D-brane Instantons from String Dualities

Ulrich Theis

Institute for Theoretical Physics
Friedrich-Schiller-University Jena

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w/ Daniel Robles-Llana, Martin Roček, Frank Saueressig and Stefan Vandoren

Introduction

- Classical string theory effective actions receive quantum corrections $\sim \alpha'$ and g_s, perturbatively and through instantons
- Important e.g. in flux compactifications, where quantum effects can lead to moduli stabilization and inflation
- An instanton calculus in string theory is still missing
- We have determined the complete perturbative, D(–1)-brane and (p,q)-string instanton contributions to the LEEA of Type IIB string theory compactified on a Calabi-Yau threefold using T- and S-duality
- Mirror symmetry maps these to perturbative and A-cycle D2-brane instanton corrections to Type IIA on mirror manifold

Calabi-Yau Compactifications of Type II Strings

Consider Type IIA/B string on background $M_{1,9} = M_{1,3} \times \mathrm{CY}_3$

$\Rightarrow$ N=2 supergravity on $M_{1,3}$:

$S = \int \big(*R + \frac{1}{2}\, g_{ij}(\varphi) *d\varphi^i \wedge d\varphi^j + \ldots \big) \supset$ NLSM with metric g_{ij}

$\varphi^i : \; M_{1,3} \to \mathcal{M} \to \mathbb{R}^d \; \in$ Vector- & Hypermultiplets

N=2 Susy $\Rightarrow$ target space $\mathcal{M} = \mathcal{M}_{\mathrm{VM}} \times \mathcal{M}_{\mathrm{HM}}$:

$\mathcal{M}_{\mathrm{VM}}$: special Kähler, $\mathcal{M}_{\mathrm{HM}}$: quaternion-Kähler

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QK: $(\mathcal{M}, g)$ $4n$-dim. space with $\mathrm{Hol}(\mathcal{M}) = K \times \mathrm{Sp}(1)$, $K \subset \mathrm{Sp}(n)$,
admits triplet of almost complex structures $\vec{J}$: $J_i\, J_j = -\delta_{ij} + \epsilon_{ijk}\, J_k$
not Kähler, Einstein ($n = 1$: Einstein & self-dual)

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Examples: $\mathbb{C}\mathrm{P}^2$, S^4, $\mathbb{H}\mathrm{P}^n$, $\dfrac{\mathrm{U}(n,2)}{\mathrm{U}(n) \times \mathrm{U}(2)}$ ($n = 1$: class. universal hyperm.)

IIA/X		IIB/X		
$\mathcal{M}^A_{\mathrm{VM}}$ ×	$\mathcal{M}^A_{\mathrm{HM}}$	$\mathcal{M}^B_{\mathrm{VM}}$ ×	$\mathcal{M}^B_{\mathrm{HM}}$	moduli spaces
$2h_{1,1}$	$4h_{1,2}+4$	$2h_{1,2}$	$4h_{1,1}+4$	$\dim\mathcal{M}$

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Quantum corrections to moduli spaces: $g_{ij} - g^{\mathrm{cl}}_{ij} = g^{\mathrm{pert}}_{ij} + g^{\mathrm{inst}}_{ij}$

- $\sim \alpha'/R^2_{\mathrm{CY}}$ worldsheet CFT
- $\sim g_s = \mathrm{e}^{\langle\phi\rangle}$ genus expansion of worldsheet + brane instantons

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α'	g_s	$-$	α', g_s	corrections

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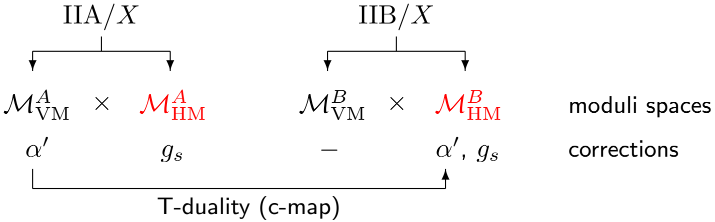

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mirror symmetry

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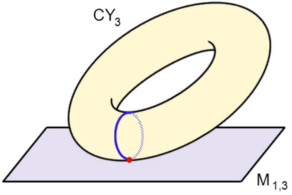
Instantons

