha_{\beta_1\dots\beta_n} X^{\beta_1}\cdots X^{\beta_n} \;.$$

Expand $X^\alpha \;=\; \sum_{n=1}^{\infty} X^\alpha_{(n)}$ around general linearized solution given by gauge functions and integration constants:

$$|X^\alpha|>0 \;:\; X^\alpha_{(1)} \;\approx\; d\lambda^\alpha \;,$$

$$|X^\alpha|=0 \;:\; X^\alpha_{(1)} \;\approx\; C^\alpha \;, \quad dC^\alpha \;=\; 0 \;.$$

Cartan integrability implies:

$$Q^\alpha_{(n)} \;\approx\; -d(\Xi^\alpha_{(n)}(C,\lambda,d\lambda)) \quad (\Xi^\alpha_{(n)} \text{ local})$$

$$X^\alpha \;\approx\; \sum_{n=1}^{\infty} \Xi^\alpha_{(n)}(C,\lambda,d\lambda) \;,$$

where higher-order homogeneous solutions have been redefined away.

Gluing compatibility

δ_ϵ acts nonlinearly on gauge functions and linearly on integration constants

$$\delta_\epsilon \lambda^\alpha \;=\; \epsilon^\alpha + \text{h.o.t.}\;, \qquad \delta_\epsilon C^\alpha \;=\; \rho(\epsilon_{[0]})^\alpha_\beta C^\beta + \text{h.o.t.}$$

Gluing compatibility: there exists a soft $\mathbb{N}$-graded principal bundle

$$\mathfrak{a}' \;\hookrightarrow\; \mathcal{P}' \;\rightarrow \mathcal{M}\;,$$

where $\mathfrak{a}' \subseteq \mathfrak{a}$ is the unbroken gauge algebra, containing transition functions t^α_{IJ} on the overlaps $\mathcal{M}_{IJ} = \mathcal{M}_I \cap \mathcal{M}_J$ of charts in $\mathcal{M}$ such that

$$\lambda^\alpha_I - \lambda^\alpha_J \;=\; t^\alpha_{IJ} + \text{h.o.t.}\;,$$

$$C^\alpha_I \;=\; \rho(t_{[0]IJ})^\alpha_\beta C^\beta_J + \text{h.o.t.}$$

Generalized Casimir invariants (amplitudes)

Invariant functions on the twisted-adjoint representation, *viz.*

$$\mathcal{I}_N[\Phi] = \sum_{n\geq N} \mathcal{I}_N^{(n)}(\Phi,\dots,\Phi) \ , \quad \Phi \in \mathfrak{R}_{[0]} := \{X^\alpha : |X^\alpha| = 0\} \ ,$$

$$d\mathcal{I}_N \approx 0 \ , \quad \mathcal{I}_N \approx \mathcal{I}_N[C] \quad (\text{in “physical gauge”}) \ ,$$

where thus $\mathcal{I}_N^{(n)}$ are n-linear invariant functions of the linearized twisted-adjoint representation.

One may think of $\mathcal{I}_N[C]$ as generalized Casimir invariants for the C-deformed “topological modules” (C,λ).

One may also think of $\mathcal{I}_N$ as generators of amplitudes – this interpretation becomes more clear (or at least less obscure) in high-spin theories with cosmological constant where the twisted-adjoint representations are irreducible and self-dual (as modules).

Cartan geometries and soldering one-forms

The unbroken phase, with Cartan gauge structure $\mathfrak{a} \hookrightarrow \mathcal{P} \rightarrow \mathcal{M}$, can be broken softly, *i.e.* without generating masses, to Cartan geometries:

- the broken gauge symmetries reside in $\mathfrak{a}_{[0]} \hookrightarrow \mathcal{P}_{[0]} \rightarrow \mathcal{M}$, and are referred to as local translations, ξ.
- the vector fields v on $\mathcal{S}$ can be identified with the local translations via the broken gauge field E, referred to as the soldering one-form, *viz.* $v = \sum_I v_I$ and $\xi_I = i_\xi v_I$ (using partition of unity).
- the softly broken phase has order parameters given by observables $\mathcal{O}[\Phi, E]$ obeying

$$\delta_{t^\alpha}\mathcal{O} \equiv 0 \quad \text{for } t^\alpha \in \mathfrak{a}' ,$$

where $\mathfrak{a}' \hookrightarrow \mathcal{P}' \rightarrow \mathcal{M}$ is the unbroken gauge structure of the Cartan geometry, and

$$\mathcal{L}_\xi \mathcal{O} \approx 0 ,$$

i.e. the observables are gauge-invariant "off-shell" and intrinsically defined on $\mathcal{M}$ on-shell.

