The geometry of Romans mass

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Introduction

String theory:

metric g accompanied by many antisymmetric fields

"RR fluxes" $F_k \in \Lambda^k T^* M_{10}$

IIA: k even
IIB: k odd

In particular, F_0 is called "Romans mass"

[Romans '86; Polchinski '95]

For $F_0 = 0$, strongly coupled IIA develops 11^{th} dimension

[Witten '95]

For $F_0 \neq 0$?

In this talk we study F_0's effects on classical $\mathrm{AdS}_4 \times M_6$ solutions.

Why AdS_4?

- they are easier to find than Minkowski solutions
- they have CFT (holographic) duals

In particular, we will study supersymmetric solutions

With $F_0 = 0$:

All vacua come from
11d supergravity

$$\begin{array}{ccc} S^1 & \hookrightarrow & M_{11} \\ & & \downarrow \\ & & M_{10} \end{array}$$

11d solutions on M_{11}

$\updownarrow$

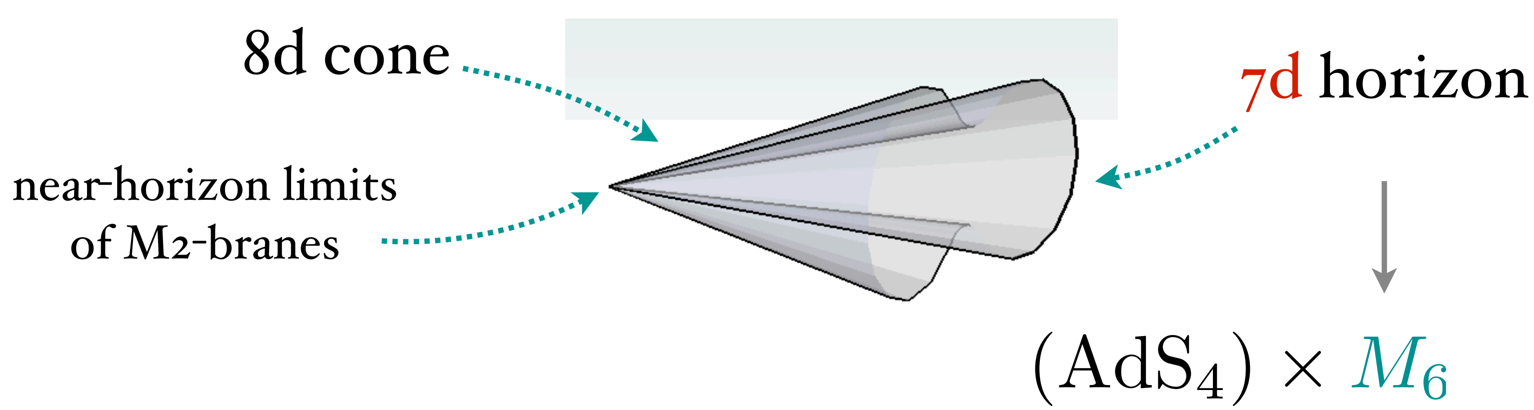
10d solutions on M_{10}

with

- $c_1 = F_2$ (RR 2-flux)
  [Chern class]
- size of $S^1 = e^{\phi}$
  [dilaton]

	8d cone	7d horizon
$\mathcal{N}=3$	hyperKähler	weakly G_2
$\mathcal{N}=2$	Calabi-Yau	Sasaki-Einstein
$\mathcal{N}=1$	Spin(7) hol.	3-Sasakian

