Remarks on a melonic field theory with cubic interactions

(based on [arXiv:2012.12238] with D. Benedetti)

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Nicolas Delporte

Okinawa Institute of Science and Technology

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[Sachdev-Ye-Kitaev '15]: quartic Fermionic with disorder.
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$\{\phi_m \in \mathbb{R} : 1 \leq m \leq N\}$, irrep of $SO(3)$ of dimension $N = 2j+1$:

$$S = \int d^d x \, \left(\frac{1}{2} \sum_m \left(\phi^m (-\partial^2)^\zeta \phi_m + \lambda_2 \, \phi_m \phi^m \right) + \sum_{m_1, m_2, m_3} \frac{\lambda}{3!} \sqrt{2j+1} \begin{pmatrix} j & j & j \\ m_1 & m_2 & m_3 \end{pmatrix} \phi^{m_1} \phi^{m_2} \phi^{m_3} \right) .$$

NB: $j \in 2\mathbb{N}$, λ can be imaginary (cf. Lee-Yang model).

$$g_j^{mm'} = g^j_{mm'} \equiv \sqrt{2j+1} \begin{pmatrix} j & j & 0 \\ m & m' & 0 \end{pmatrix} = (-1)^{j-m} \delta_{m,-m'}$$

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Goals: Renormalization group fixed points, associated CFTs.

Long-range (LR) parameter $0 < \zeta \leq 1$:

$$C(p) = \frac{1}{p^{2\zeta}}, \quad C(x,y) \sim \frac{1}{|x-y|^{2\Delta_\phi^{(0)}}}, \quad \Delta_\phi^{(0)} = \frac{d-2\zeta}{2}.$$

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Long-range melonic CFTs can have real conformal data at large-N [Benedetti et al. '19, '20] (quartic $O(N)^3$, $\zeta = d/4$) and also real couplings [Benedetti et al. '19] (sextic $U(N)^3$, $\zeta = d/3$).

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Regularizations:

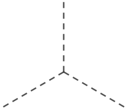
$$d = 6\zeta - \epsilon \quad \text{or} \quad \zeta = \frac{d+\epsilon}{6} .$$

Amplitude of a Feynman graph γ:

$$\mathcal{A}_\gamma = c_\gamma \left(\frac{\lambda}{3!} \sqrt{N} \right)^{v(\gamma)} I_\gamma \, A_\gamma \, .$$

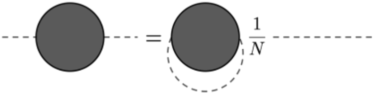
Feynman rules ($SO(3)$ index):

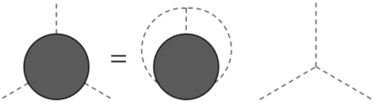
$\delta_{m,m'}$



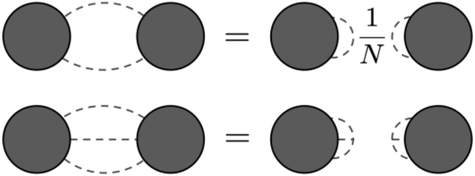
$\lambda (3jm) \sqrt{N}$

$SU(2)$ recoupling properties:

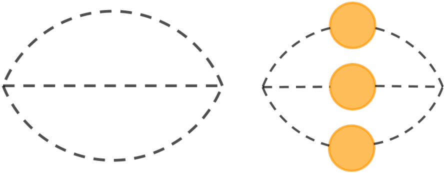




2- and 3-Particle Reducible (PR) diagrams:



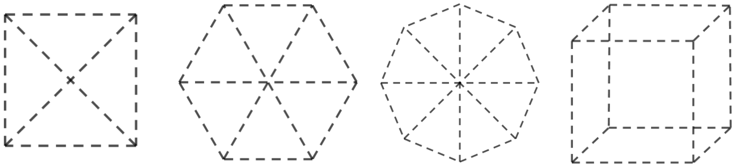
Fully-2PR diagrams (primary and with melonic insertions):



Diagrammatics

Decomposition into irreducible $\{3n_i j\}$ components

$$A_\gamma = N^{-n_0} \prod_{i=1}^{k} A_{\{3n_i j\}} \,, \qquad n = 1 + n_0 - k + \sum_{i=1}^{k} n_i \,,$$



$$\{6j\}, \{9j\}, \{12j - I\}, \{12j - II\}.$$

[Amit and Roginsky '79] showed an asymptotic bound

$$|\{3nj\}| \lesssim N^{-n+1-\alpha'} \,,$$

for $N = (2j+1)$ large, with $\alpha' > 0$.

