Quantum error correction and large N

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Historial references and motivation

- Quantum error correction seem to be tightly related with holography

Almheiri, Dong, Harlow'14

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- Scar states from singlets

  Pakrouski, Pallegar, Popov, Klebanov'20

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- No locality: stringy geometry

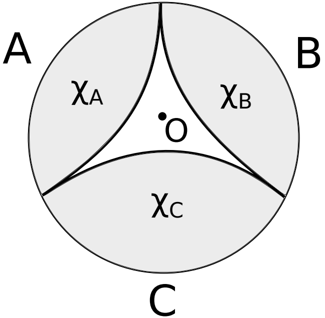
Berenestein'04; Itzhaki, McGreevy'04; ...

How robust are singlet states in these models against errors, such as erasures?

Outline

Consider HKLL and causal wedge reconstruction.

Almheiri, Dong, Harlow '15



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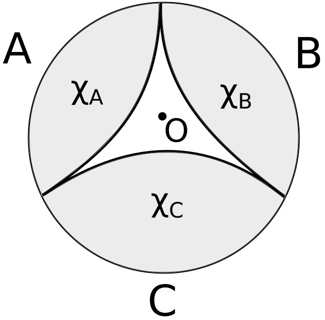

Reconstruction of O requires AB or AC or BC. No need for the whole ABC.

Interpretation: quantum error correction.
Holographic states are robust: loss of A does not prevent us from reconstructing O.

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Comment: Gauge symmetry vs global symmetry: singlet errors vs arbitrary errors?

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- Hence $|\xi_{1,2}\rangle$ are orthogonal in the large N limit.

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1. Bound on matrix elements of singlet operators
2. State orthogonality and quantum error correction
3. What if we act with too many operators/have too many errors?

Planar limit

- Start from two Majoranas ψ^1, ψ^2 and from creation/annihilation operators

$$a_{ij} = \psi^1_{ij} + i\psi^2_{ij}, \quad a^\dagger_{ij} = \psi^1_{ij} - i\psi^2_{ij}$$

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$$\langle s|\mathrm{Tr}\psi^m|s\rangle \sim N^{m/2+1}$$

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Unfortunately, m_i has to be parametrically smaller than N

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4. Repeat to get $\mathrm{Tr}\left(\psi^1\psi^1\psi^1\psi^1\right)$

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Bounds from representation theory

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$$\left.\begin{aligned} &|\langle s|\mathcal{O}_{3k}|s\rangle| \leq 2^{k/2}N^{5k/2} \\ &|\langle s|\mathcal{O}_{3k+1}|s\rangle| \leq \sqrt{2}N \times 2^{k/2}N^{5k/2} \\ &|\langle s|\mathcal{O}_{3k+2}|s\rangle| \leq 2N^2 \times 2^{k/2}N^{5k/2} \\ &1 \leq k \leq N^{1/4}/3 \end{aligned}\right\} \mathcal{O}_m \sim N^{9m/10}$$

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For $\mathcal{O}_3, \mathcal{O}_4$ coincides with the answer in the planar limit, otherwise gives much bigger value

How do we describe errors

Unitarity evolution U:

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Example: spin flip with probability p:

$$E_1 = \sqrt{1-p}1$$

$$E_2 = \sqrt{p}X$$

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General quantum operations are not always invertible. $\{E_\alpha\}$ is invertible for ρ if there is another $\{R_\alpha\}$ such that

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Approximate quantum error correction:

$$\rho \to \mathcal{E}(\rho) = \sum_\alpha E_\alpha^\dagger \rho E_\alpha$$

$$\mathcal{E}(\rho) \to \mathcal{R}(\mathcal{E}(\rho)) = \sum_\alpha R_\alpha^\dagger \mathcal{E}(\rho) R_\alpha$$

$$||\rho - \mathcal{R}(\mathcal{E}(\rho))|| \leq \epsilon$$

What does erasure mean?