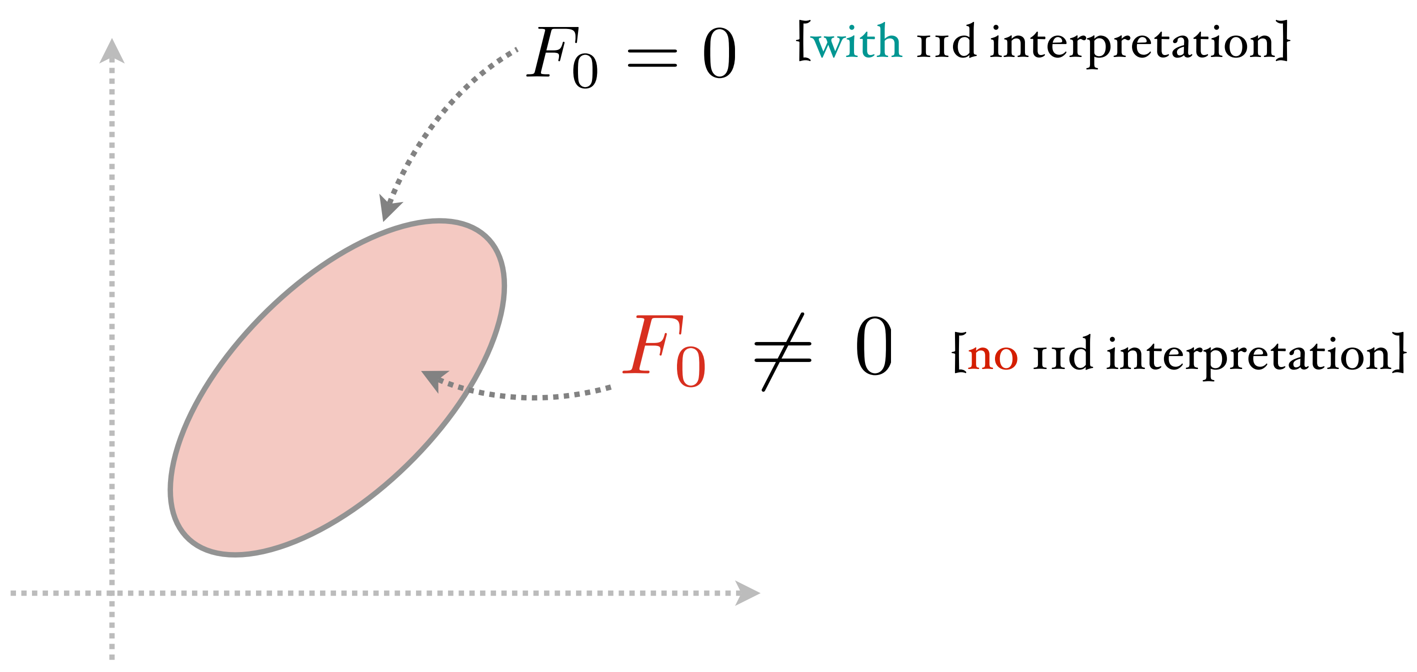
the 11d solutions originate
from "M2" membranes:



With $F_0 \neq 0$: no such a relation to 11d

Recently: introducing F_0 has produced many new solutions

An example of
a phenomenon
we'll see:



Plan

- (Generalized Complex) Geometry of AdS supersymmetric solutions
- An Ansatz: SU(3) structure solutions
- Comments on the general case

Supersymmetry

We look for solutions $\mathrm{AdS}_4 \times M_6$ in type II supergravity

'Vacuum': • metric is 'warped product': $ds^2_{10} = e^{2A} ds^2_{\mathrm{AdS}_4} + ds^2_{M_6}$

• fluxes don't break symmetries: $F = f + \mathrm{vol}_{\mathrm{AdS}_4} \wedge *_6 f$

and closed three-form H

diff. forms on M_6

such that $(d - H\wedge) f = 0\,.$

susy equations contain susy parameters $\epsilon_{1,2}$ ← type II

decompose: $\epsilon_{1,2} = \zeta \otimes \eta_{1,2} + \text{c.c.}$

(ζ: 4d; $\eta_{1,2}$: 6d)

- each η defines almost complex structure: $I^m{}_n = \eta^\dagger \gamma^m{}_n \eta$ $(I^2 = -1)$

- I is hermitian: $gI \equiv J$ is antisymmetric. (g: metric) [U(3) structure on M_6.]

- Chern class $c_1(I) = 0 \;\Leftrightarrow\; \exists\, \Omega_{3,0}$ $\qquad \Omega_{mnp} = \eta^t \gamma_{mnp} \eta$

- they also satisfy $3i\Omega \wedge \bar{\Omega} = 4J^3$. [SU(3) structure on M_6.]