At leading order in $1/N$:

$$G(p)^{-1} = Z\left(p^{2\zeta} + \lambda_2\right) - \frac{\lambda^2}{2}\int_q G(q)G(p+q)\,, \quad G(p) = p^{-d/3}\mathcal{Z}_{\mathrm{S/LR}}\,.$$

Assume $\lambda_2 = 0$.
Z: wave function renormalization (finite in the long-range variant).
$(\zeta = 1)$ IR limit, p^2 term is negligible wrt $p^{d/3}$

$$\mathcal{Z}_{\mathrm{SR}} = \left(\frac{d(4\pi)^{d/2}}{3\lambda^2}\frac{\Gamma(d/6)^2\Gamma(2d/3)}{\Gamma(d/3)^2\Gamma(1-d/6)}\right)^{\frac{1}{3}}\,.$$

$(\zeta = d/6)$ The SDE reduces to

$$1 = \mathcal{Z}_{\mathrm{LR}} + \underbrace{\frac{3\lambda^2}{d(4\pi)^{d/2}}\frac{\Gamma(d/3)^2\Gamma(1-d/6)}{\Gamma(d/6)^2\Gamma(2d/3)}}_{a(\lambda)}\mathcal{Z}_{\mathrm{LR}}^3\,,$$

with explicit solution in terms of $a(\lambda)$

$$\mathcal{Z}_{\mathrm{LR}} = \frac{1}{6^{2/3}}\left(\sqrt{3}\sqrt{a^3(27a+4)} + 9a^2\right)^{\frac{1}{3}}\left(\frac{2^{1/3}}{a} - \frac{2\times 3^{1/3}}{\left(\sqrt{3}\sqrt{a^3(27a+4)} + 9a^2\right)^{2/3}}\right)\,.$$

$\mathcal{Z}_{\rm LR}$ generating function of 3-Catalan numbers:

$$\mathcal{Z}_{\rm LR}(a) = \sum_n \frac{1}{3n+1}\binom{3n+1}{n}(-a)^n \,.$$

If $\lambda^2 > 0$ and $d < 6$:

$$a > 0\,, \quad \mathcal{Z}_{\rm LR}(a) > 0\,, \quad \lim_{a\to+\infty} \mathcal{Z}_{\rm LR}(a) = \mathcal{Z}_{\rm SR}(= a^{-1/3})\,.$$

If $\lambda^2 < 0$ and $d < 6$:

$$a < 0\,, \quad \lim_{a\to -4/27^+} \mathcal{Z}_{\rm LR}(a) = 3/2\,,$$

analogous to LR quartic $O(N)^3$ model [Benedetti et al. '19].
In terms of rescaled coupling $g^2 := \lambda^2 \mathcal{Z}_{\rm LR}^3$

$$-\frac{1}{2}g_c^2 < g^2 < g_c^2 = d(4\pi)^{d/2}\frac{\Gamma(d/6)^2\Gamma(2d/3)}{3\Gamma(d/3)^2\Gamma(1-d/6)}\,.$$

$(\zeta = 1)$ Introduce renormalized coupling and renormalization condition:

$$g := \mu^{-\epsilon/2}\lambda Z^{3/2}, \quad \lim_{\epsilon\to 0}\frac{d\Gamma^{(2)}(p)}{dp^2}|_{p^2=\mu^2} = 1, \quad \Gamma^{(2)}(p) = G(p)^{-1}.$$

At leading order, no vertex correction:

$$\beta(g) = \mu\frac{\partial g}{\partial\mu} = \frac{g}{2}(-\epsilon + 3\eta(g)), \quad \eta(g) = \mu\partial_\mu \ln Z.$$

Fixed point imposes $\eta(g^\star) = \epsilon/3$, or

$$g^\star_\pm = \pm 8\sqrt{2\pi^3\epsilon} + \mathcal{O}\left(\epsilon^{3/2}\right),$$
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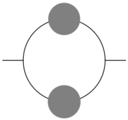

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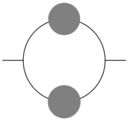


$(\zeta = d/6)$

$$\beta(g) = -\epsilon\frac{g}{2}\,,$$

at $\epsilon = 0$, we have a line of IR fixed points.