Non-commutative correspondence space

- Soft bundles $\rightsquigarrow$ non-commutative correspondence spaces
- "4D" gauge theories: twisted product of a non-commutative phase-space and non-commutative twistor spaces, locally

$$T^*(M_X) \times T^*_Y \times T^*_Z$$

with two-form $(\alpha, \dot{\alpha} = 1, 2;\, M = 1, 2, \dots)$

$$\Omega = dX^M dP_M + dy^\alpha dy_\alpha + d\bar{y}^{\dot{\alpha}} d\bar{y}_{\dot{\alpha}} - dz^\alpha dz_\alpha - d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}} \ .$$

- Closed and twisted-central element

$$J = b\ dz^2 (-1)_\star^{N_y + N_z} + \mathrm{h.c.}$$

i.e. $d^4 Y \star dJ = 0$ (treating dY as fermionic zero-modes) and

$$d^4 Y \star [f, J]_\pi \equiv 0 \quad \forall\ f(Y; X, P, Z; dX, dP, dZ)$$

with $[f, g]_\pi := f \star g - g \star \pi(f)$ and

$$\pi : y, dy, z, dz \to -(y, dy, z, dz) \ .$$

Simplest example: Vasiliev's equations

- One-form A and zero-form Φ obeying

$$d^4Y \star (F + \Phi \star J) \;\approx\; 0 \;, \qquad d^4Y \star D\Phi \;\approx\; 0 \;,$$

$$F := dA + A \star A \;, \qquad D\Phi := d\Phi + [A, \Phi]_\pi$$

- Minimal bosonic models with spin $s = 0, 2, 4, \ldots$:

$$\tau(A, B) = (-A, \pi(B)) \;, \qquad (A, B)^\dagger = (-A, \pi(B))$$

  where $\tau : y, dy, z, dz \to i(y, dy, -z, -dz)$ is a graded anti-automorphism.

- Linear-in-Φ coupling $\Leftarrow$ manifest Lorentz invariance plus fixed parity of the scalar field.

Manifest Lorentz invariance

Embed canonical Lorentz connection $(\omega^{\alpha\beta}, \bar{\omega}^{\dot{\alpha}\dot{\beta}})$

$$W \; := dX^M A_M + dP_M A^M \; := \; V + \frac{1}{4i}\omega^{\alpha\beta} M_{\alpha\beta} - \mathrm{h.c.}$$

where $M_{\alpha\beta} := y_\alpha \star y_\beta - z_\alpha \star z_\beta + S_{(\alpha} \star S_{\beta)}$ with $S_\alpha := z_\alpha - 2iA_\alpha$.

Manifestly Lorentz-covariant constraints

$$\nabla V + V \star V + \frac{1}{4i} R^{\alpha\beta} M_{\alpha\beta} - \mathrm{h.c.} \; \approx \; 0 \; ,$$

$$\nabla \Phi + [V, \Phi]_\pi \; \approx \; 0 \; , \quad \nabla S_\alpha + [V, S_\alpha]_\star \; \approx \; 0 \; ,$$

$$[S_\alpha, S_\beta]_\star \; \approx \; -2i\epsilon_{\alpha\beta}(1 - \Phi \star \kappa) \; , \quad [S_\alpha, S_{\dot{\alpha}}]_\star \; \approx \; 0 \; ,$$

$$S_\alpha \star \Phi + \Phi \star \pi(S_\alpha) \; = \; 0$$

Field redefinition $\rightsquigarrow$ shift-symmetry $\rightsquigarrow$ can impose

$$\frac{\partial^2}{\partial y^\alpha \partial y^\beta} V|_{Y=Z=0} \; = \; 0 \; .$$