Two $\mathrm{SU}(3)$ structures η_1, η_2 on TM_6

$\Updownarrow$

better to work with:

$\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure on $TM_6 \oplus T^*M_6$

$\parallel$

Two differential forms $\Phi_{1,2}$ obeying algebraic conditions:

[Hitchin '02;Gualtieri '04]

1. each Φ is pure: [$\mathrm{Ann}(\Phi) \subset T \oplus T^*$ has dimension 6]

2. Compatibility:

$$(\Phi_1, X \cdot \Phi_2) = 0 \qquad (\Phi_1, X \cdot \bar{\Phi}_2) = 0 \qquad \forall X \in T \oplus T^*$$

[$X \cdot = \{1-\text{form} \wedge, \text{vector} \llcorner\}$]

The map

$(\eta_1, \eta_2) \mapsto (\Phi_1, \Phi_2)$ is given by

$$\Phi_1 = \eta_1 \otimes \eta_2^\dagger$$

$$\Phi_2 = \eta_1 \otimes \eta_2^t$$

Example:

(' SU(3) structure')

two-form

$$\Phi_1 = e^{-iJ}$$

$$\Phi_2 = \Omega$$

three-form

[when $\eta_1 = \eta_2$]

1. $\mathrm{Ann}(e^{-iJ}) = \{v\llcorner + i(v\llcorner J)\wedge, \forall v \in T\}$ $\quad$ $\mathrm{Ann}(\Omega) = \{T^*_{1,0}\wedge, T_{0,1}\llcorner\}$

2. $(\Omega, \alpha \wedge e^{-iJ}) = -i\,\Omega \wedge v \wedge J = 0\,.$

the susy equations on $\Phi_{1,2}$ now read

[suppressed a few details]

$$\begin{cases} d_H \Phi_1 = -2\mu \,\mathrm{Re}\Phi_2 \\ d_H \mathrm{Im}\Phi_2 = -3\mu \,\mathrm{Im}\Phi_1 + *f \end{cases}$$

[recall: H, f fluxes]

$$d_H = d - H\wedge$$

[Graña,Minasian,Petrini,AT '06]

for some constant $\mu (= -\sqrt{\Lambda_{\mathrm{AdS}_4}}\,)$

SU(3) structure

We now use

$$\Phi_1 = e^{-iJ}$$
$$\Phi_2 = \Omega$$

('SU(3) structure')

in

$$\begin{cases} d_H \Phi_1 = -2\mu\, \mathrm{Re}\Phi_2 \\ d_H \mathrm{Im}\Phi_2 = -3\mu\, \mathrm{Im}\Phi_1 + *f \end{cases}$$

$$H + idJ = 2\mu e^{-i\theta}\mathrm{Re}\Omega$$

$$F_0 = 5\mu \cos(\theta)$$

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Hence $F_0 = 0 \;\Rightarrow\; \theta = \frac{\pi}{2}$

but $F_0 \neq 0$ allows any θ

All the geometric conditions:

$$dJ = \alpha\, \mathrm{Re}\Omega \qquad i\Omega \wedge \bar{\Omega} = \frac{3}{4} J^3$$

$$\Delta \mathrm{Re}\Omega = \beta\, \mathrm{Re}\Omega$$

<u>Example</u>: $\mathrm{AdS}_4 \times S^7$ (round)

$\downarrow$ [Hopf]

$\mathrm{AdS}_4 \times \mathbb{CP}^3$ (Fubini-Study)

[Nilsson,Pope'84; Watamura'84; Sorokin,Tkach,Volkov'85]

we know we should have a IIA solution on this space.