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Example: one qubit

$$\rho \to \frac{1}{4}(1\rho 1 + X\rho X + Y\rho Y + Z\rho Z)$$

Knill–Laflamme condition

$\{E_\alpha\}$ *is exactly correctable iff*

$$P_{\mathrm{code}} E_\alpha^\dagger E_\beta P_{\mathrm{code}} = N_{\alpha\beta} 1 P_{\mathrm{code}}$$

P_{code} projector on code subspace.
$N_{\alpha\beta}$ - hermitian matrix

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Requires $\sqrt{\langle s_1|E_\alpha^\dagger E_\beta|s_2\rangle} \to$ bound norms/eigenvalues

Matrix models: errors E_α and field content

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Normalization:

$$\psi^a_{(ij)}\psi^b_{(kl)} + \psi^b_{(kl)}\psi^a_{(ij)} = 2 \times 1 \times \delta^a_b \times \delta^{(ij)}_{(kl)}$$

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- Suppose $E^\dagger_\alpha E_\beta$ has k fermions:

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Conditions on error correction

- Problem 1(bilinear problem):

$$\psi^1_{(ij)}\psi^2_{(ij)} \propto \frac{1}{N^2}\mathrm{Tr}\left(\psi^1\psi^2\right)\mathrm{Tr}\left(T_{(ij)}T_{(ij)}\right) = \mathcal{O}\left(N^0\right)$$

- Can not distinguish them. Blunt solution: avoid them.
- Problem 2: $\mathcal{O}_{\alpha\beta}$ might contain a lot($\sim N$) of operators.
- Suppose $E^\dagger_\alpha E_\beta$ has k fermions: suppressed at least by

$$\sim N^{k/10}$$

- Total number of different singlet operators $\sim k!$

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(probably could be improved)

Carrozza–Tanasa–Klebanov–Tarnopolsky(CTKT) tensor model

Field content: Majorana ψ_{abc} in the fundamental of $O(N)^3$

Gurau'11; Witten'16; Carrozza, Tanasa'15; Klebanov, Tarnopolsky'16

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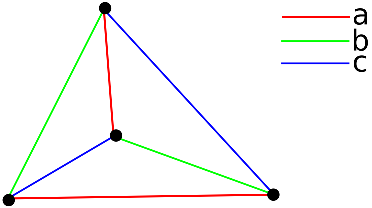

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$$H_{\mathrm{GW}} = J \sum_{abca'b'c'} \psi^1_{abc}\psi^2_{ab'c'}\psi^3_{a'bc'}\psi^4_{a'b'c}$$

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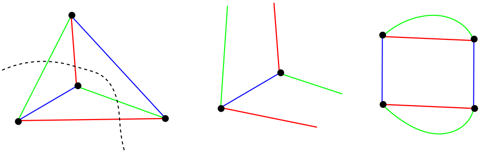

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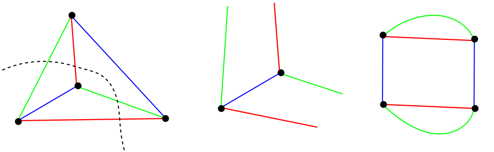

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Last term is Casimir for $O(N^2)$(sic!): $\psi_{abc} \to \psi_{Ac}$ with $O(N^2) \times O(N)$

$$(c) \leq N^5$$

General bound

$\mathcal{O}_{2k}$ build from $2k \leq N^{3/4}$ fermions:

$$|\langle s|\mathcal{O}_{2k}|s\rangle| \leq N^{5k/2} \times \begin{cases} 1, & \text{k even} \\ \frac{1}{\sqrt{N}}, & \text{k odd} \end{cases}$$

This result depends on field content only(Hilbert space).
Holds for any singlet $|s\rangle$

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- Correct any erasure of size $|E| \leq N^{1/6}$ at known locations:

$$||\rho - \mathcal{R}(\mathcal{E}(\rho))|| \lesssim \frac{|E|^5}{N^2}$$

Summary

- For matrix and tensor models, large N and singlet condition automatically imply correction of certain erasures.

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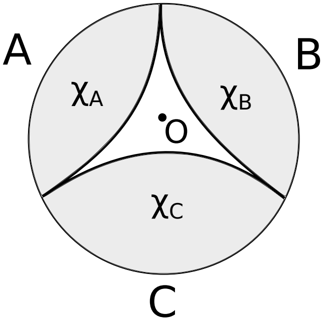
- As long as erasures are not too large.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Thank you!

Extra slides

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$$\langle \mathrm{Tr} \left(\chi^k \right) \rangle \sim N$$

But $\chi \sim \sqrt{N} \psi$, so

$$\langle \mathrm{Tr} \left(\psi^k \right) \rangle \sim N^{1+k/2}$$

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Conjecture: non-singlets are gapped.
Consistent with Monte–Carlo data.

Berkowitz, Hanada, Rinaldi, Vranas'17

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