Euclidean worldvolume of string/brane wrapping around supersym. cycle γ_{p+1} in CY_3



Becker[2], Strominger '95

Type $\mathrm{II}/\mathrm{CY}_3$:

- F1-string $\sim \mathrm{e}^{-1/\alpha'}$
- Dp-brane $\sim \mathrm{e}^{-1/g_s}$
  IIA: $p = 2$, IIB: $p = -1, 1, 3, 5$
- NS5-brane $\sim \mathrm{e}^{-1/g_s^2}$

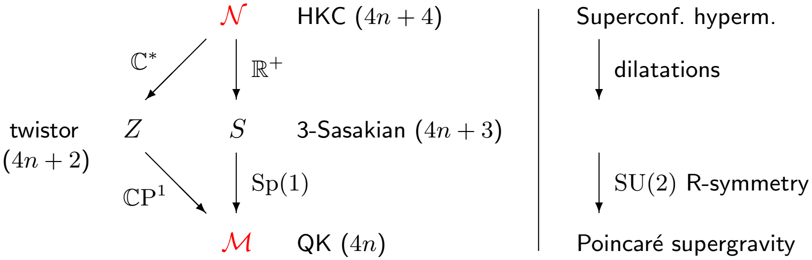
In this talk, I will determine the contributions to the LEEAs from D(–1)-brane, (p, q)-string and A-cycle D2-brane instantons

Ingredients:

- Full α' corrections to IIA vector multiplets (3-loop & worldsheet instantons)
- Perturbative g_s corrections to hypermultiplets (1-loop)

Hyperkähler Cone/Off-shell Formulation

Swann bundle:



HKC: Hyperkähler w/ homothetic Killing vector ξ ($\nabla_A \xi^B = \delta_A^B$) and isometric $\mathrm{SU}(2)$ action $\vec{k} = \vec{J}\xi$ (not triholomorphic)

HK potential $\chi = \frac{1}{2} g(\xi, \xi) \;\Rightarrow\; ds^2_{\mathcal{N}} = d\chi^2/2\chi + 2\chi\, ds^2_{\text{3-S}}$

Complete low-energy effective action determined by HK potential χ!

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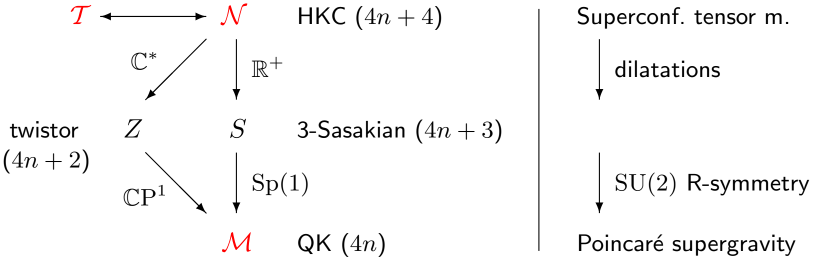

No 3- and 5-brane instantons $\Rightarrow$ HKC has $n+1$ commuting isometries,
consider space of orbits $\mathcal{T} = \mathcal{N}/G$ $(\dim \mathcal{T} = 3n+3)$

Physics: dualization of hyper- into tensor multiplets: $*d\varphi^I \sim dB_I$

$\mathcal{T}$ also determined by real potential $\chi_T(\vec{r}^{\,I})$, $I = 1, \ldots, n+1$ *de Wit, Saueressig '06*

Physical tensor multiplet scalars of Poincaré supergravity are dilatation and $\mathrm{SU}(2)_R$-invariant functions of the $\vec{r}^{\,I}$:

$\mathrm{IIA}/\tilde{X}$	IIB/X
$\mathrm{e}^{-\phi_{\mathrm{IIA}}}$	$\tau_2 = \mathrm{e}^{-\phi_{\mathrm{IIB}}}$
$\int_{\gamma_3^a} \Omega \Big/ \int_{\gamma_3^1} \Omega = z^a_{\mathrm{IIA}}$	$z^a_{\mathrm{IIB}} = b^a + \mathrm{i} t^a = \int_{\gamma_2^a} (B + \mathrm{i} J)$
$\int_{\gamma_3^1} C_3 = \xi^1$	$\tau_1 = C_0$
$\int_{\gamma_3^a} C_3 = \xi^a$	$\tau_1 b^a - c^a = -\int_{\gamma_2^a} \mathrm{e}^{-B} \wedge (C_0 + C_2)$
$\delta^a_a = h_{1,2}(\tilde{X})$	$\delta^a_a = h_{1,1}(X)$

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↑

Mirror symmetry

Böhm, Günther, Herrmann, Louis '99
RSTV '08

SL(2,Z) Invariance of IIB String Theory

Any isometry of $\mathcal{M}_{\mathrm{HM}}$ is lifted to a symmetry of the tensor potential χ_T

$\Rightarrow$ χ_T must be modular invariant!

$\mathrm{SL}(2,\mathbb{Z})$ transformations of tensor multiplet scalars:

Böhm, Günther, Herrmann, Louis '99

- dilaton/axion $\quad \tau \mapsto \dfrac{\alpha\tau+\beta}{\gamma\tau+\delta}\,, \qquad \alpha\delta-\beta\gamma=1$
- Kähler moduli $\quad t^a \mapsto |\gamma\tau+\delta|\, t^a$
- 2-form moduli $\quad \begin{pmatrix} b^a \\ c^a \end{pmatrix} \mapsto \begin{pmatrix} \delta & \gamma \\ \beta & \alpha \end{pmatrix} \begin{pmatrix} b^a \\ c^a \end{pmatrix}$
- conf. compensator $\quad r^0 \mapsto |\gamma\tau+\delta|\, r^0$

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$\mathrm{SL}(2,\mathbb{Z})$ generated by $T:\ \tau\mapsto\tau+1\;,\quad S:\ \tau\mapsto-\dfrac{1}{\tau}$

string coupling constant $g_s = 1/\langle\tau_2\rangle$ $\Rightarrow$ strong-weak duality

IIB Tensor Multiplet Potential

Modular invariant tensor potential: $\chi_T^{\mathrm{IIB}}(\vec{r}^{\,I}) \equiv r^0 \sqrt{\tau_2}\, \Psi^{\mathrm{IIB}}(\tau, t, b, c)$

Classical action from

$$\Psi_{\mathsf{cl}}^{\mathrm{IIB}} = \frac{4}{3!}\, \tau_2^{3/2} \kappa_{abc}\, t^a t^b t^c$$

Perturbative, D(–1)-brane and (p, q)-string instanton corrections: *RRSTV '07*

$$\Psi_{\mathsf{qu}}^{\mathrm{IIB}} = -\frac{1}{(2\pi)^3} \sum_{k \in H_2, 0} n_k \sum_{m,n}{}' \frac{\tau_2^{3/2}}{|m\tau + n|^3} \left(1 + 2\pi |m\tau + n| k_a t^a\right) \mathrm{e}^{-S_{m,n}}$$

n_k: genus zero GV invariants, # rational curves of class k_a in X *Gopakumar, Vafa '98*

$n_0 \equiv -\chi_E(X)/2 = h_{1,2}(X) - h_{1,1}(X)$

(p, q)-string instanton action: $S_{m,n} = 2\pi k_a \big(|m\tau + n| t^a - \mathrm{i} m c^a - \mathrm{i} n b^a\big)$

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$$\Psi^{\rm IIB}_{\sf qu} = -\frac{1}{(2\pi)^3} \sum_{k \in H_2, 0} n_k \sum_{m,n}{}' \frac{\tau_2^{3/2}}{|m\tau + n|^3} \big(1 + 2\pi |m\tau + n| k_a t^a\big)\, {\rm e}^{-S_{m,n}}$$

Can also be obtained from 1-loop graviton/M2-brane amplitude in M-theory compactified on $X \times T^2$ *Collinucci, Soler, Uranga '09*