Usually on $\mathbb{CP}^3$: Fubini-Study (Kähler, Einstein) $I_{FS},\ g_{FS}$

It can't be useful for us:

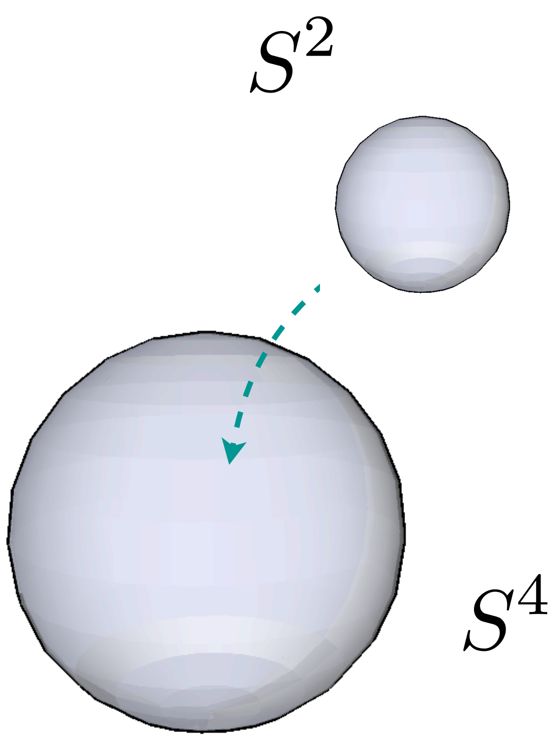
- susy ⇨ $dJ \propto \mathrm{Re}\Omega$ ⇨ I not integrable

  dJ: $(1,1)$; $\mathrm{Re}\Omega$: $(3,0)+(0,3)$

- $I_{\rm FS}$ has no globally defined (3,0)-form: $\nexists\, \Omega_{FS}$

We need a different almost complex structure I.

$\mathbb{CP}^3$ is a sphere fibration.



not

$$I_{\rm FS} = \begin{pmatrix} I_2 & \\ & I_4 \end{pmatrix}$$

but

$$I_{\rm susy} = \begin{pmatrix} -I_2 & \\ & I_4 \end{pmatrix}$$

[Eells,Salamon '85]