Minimal Type A/B models

- Parity assignments (P exchanges dotted and undotted spinors):

$$\text{Type A model}: \; b = 1 \text{ and } P(A,\Phi) = (A,\Phi) \rightsquigarrow \text{scalar } 0_+$$

$$\text{Type B model}: \; b = i \text{ and } P(A,\Phi) = (A,-\Phi) \rightsquigarrow \text{pseudo-scalar } 0_-$$

- Conjectured AdS/CFT correspondence:

Type A / B models in asymptotic AdS_4 backgrounds with coupling $\hbar$

$$\leftrightarrow$$

$O(N)$ model / Gross-Neveu model with $N = \hbar^{-2}$ scalars / fermions.

  - few $\hbar$-corrections to bulk fields
  - dynamical symmetry breaking $\rightsquigarrow$ anomalous dimensions for spin $s = 0, 4, 6, \ldots$

Question: Are the Type A/B models UV completions of (effective) quantum gravity?

Gauge function method

- Contract $T^*M_X \to M_X$ ("trivial sector" of phase-space functions)
  $\rightsquigarrow$ Maurer-Cartan system on M_X
  $\rightsquigarrow$ possibly multi-valued gauge function:

  $$A_M \approx g^{-1} \star \partial_M g \ , \quad \Phi \approx g^{-1} \star \Phi' \star \pi(g) \ ,$$

  $$A_\alpha \approx g^{-1} \star (\partial_\alpha + A'_\alpha) \star g$$

  where "initial data" obey $\partial_M \Phi' \approx \partial_M A'_{\underline{\alpha}} \approx 0$ and twistor-space equations

  $$F' + \Phi' \star J \approx 0 \ , \quad D'\Phi' \approx 0 \ .$$

- Equivalent deformed oscillator with "anyon-statistics parameter" Φ':

  $$[S'_\alpha, S'_\beta]_\star \approx -2i\epsilon_{\alpha\beta}(1 - \Phi' \star \kappa) \ , \quad S'_\alpha \star \Phi' + \Phi' \star \pi(S'_\alpha) = 0 \ ,$$

  $\rightsquigarrow$ exact solution-generating methods, *e.g.* Type D moduli space.

- Contract T^*_Z in physical gauge $Z^{\underline{\alpha}} S'_{\underline{\alpha}} = 0$ (again trivial sector)
  $\rightsquigarrow$ unique perturbative expansion in $C(Y) := \Phi'|_{Z=0}$

Observables in unbroken phase

Decorated Wilson loops

$$Tr_{\kappa\bar{\kappa}}\left[P\left\{\prod_i X_{2N_i}|_{p_i}\exp\oint_L W\right\}\right]$$

where $Tr_\Gamma[\cdot]\equiv Tr[\Gamma\star(\cdot)]$, Tr is chiral trace and adjoint impurities

$$X_{2N} \ := \ X^{\star N}\ , \quad X \ := \ \Phi\star\pi(\Phi)\ .$$

Trivial L and removal of point-split $\rightsquigarrow$ generalized Casimir invariants

$$\mathcal{I}_{2N} \ := \ Tr_{\kappa\bar{\kappa}}[X^{\star N}]\ .$$

Contract $T^*M_X\times T^*_Z$ perturbatively yields

$$\mathcal{I}_{2N} \ = \ STr[(C\star\pi(C))^{\star N}]+\sum_{n\geq 2N+1}\mathcal{I}^{(n)}_{2N}(C)$$

where $\mathcal{I}^{(n)}_{2N}$ are n-linear invariant functions on the linearized twisted-adjoint representation given by closed-contour homotopy integrals in T^*_Z.