Poisson resummation gives

$$\Psi^{\mathrm{IIB}}_{\mathsf{qu}} = \frac{\chi_E(X)}{(2\pi)^3}\Big(\zeta(3)\,\tau_2^{3/2} + 2\zeta(2)\,\tau_2^{-1/2}\Big)$$

perturbative α' and g_s corrections

$$-\frac{2\tau_2^{3/2}}{(2\pi)^3}\sum_{k\in H_2} n_k\ \mathrm{Re}\Big(\mathrm{Li}_3(\mathrm{e}^{2\pi\mathrm{i}k_a z^a}) + 2\pi k_a t^a\,\mathrm{Li}_2(\mathrm{e}^{2\pi\mathrm{i}k_a z^a})\Big)$$

worldsheet instantons

$$-\frac{\sqrt{\tau_2}}{2\pi^2}\sum_{k\in H_2,0} n_k \sum_{m\neq 0, n\in\mathbb{Z}} \left|\frac{k_a z^a + n}{m}\right| \mathrm{e}^{2\pi\mathrm{i}m(k_a(c^a - \tau_1 b^a) - n\tau_1)} \times \times K_1(2\pi\tau_2|m||k_a z^a + n|)$$

$k_a = 0$: D(–1) instantons, $k_a \neq 0$: D1 instantons

A-cycle D2-brane Instanton Corrections in IIA

Mirror symmetry :

IIB/X		$\mathrm{IIA}/\tilde{X}$
$H_0(X,\mathbb{Z})$ $\mathrm{D}(-1)/\gamma_0$	$\Longleftrightarrow$	$H_{0,3}(\tilde{X},\mathbb{Z})$ $\mathrm{D}2/\gamma_3^1$
$H_{1,1}(X,\mathbb{Z})$ $\mathrm{D}1/k_a\gamma_2^a$ with $\int F/2\pi = n$	$\Longleftrightarrow$	$H_{1,2}(\tilde{X},\mathbb{Z})$ $\mathrm{D}2/k_a\gamma_3^a + n\gamma_3^1$

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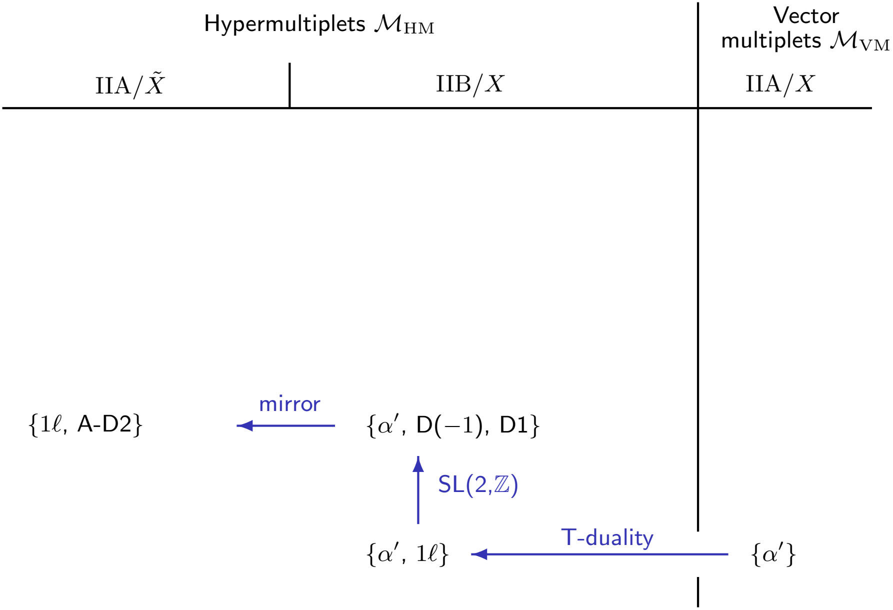
$H_{1,1}(X,\mathbb{Z})$ $\mathrm{D1}/k_a\gamma_2^a$ with $\int F/2\pi = n$	$\Longleftrightarrow$	$H_{1,2}(\tilde{X},\mathbb{Z})$ $\mathrm{D2}/k_a\gamma_3^a + n\gamma_3^1$

$$\Rightarrow \quad \Psi^{\mathrm{IIA}}_{\text{A-D2}} = -\frac{\sqrt{\tau_2}}{2\pi^2} \sum_{k_\Lambda \neq 0} n_{k_\Lambda} \sum_{m \neq 0} \left| \frac{k_\Lambda z^\Lambda}{m} \right| \mathrm{e}^{-2\pi \mathrm{i} m k_\Lambda \xi^\Lambda} K_1(2\pi\tau_2 |m k_\Lambda z^\Lambda|)$$

with $k_\Lambda = (n, k_a)\,,\ z^\Lambda = (1, z^a)\,,\ \xi^\Lambda = (\xi^1, \xi^a)$ *RSTV '08*