integrable

not integrable

$c_1 = 4$

$c_1 = 0$

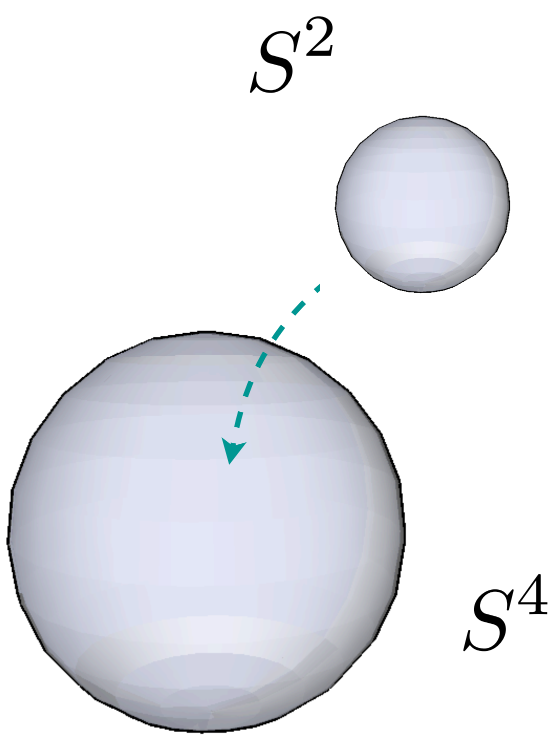
$\Downarrow$

$\Downarrow$

$\nexists\ \Omega_{\rm FS}$

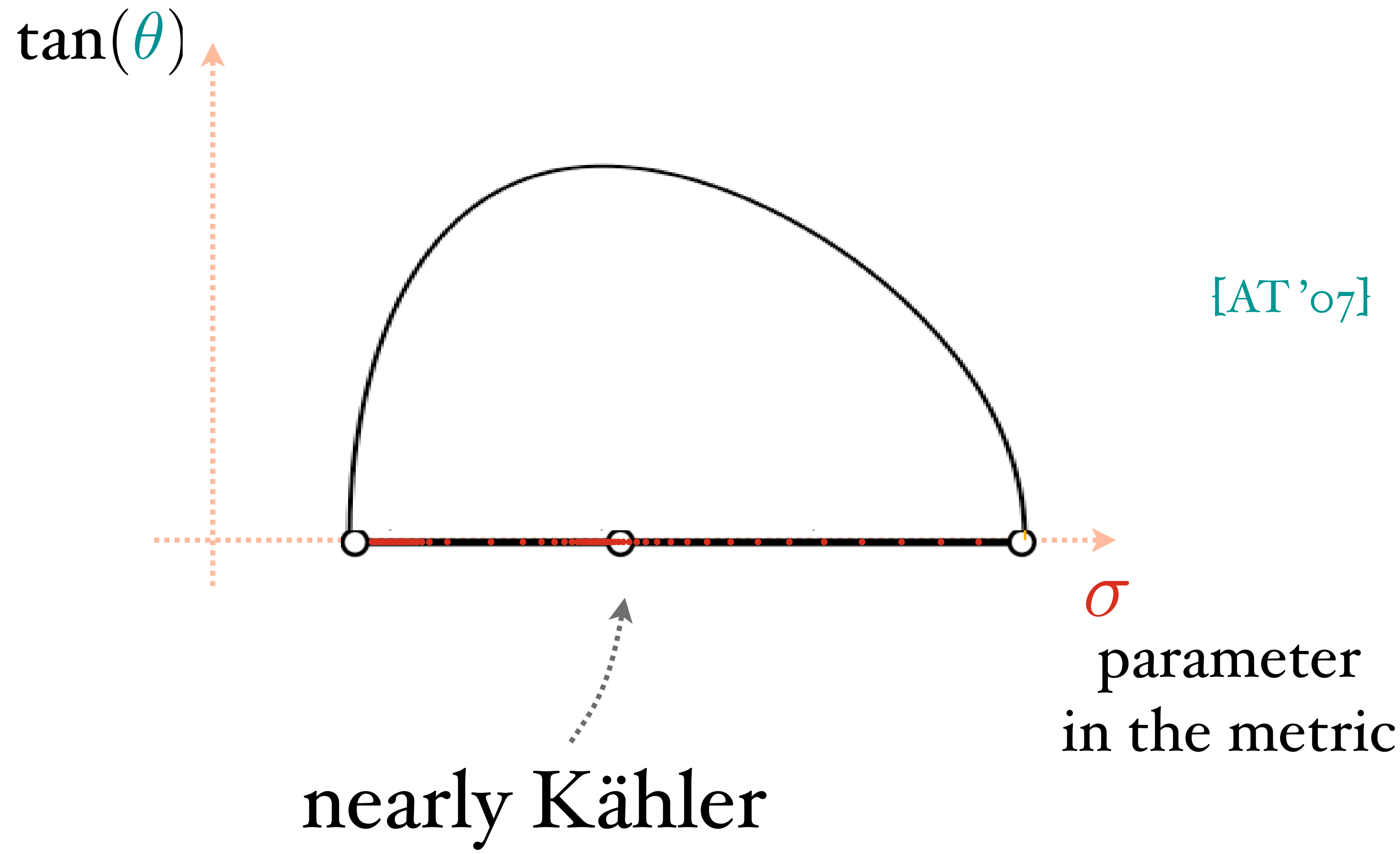
$\exists\ \Omega_{\rm susy}$

$\mathbb{CP}^3$ is a sphere fibration.



supersymmetry equations
boil down to:

$$\tan(\theta) = \frac{\sqrt{(\sigma - \frac{2}{5})(2-\sigma)}}{\sigma + 2}$$

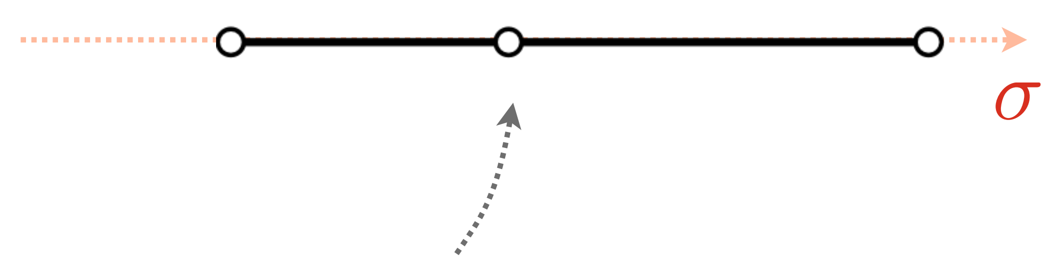


[AT '07]