High-spin Cartan geometry

Define $P_\pm = \frac{1}{2}(1 \pm \pi)$ and split V into $E \ := \ P_- V \ , \quad \Omega \ := \ P_+ V \rightsquigarrow$

$$\nabla E + [\Omega, E]_\star + \frac{1}{4i} R^{\alpha\beta} P_- M_{\alpha\beta} - \mathrm{h.c.} \ \approx \ 0 \ ,$$

$$\nabla \Omega + \Omega \star \Omega + E \star E + \frac{1}{4i} R^{\alpha\beta} P_+ M_{\alpha\beta} - \mathrm{h.c.} \ \approx \ 0 \ ,$$

$$\nabla \Phi + [\Omega, \Phi]_\star + \{E, \Phi\}_\star \ \approx \ 0 \ ,$$

that can be examined using geometric methods:

- Real and imaginary parts of on-shell Chern classes

$$H_{[2N]} \ := \ Tr_\kappa [E^{\star 2N}] + R^{\alpha\beta}\text{-corrections} \ , \quad dH_{[2N]} \ \approx \ 0 \ .$$

- High-spin metrics of ranks $s = 2, 4, 6, \ldots$:

$$G_{M_1 \ldots M_s} \ = \ Tr_{\kappa\bar{\kappa}} [E_{(M_1} \star \cdots \star E_{M_s)}] \ ,$$

  that define norms, geodesics and minimal areas.

Question: Calibrations $(H_{2N}, G_{2N}) \overset{?}{\rightsquigarrow}$ Brane actions/partition functions

Generalized Poisson sigma model

- Classical unfolded system $df^* + f^* Q \approx 0$ with $f : \mathcal{M} = T[1]\mathcal{S} \to \mathcal{N}$.
- Embed $\mathcal{M} \hookrightarrow \mathcal{B} = T[1]\mathcal{D}$ and $\mathcal{N} \hookrightarrow T^*[D]\mathcal{N}$ with $D = \dim(\mathcal{D}) - 1$.
- Extend $f : \mathcal{B} \to T^*[D]\mathcal{N}$ and consider action $S_{\rm tot} = S_{\rm bulk} + S_{\rm marg}$.
- Bulk action $S_{\rm bulk} = \int_{\mathcal{B}} f^*(R^\alpha P_\alpha + \Pi_2^{\alpha\beta} P_\beta P_\alpha + \Pi_3^{\alpha\beta\gamma} P_\gamma P_\beta P_\alpha + \cdots)$ $\equiv \int_{\mathcal{B}} f^*(dX^\alpha P_\alpha + H(X,P))$
- Gauge invariance requires $\{H,H\}^{[-D]} = \partial_\alpha H \partial^\alpha H \equiv 0$,*i.e.*

$$\mathcal{L}_Q Q \equiv 0 \ , \quad \mathcal{L}_Q \Pi_2 \equiv 0 \ , \quad \{\Pi_2, \Pi_2\}_{S.B.} + \mathcal{L}_Q \Pi_3 \equiv 0 \ ,$$

  which implies $Q^\alpha = \Pi_2^{\alpha\beta}\partial_\beta h$ (Hamiltonian Q-structure).
- Semi-classically marginal deformation $S_{\rm marg} = \int_{\mathcal{M}} f^* M[X, dX]$

$$\delta_{\rm gauge} M \equiv 0 \ , \quad \delta M \approx 0 \ ,$$

  so that $\int_{\mathcal{M}} M \approx$ generator of semi-classical amplitudes ("tree diagrams").

Duality extended Vasiliev system

Extend (A, Φ, J) by forms in higher degrees:

$$(\widehat{A}, \widehat{B}, \widehat{U}, \widehat{V}; \widehat{J}) \ \in \ \widehat{\mathcal{U}}[X, P, Y, Z; dX, dP, dY, dZ; k, \bar{k}] \ ,$$

$$k \star (y, dy, z, dz) = -(y, dy, z, dz) \star k \ , \quad k \star k \ = \ 1 \ ,$$

and degree assignments ($D \in 2\mathbb{N}$):

$$\widehat{A} = A_{[1]} + A_{[3]} + \cdots + A_{[D-1]}$$

$$\widehat{V} = V_{[D-1]} + V_{[D-3]} + \cdots + V_{[1]}$$

$$\widehat{U} = U_{[2]} + U_{[4]} + \cdots + U_{[D]}$$

$$\widehat{B} = B_{[D-2]} + B_{[D-4]} + \cdots + B_{[0]}$$

and $\widehat{J} = J_{[2]} + J_{[4]}$ is real, closed, central and $i_M \widehat{J} = i^M \widehat{J} = 0$ ($J_{[4]}$ brings in four new parameters).

