

TD 2 TMB :

Exo 1:

$$a) \begin{cases} 2x_1 + x_2 = 1 & \textcircled{1} \\ -x_1 + x_3 = -1 & \textcircled{2} \\ x_1 + x_2 + x_3 = 0 & \textcircled{3} \end{cases} \Leftrightarrow \begin{cases} 2x_1 + x_2 = 1 & \textcircled{1} \\ \frac{1}{2}x_2 + x_3 = -\frac{1}{2} & \textcircled{2} + \frac{1}{2}\textcircled{1} \\ \frac{1}{2}x_2 + x_3 = -\frac{1}{2} & \textcircled{3} - \frac{1}{2}\textcircled{1} \end{cases} \Leftrightarrow \begin{cases} 2x_1 + x_2 - 1 = 1 \\ \frac{1}{2}x_2 + x_3 = -\frac{1}{2} \\ 0 = 0 \end{cases}$$

Le rang du système est 2, et

$$S = \left\{ \left(\frac{1-x_2}{2}, x_2, -\frac{1}{2} - \frac{1}{2}x_2 \right) / x_2 \in \mathbb{R} \right\}$$

$$= \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix} + \mathbb{R} \begin{pmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{pmatrix}$$

$$b) \begin{cases} 2x_1 + x_2 = 2 & \text{la même} \\ -x_1 + x_3 = -1 \\ x_1 + x_2 + x_3 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x_1 + x_2 = 2 \\ -\frac{1}{2}x_2 + x_3 = 0 \\ \frac{1}{2}x_2 + x_3 = -1 \end{cases} \quad \text{impossible}$$

rg = 2, mais pas de solutions.

Exo 2:

$$\begin{cases} -3x_1 - 2x_2 + 2x_3 = a \\ -2x_1 - x_2 + x_3 = b \\ 7x_1 + 5x_2 - 4x_3 = c \end{cases} \Leftrightarrow \begin{cases} -3x_1 - 2x_2 + 2x_3 = a & \textcircled{1} \\ x_1 - x_3 = 3b - 2a & \textcircled{2} - \frac{2}{3}\textcircled{1} \times 3 \\ x_2 + 2x_3 = 3c + 7a & \textcircled{3} + \frac{7}{3}\textcircled{1} \times 3 \end{cases}$$

$$\Leftrightarrow \begin{cases} -3x_1 - 2x_2 + 2x_3 = a \\ x_2 - x_3 = 3b - 2a \\ x_3 = 3a - b + c & \textcircled{3} - \textcircled{2} \end{cases} \Leftrightarrow \begin{cases} -3x_1 - 2x_2 = -5a + 1b - 1c & \textcircled{1} - 2\textcircled{2} \\ x_2 = a + 2b + c & \textcircled{2} + \textcircled{3} \\ x_3 = 3a - b + c \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = a - 2b \\ x_2 = a + 2b + c \\ x_3 = 3a - b + c \end{cases} \quad \begin{matrix} \textcircled{1} + 2\textcircled{2} \\ -3 \\ \text{rg} = 3 \\ \text{1 produit} \end{matrix} \quad \begin{matrix} \textcircled{a} & x_1 = 1, x_2 = 1, x_3 = 3 \\ \textcircled{b} & x_1 = -4, x_2 = 1, x_3 = -5 \\ \textcircled{c} & x_1 = -1, x_2 = 4, x_3 = 3 \end{matrix}$$

Exo 3:

$$a) \begin{cases} 2x_1 + 3x_2 + x_3 = 3 \\ x_1 + 4x_2 + 3x_3 = -1 \\ -3x_1 - 2x_2 + x_3 = -7 \\ x_1 - 4x_2 - 5x_3 = 7 \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ x_1 + x_3 = -1 \\ \frac{5}{2}x_2 + \frac{5}{2}x_3 = \frac{5}{2} \\ \frac{11}{2}x_2 - \frac{11}{2}x_3 = \frac{11}{2} \end{cases} \Leftrightarrow \begin{cases} x_1 + 8x_2 = 2 \\ x_2 + x_3 = -1 \end{cases}$$