Type IIA instanton numbers: $n_{(n,0)} = \chi_E(\tilde{X})/2\ , \quad n_{(n,k_a)} = n_{k_a}(X)$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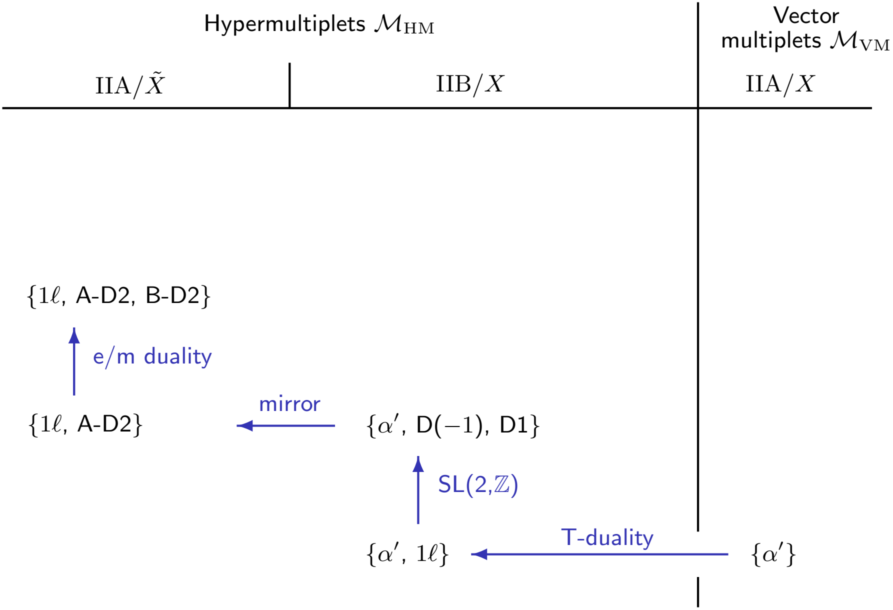
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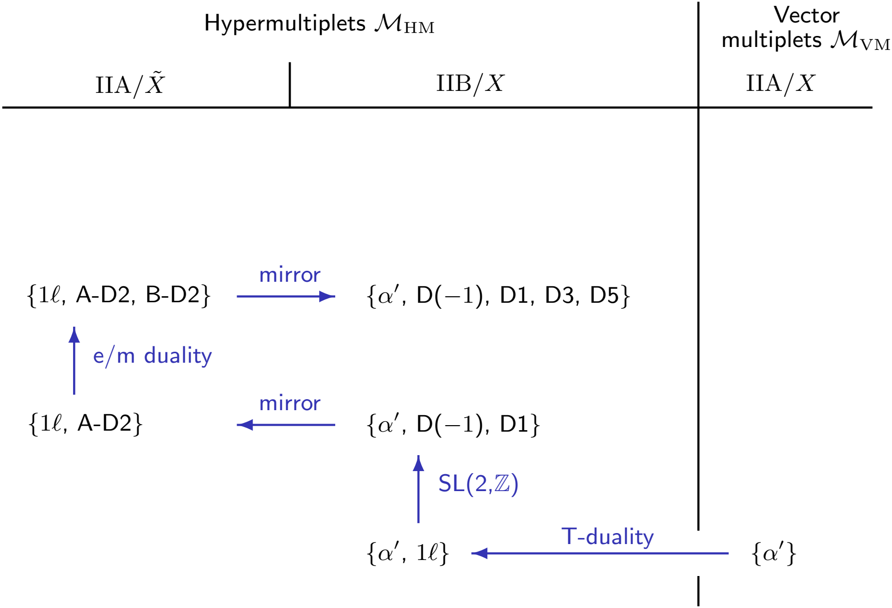
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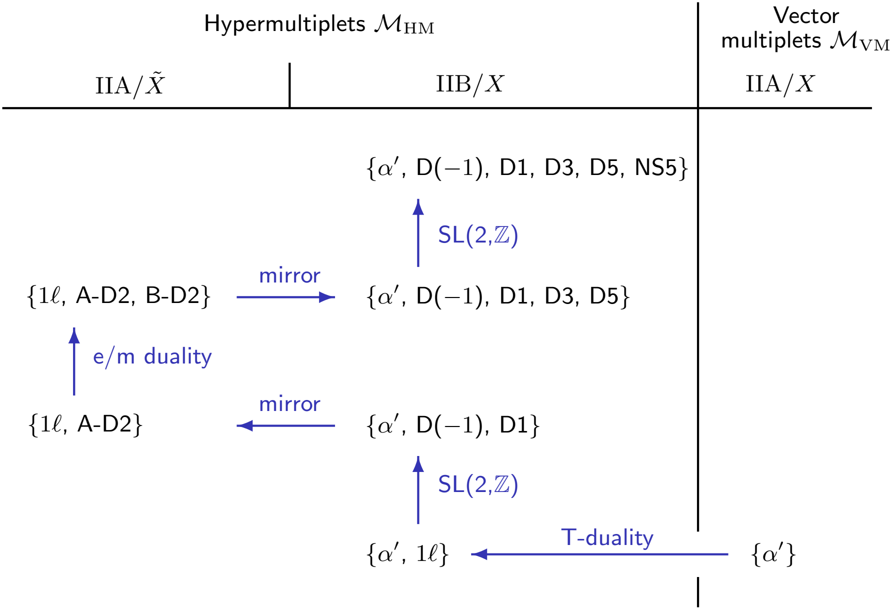