$(\zeta = 1)$ [Amit, de Alcantara Bonfim et al., Gracey et al., Bellon et al.] considered general cubic potential

$$d_{m_1 m_2 m_3} \phi^{m_1} \phi^{m_2} \phi^{m_3} \ ,$$

assuming single rank-3 invariant, and computed finite-N beta functions at fixed number of loops, e.g. two-loops:

$$\begin{aligned}\beta(\bar{g}) = &- \frac{\epsilon}{2}\bar{g} + \left(\frac{T_2}{4} - T_3\right)\bar{g}^3 \\ &+ \frac{1}{144}\left(-11\,T_2^2 + 66\,T_2\,T_3 - 108\,T_3^2 - 72\,T_5\right)\bar{g}^5 + \mathcal{O}(\bar{g}^7)\,,\end{aligned}$$

$$T_2 = 1\,, \quad T_3 = (2j+1)\{6j\} \approx N^{-1/2}\,, \quad T_5 = (2j+1)^2\{9j\} \lesssim N^{-1/2}\,,$$
$$\bar{g}^2 = g^2 S_d/(2\pi)^d\ , \quad S_d = 2\pi^{d/2}/\Gamma(d/2)\ .$$

At large-N and four-loops [Gracey '15]:

$$\beta(\bar{g}) = -\frac{\epsilon}{2}\bar{g} + \frac{1}{4}\bar{g}^3 - \frac{11}{144}\bar{g}^5 + \frac{821}{20736}\bar{g}^7 - \frac{20547}{746496}\bar{g}^9 + \mathcal{O}(\bar{g}^{11}) \,,$$

$$\bar{g}^\star_\pm = \pm\left(\sqrt{2\epsilon} + \frac{11\,\epsilon^{3/2}}{18\sqrt{2}} + \frac{13\,\epsilon^{5/2}}{648\sqrt{2}} + \frac{623\epsilon^{7/2}}{7776\sqrt{2}}\right) + \mathcal{O}\left(\epsilon^{9/2}\right) \,,$$

$$\omega = \beta'(\bar{g}^\star_\pm) = \epsilon - \frac{11\,\epsilon^2}{18} + \frac{337\,\epsilon^3}{648} - \frac{16013\epsilon^4}{23328} + \mathcal{O}\left(\epsilon^5\right) \,.$$

At finite-N and two-loops:

$$\bar{g}^\star_\pm = \pm\left(\frac{2\epsilon}{T_2 - 4\,T_3}\right)^{1/2} + \mathcal{O}(\epsilon) \,,$$

$$\omega \simeq \epsilon\left(1 + 6\frac{2^{5/4}}{\sqrt{\pi N}}\cos\left(3N\arccos\left(-1/3\right) + \frac{\pi}{4}\right) + \mathcal{O}(N^{-3/2})\right) + \mathcal{O}\left(\epsilon^2\right) \,.$$

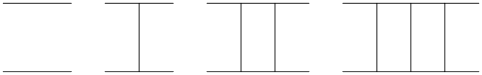
Beta functions: higher loops and finite-N

$(\zeta = 1)$ Real IR fixed point is preserved (except $N = 13$ that needs $g = \mathrm{i}\,|g|$).
NB: In the quartic $O(N)^3$ at short-range, $\sqrt{\epsilon}N \gg 1$ was needed [Benedetti et al. '20].

$(\zeta = d/6)$ $\epsilon\sqrt{N} \ll 1$ is needed, as in the quartic $O(N)^3$ at long-range [Benedetti et al. '20].

4-point function in $(12 \to 34)$ channel:

$$\mathcal{F}_{m_1,m_2;m_3,m_4}(x_1,x_2;x_3,x_4) = \langle \phi_{m_1}(x_1)\phi_{m_2}(x_2)\phi_{m_3}(x_3)\phi_{m_4}(x_4)\rangle - \langle \phi_{m_1}(x_1)\phi_{m_2}(x_2)\rangle \langle \phi_{m_3}(x_3)\phi_{m_4}(x_4)\rangle$$