~~rg~~ $rg = 2, S = \{(2 - x_2, x_2, -1 - x_2) \mid x_2 \in \mathbb{R}\}$

$$= \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \mathbb{R} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

$$b) \begin{cases} 7x_1 + 2x_2 - 4x_3 = 2 \\ -2x_1 + 3x_2 + 4x_3 = 3 \\ -3x_1 - 3x_2 + 6x_3 = -3 \\ x_1 + 2x_2 - 2x_3 = 2 \end{cases} \Leftrightarrow \begin{cases} \cancel{x_1 - 2x_3} = 0 \\ 8x_2 = 1 \quad \text{①} \\ \cancel{-3x_1 - 2x_2} = 0 \quad \times \frac{1}{3} \\ \cancel{x_1 - 2x_3} = 0 \end{cases} \Leftrightarrow \begin{cases} x_2 = 1 \\ x_1 = 2x_3 \end{cases}$$

$rg = 2, S = \{(2x_3, 1, x_3) \mid x_3 \in \mathbb{R}\}$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mathbb{R} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

Exo 4:

$$\begin{cases} x_1 + 4x_2 + x_3 = a \\ 5x_1 + 9x_2 + 2x_3 = b \\ 6x_1 + 5x_2 + 3x_3 = b \\ 5x_1 + 8x_2 + 3x_3 = c \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ -11x_2 - 3x_3 = b - 5a \\ -19x_2 - 3x_3 = b - 6a \\ -12x_2 + 4x_3 = c - 5a \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ 8x_2 = -\frac{a}{8} \\ 4x_3 = \frac{c - 5a - 3}{8} \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = a + \frac{a}{2} - \frac{1}{4}(c - 5a - \frac{3}{2}) = \frac{11}{4}a - \frac{1}{4}c - \frac{3}{8} \\ 5x_1 = b + \frac{9a}{8} - \frac{1}{2}(c - 5a - \frac{3}{2}) = -\frac{11}{8}a + b - \frac{1}{2}c + \frac{3}{4} \\ x_2 = -\frac{a}{8} \\ x_3 = \frac{1}{4}(c - 5a - \frac{3}{2}) \end{cases}$$

① $x_2 = \frac{17}{8} = -\frac{1}{40}$ pas de sol
 ② 1 seule solution:
 $(1, 0, -1)$

Exo 5:

$$\begin{cases} 2x_1 + x_2 + x_3 - 3x_4 = a \\ 4x_1 + 2x_2 - 4x_3 + 6x_4 = b \\ -2x_1 + x_2 + x_3 + x_4 = c \\ 6x_1 + 5x_2 - x_3 + x_4 = d \end{cases} \Rightarrow \begin{cases} -6x_3 + 12x_4 = b - 2a \\ 2x_2 + 2x_3 - 2x_4 = c + a \\ 2x_2 - 4x_3 - 8x_4 = d - 3a \end{cases} \Rightarrow \begin{cases} 2x_2 + 2x_3 - 2x_4 = c + a \\ -6x_3 + 12x_4 = b - 2a \\ -6x_3 - 6x_4 = d - c + 4a \end{cases}$$

$$\Leftrightarrow \begin{cases} \dots \\ \dots \\ \dots \\ -18x_4 = -2a - b - c + d \end{cases} \Rightarrow \text{rg} = 4, \text{ 1 seule solution}$$

a) $18x_4 = 0 \Rightarrow x_4 = 0$

$$\leadsto \begin{cases} 2x_1 = 1 \\ 2x_2 = -2 \\ 4x_3 = -1 \end{cases}$$

$$S = \{(1, -2, -1, 0)\}$$

b) $-18x_4 = -3 \Rightarrow x_4 = \frac{1}{6}$

$$\begin{cases} 2x_1 + \frac{11}{6} + \frac{2}{6} - \frac{3}{6} = 1 \Rightarrow x_1 = -\frac{1}{3} \\ 2x_2 + \frac{2}{3} - \frac{1}{3} = 4 \Rightarrow x_2 = \frac{11}{6} \\ -6x_3 - 1 = -3 \Rightarrow x_3 = \frac{1}{3} \end{cases}$$

$$S = \left\{ \left(-\frac{1}{3}, \frac{11}{6}, \frac{1}{3}, \frac{1}{6} \right) \right\}$$

Exo 6:

1) $\begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

2) $\begin{pmatrix} -3 & -2 & 2 \\ -2 & -1 & 1 \\ 7 & 5 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

3) $\begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 3 \\ -3 & 2 & 1 \\ 1 & -4 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -7 \\ 7 \end{pmatrix}$

4) $\begin{pmatrix} 1 & 4 & 1 \\ 5 & 9 & 2 \\ 6 & 5 & 3 \\ 5 & 8 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

5) $\begin{pmatrix} 2 & 1 & 1 & -3 \\ 4 & 2 & 4 & 6 \\ -2 & 1 & 1 & 1 \\ 6 & 5 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -5 \\ -3 \end{pmatrix}$