[Behrnd-Cvetic '04]

nearly Kähler:
$$\begin{cases} dJ = \alpha\,\mathrm{Re}\Omega \\ d\mathrm{Im}\Omega = -\frac{2}{3}\alpha\, J^2 \end{cases}$$

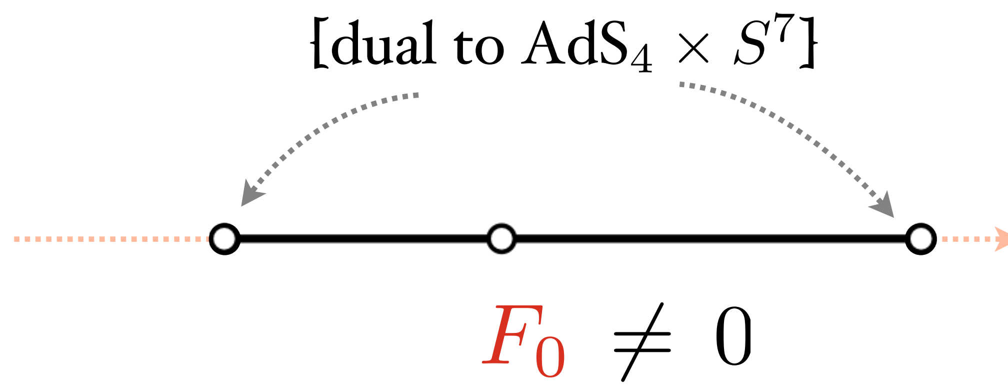
in a sense, this is the closest
one can get to a Calabi-Yau



nearly Kähler

[Behrnd-Cvetic '04]

Bottom line for $\mathrm{AdS}_4 \times \mathbb{C}P^3$:



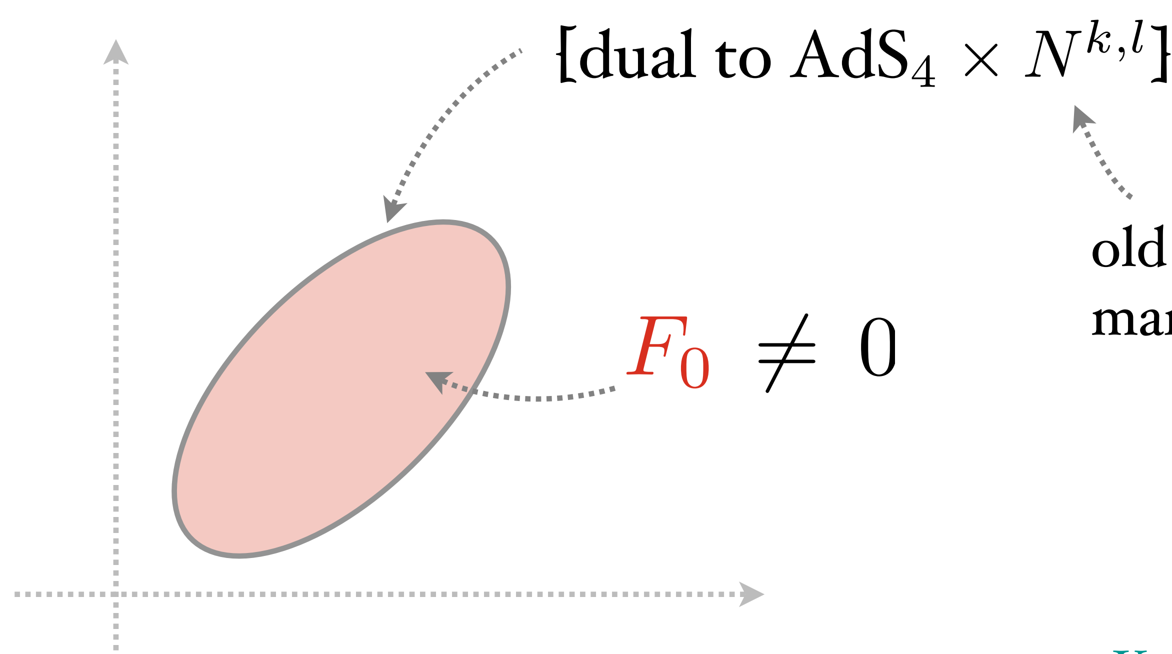
[AT '07]