Semi-classical Type A/B bulk action

$$S_{\rm bulk} \ = \ \int_{\mathcal{B}} \widehat{T}r_{d^4Ydkd\bar{k}} \left[\widehat{U}\star \widehat{D}\widehat{B} + \widehat{V}\star(\widehat{F}+\widehat{B}\star\widehat{J}) + \widehat{V}\star f(\widehat{U})\right] \ ,$$

where $f(U) := f_1 U + f_2 U^{\star 2} + \cdots$ and the linear dependence on $\widehat{B}$ is required by gauge invariance.

Type A/B projection: insert $P_{\pm} := \frac{1}{2}(1+k\bar{k})$ into the trace and impose

$$B_{[0]} \ = \ P_+ k \star \Phi \ , \quad A_{[1]} \ = \ P_+ \star A \ ,$$

$$J_{[2]} \ = \ P_+ k \star J$$

plus consequences in higher degrees $\Rightarrow D \in 4\mathbb{N}$ and $f(-\widehat{U}) = -f(\widehat{U})$ and only two parameters remain in $J_{[4]} = P_+(b_1 + b_2\kappa\bar{\kappa})dZ^4$.

Classically marginal deformations

- Broken phase: split W into $\widetilde{E} = P_- W$ and $\widetilde{\Omega} = P_+ W$
  $\rightsquigarrow$ candidate full Fradkin-Vasiliev-like action ($\widetilde{R} := d\widetilde{\Omega} + \widetilde{\Omega} \star \widetilde{\Omega}$):

  $$S_{\rm FV} \ := \ {\rm Im} \int_{\mathcal{M}} \widehat{T}r_{d^4 Y J_{[4]} dk d\bar{k} \kappa} \left[c_1 \widetilde{R} \star \widetilde{R} + c_2 (\widetilde{R} + \frac{1}{2} \widetilde{E} \star \widetilde{E}) \star \widetilde{E} \star \widetilde{E} \right] \ .$$

  Question: Lorentz-covariant perturbative expansion on-shell (with fixed parameters)$\stackrel{?}{\leftrightarrow}$ CFT correlators

- Unbroken phase: $\widehat{T}r_{d^4 Y dk d\bar{k}} \left[c_1' F \star F + c_2' (F + \frac{1}{2} B \star J) \star B \right] |_{p \in T^* M_X}$

  Question: Perturbative expansion on-shell (with fixed parameters) $\stackrel{?}{\leftrightarrow}$ correlation functions of topological open "singleton" string

Generalized AKSZ sigma models

Starting from generalized Poisson sigma model $S_{\rm bulk}$, the minimal BRST-BV classical master action is given by

$$S_{\rm min} \;=\; \int_{\mathcal{B}} \mathbf{f}^*(dX^\alpha P_\alpha + H)|^{[0]} \;,$$

where $\mathbf{f} : \mathcal{B} \to T^*[D]\mathcal{N}$ has non-vanishing first-quantized and second-quantized ghost numbers gh_1 and gh_2, respectively, but vanishing intrinsic total degree $gh_1 + gh + 2$, where gh_1 is identified as the form degree on $\mathcal{B}$:

$$\mathbf{X}^\alpha \;=\; \sum_{\substack{gh_1 \in \mathbb{N} \\ gh_2 \in \mathbb{Z}}} (\mathbf{X}^\alpha)^{[gh_2]}_{[gh_1]} \;.$$

Extra assumptions on auxiliary volume form $\rightsquigarrow \Delta_{BV} S_{\rm min} = 0$.

Question: Precise generalization to systems with non-commutative base manifold, and central+closed elements?

Gauge fixing

Add canonical (albeit in general non-minimal) gauge fixing sector (ghost momenta with $gh_2 < 0$ and Lagrange multipliers, and then additional layers of ghosts, ghost momenta and multipliers):

$$S_{\rm g.f.} \;=\; S_{\rm kin} + S_{\rm int} \;,$$

$$S_{\rm kin} \;=\; \int_{\mathcal{B}} (d\mathbf{X}^\alpha \mathbf{P}_\alpha + \cdots)^{[0]} \;,$$

$$S_{\rm int} \;=\; \int_{\mathcal{B}} (Q^\alpha(\mathbf{X})\mathbf{P}_\alpha + \Pi_2^{\alpha\beta}(\mathbf{X})\mathbf{P}_\beta \mathbf{P}_\alpha + \cdots)^{[0]} \;,$$

exhibiting tensorial supersymmetry in the sense that

$$\int D({\rm fields}) e^{\frac{i}{\hbar_2} S_{\rm kin}} \;=\; 1 + \text{auxiliary-curvature corrections} \;.$$

Topological sum

Expand around $\langle E \rangle = \overline{E}$ and adapt auxiliary vielbein to $\overline{E}$ $\rightarrow$ additional 1-loop corrections from $\rho(\overline{E})$.