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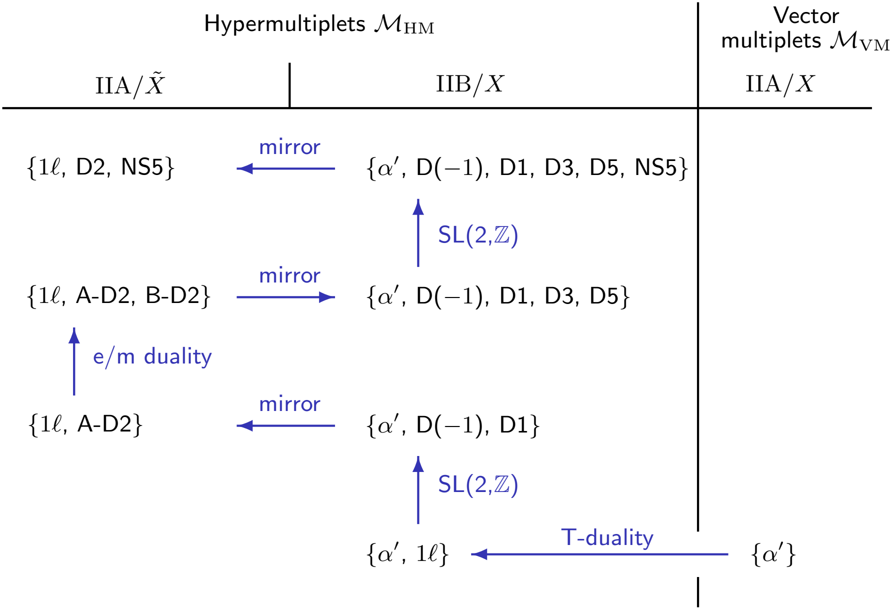
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Outlook

- Inclusion of D3, D5 and NS5-brane instantons
- Orientifolding to obtain N=1 theory *RRSTV '07*
- Instanton corrections to flux-induced scalar potential
  $\Rightarrow$ moduli stabilization, uplift to de Sitter vacua, inflation?
- Dualization of IIA/CY_3 into heterotic string on $K3 \times T^2$