Another example:

$\mathrm{AdS}_4 \times \mathbb{F}(1,2;3)$:

"flag manifold"

two relevant parameters in the metric



old 3-Sasakian manifold: $\mathrm{SU}(3)/\mathrm{U}(1)$

[AT '07; Koerber,Lüst,Tsimpis '08]

Open question: Does this happen for all 3-Sasakian manifolds?

The general case

Supersymmetry: $$\begin{cases} d_H \Phi_1 = -2\mu\, \mathrm{Re}\Phi_2 \\ d_H \mathrm{Im}\Phi_2 = -3\mu\, \mathrm{Im}\Phi_1 + *f \end{cases}$$

The SU(3) structure Ansatz $$\Phi_1 = e^{i\theta} e^{-iJ} \qquad \Phi_2 = \Omega$$ is not exhaustive.

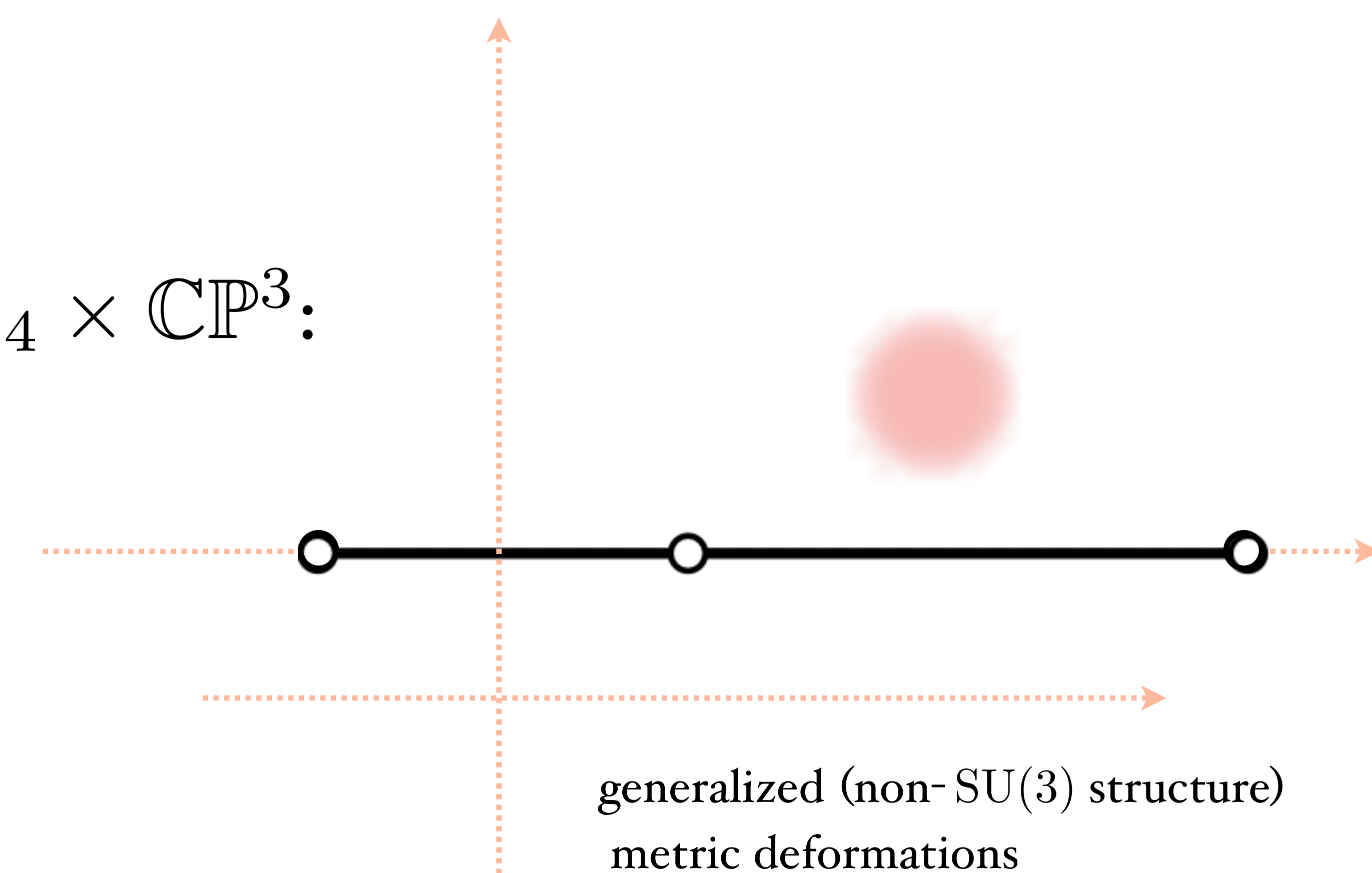
For example: $F_0 \neq 0$ & extended (i.e. $\mathcal{N} > 1$) susy $\Rightarrow$ not SU(3)