$\Lambda_{\rm CC} = 0 = Mass^2 \rightsquigarrow$ all vacuum bubbles cancel.

Question: What happens for critical masses if $\Lambda_{\rm CC} \neq 0$?

Topological sum: If all one-loop corrections combine into curvature invariants then it makes sense to examine for which topologies the partition function is actually well-defined and to sum over these

$\rightsquigarrow$ notion of "third quantization".

de Rham-like BRST operator and non-commutativity

The prototype bulk action

$$S_{\rm bulk} \;\sim\; \int_{\mathcal{B}} f^*(dX_{[p]}\cdot P_{[D-p]} + d\widetilde{X}_{[D-p-1]}\cdot \widetilde{P}_{[p+1]} + P_{[D-p]}\cdot \widetilde{P}_{[p+1]} + \cdots)\;,$$

induces de Rham-like BRST operator acting on non-commutative zero-modes:

$$\delta P \;=\; d\epsilon + \cdots\;,\quad \delta\widetilde{P} \;=\; d\widetilde{\epsilon} + \cdots\;,$$

$$\delta X \;=\; -(-1)^{D(p+1)}\widetilde{\epsilon}\,,\quad \delta\widetilde{X} \;=\; -\epsilon + \cdots\;.$$

$$\rightsquigarrow \quad Q_{BRST} \sim C_{[0]}^{[D-p]}\cdot \frac{\partial}{\partial \widetilde{X}_{[0]}^{[D-p-1]}} + \widetilde{C}_{[0]}^{[p+1]}\cdot \frac{\partial}{\partial X_{[0]}^{[p]}}$$

Russian doll structure: Quantum Gauge Principle

Unify Einstein-Weyl's Gauge Principle and Quantum Mechanics

$\rightarrow$ Quantum Gauge Principle

Fundamental interactions in Nature form hierarchic structure
$(n = \ldots, 1, 2, 3, \ldots)$

SigmaModel$_n$[CartanGeom$_n$, BRST-BVGeom$_n$; LoopExp$_n$; TopSum$_n$]

$$\mathcal{Z}_n[\alpha'_n, \hbar_n, g_n] = \sum_{Topologies_n} (g_n)^{Index_n} \int_{Maps_n : Base_n \rightarrow Target_n} e^{\frac{i}{\hbar_n} S[Map_n; \alpha'_n; \hbar_n]}$$

SigmaModel$_n$ = Master theory of SigmaModel$_{n-1}$

CartanGeom$_n$ = BRST-BVGeom$_{n-1}$ ($\alpha'_n = \hbar_{n-1}$)

LoopExp$_n$ = TopSum$_{n-1}$ ($\hbar_n = g_{n-1}$)

Base$_n$: zero-modes of $(n-1)$th unfolded system

Target$_n$: composite operators of the $(n-1)$th unfolded system

High-spin implementation

Limit $\hbar_1 = 1$, $\hbar_2 = \hbar_3 = 0$

$\rightsquigarrow$ classical unfolded system on noncommutative $\mathbb{Z}$-graded manifold
$\supset$ $\mathbb{N}$-graded correspondence space

Natural arena for High-Spin Gauge Theory !

$\rightsquigarrow$ Starting point for examining QGP :

- 1^{st}-quantized TopOpenString on the correspondence space for massless fields in four space-time dimensions
- 2^{nd}-quantized Vasiliev Systems in correspondence space
- 3^{rd}-quantized moduli space (geometric quantization of high-spin invariant observables)

Conclusions

- Existence of an action principle for Vasiliev's equations
- Germ of geometric framework for quantization as well as exact solution finding

Merci beaucoup!