⇨ we should also consider general Φ_1, Φ_2 ($SU(3) \times SU(3)$ structure)

General solution to algebraic conditions is known;
but differential equations are hard

<u>Example</u>:

again on $AdS_4 \times \mathbb{CP}^3$:



[Gaiotto,AT '09]

so far, only
perturbative analysis

- all-orders (numerical) $\mathcal{N} = 2$ solution

on $\mathrm{AdS}_4 \times M_{111}$

$$\begin{array}{ccc} S^2 & \hookrightarrow & M_{111} \\ & & \downarrow \\ & & \mathbb{CP}^2 \end{array}$$

[Petrini,Zaffaroni '09]

- in turn, generalized to $\mathrm{AdS}_4 \times Y_6$

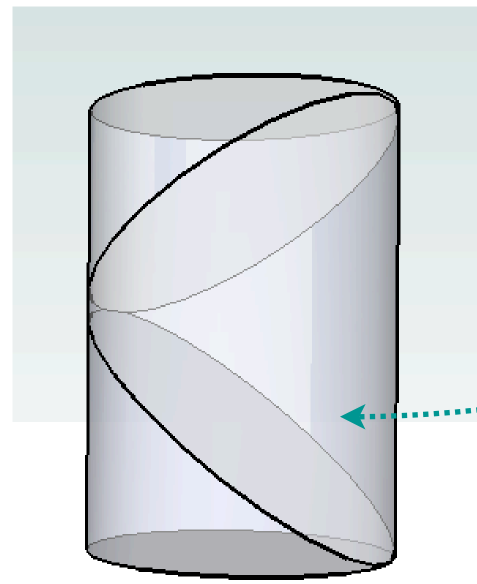
$$\begin{array}{ccc} \mathrm{SE}_5 & \hookrightarrow & Y_6 \\ & & \downarrow \\ & & I \end{array}$$

[Lüst, Tsimpis '09]

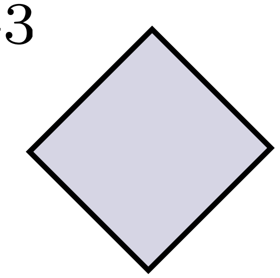
Fortunately, there is an argument
that predicts many such solutions!

In string theory,

$\mathrm{AdS}_4 \times M_6$ solutions are dual to



conformal field theories in Minkowski$_3$



It has been found recently that these CFTs are

[Aharony, Bergman, Jafferis, Maldacena '08]

Chern-Simons theories with $G_{\text{gauge}} = \Pi_{i=1}^{n} \mathrm{U}(N_i)$

[+ various matter fields]

But: $F_0 = \sum_{i=1}^{n} k_i$

[Gaiotto,AT '09; Fujita,Li,Ryu,Takayanagi '09]

In most cases, $F_0 \neq 0$ does not seem to destroy the CFT.

Conclusions

- F_0 enhances the supersymmetric geometry of IIA sugra.

- Some examples explicitly known.

- AdS/CFT seems to predict rich many new examples.