From the 2PI formalism (e.g. [Benedetti '20]):

$$\mathcal{F}_{m_1\, m_2;m_3\, m_4}(x_1,x_2;x_3,x_4) = \int_{uv} (1-K)^{-1}_{m_1\, m_2;a\, b}(x_1,x_2;u,v) \times \left(G_{am_3}(u,x_3)G_{bm_4}(v,x_4) + G_{am_4}(u,x_4)G_{bm_3}(v,x_3)\right),$$

$$G_{ab}(x,y) = \langle \phi_a(x)\phi_b(y)\rangle \ ,$$

$$K_{m_1\, m_2;m_3\, m_4}(x_1,x_2;x_3,x_4) = \int_{uv} G_{m_1 a}(x_1,u)G_{m_2 b}(x_2,v)\frac{\delta^2\Gamma^{2PI}_{\text{loops}}}{\delta G_{ab}(u,v)\delta G_{m_3 m_4}(x_3,x_4)} \, .$$

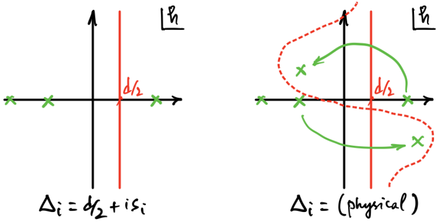
Conformal partial wave expansion [Simmons-Duffin et al. '17]:

$$\sum_{m,m'} \mathcal{F}_{m}{}^{m}{}_{;m'}{}^{m'}(x_1, x_2; x_3, x_4) = N \sum_{J \geq 0} \int_{\frac{d}{2} - \mathrm{i}\,\infty}^{\frac{d}{2} + \mathrm{i}\,\infty} \frac{dh}{2\pi\,\mathrm{i}} \, \frac{1}{1 - k(h, J)} \, \mu_{h,J}^{\Delta_\phi} \mathcal{G}_{h,J}^{\Delta_\phi}(x_i)$$
$$= N \sum_{n,J} c_{n,J}^2 \, \mathcal{G}_{h_{n,J},J}^{\Delta_\phi}(x_i)\,.$$

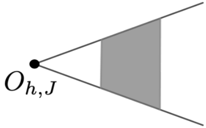
OPE coefficients:

$$c_{n,J}^2 = -\mu_{h_{n,J},J}^{\Delta_\phi} \mathrm{Res}\left[\frac{1}{1 - k(h, J)}\right]_{h = h_{n,J}}$$

Deformation of the contour:



OPE of bilinears:



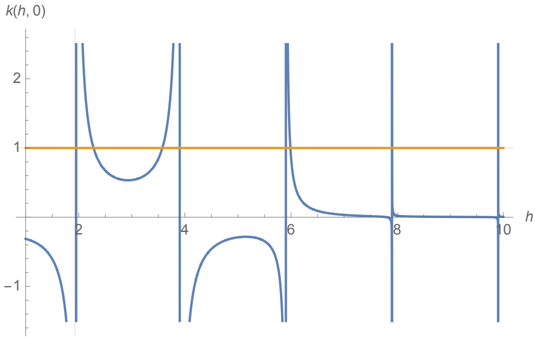
$$k(h, J)v_{h,J}(x_1, x_2, x_3) = \int_{yz} v_{h,J}(x_1, y, z)K_m{}^m{}_{;m'}{}^{m'}(x_2, x_3; y, z)$$

$$v_{h,J}(x_1, x_2, x_3) := \langle O_{h,J}(x_1)\phi^m(x_2)\phi_m(x_3)\rangle$$

$$O_{h,J} = \phi^m(\partial^2)^n\partial_{\mu_1}\dots\partial_{\mu_J}\phi_m$$

$(\zeta = 1)$

$$k(h,J) = -2\frac{\Gamma\left(1-\frac{1}{6}\epsilon\right)\Gamma\left(4-\frac{2}{3}\epsilon\right)\Gamma\left(2-\frac{1}{3}\epsilon-\frac{h-J}{2}\right)\Gamma\left(-1+\frac{1}{6}\epsilon+\frac{h+J}{2}\right)}{\Gamma\left(-1+\frac{1}{6}\epsilon\right)\Gamma\left(2-\frac{1}{3}\epsilon\right)\Gamma\left(4-\frac{2}{3}\epsilon-\frac{h-J}{2}\right)\Gamma\left(1-\frac{1}{6}\epsilon+\frac{h+J}{2}\right)}$$



$\epsilon = 0.145$

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$h_{n,J} = 2\Delta_\phi^{(0)} + 2n + J + z_{n,J}$, J even:

$$z_{0,2} = -\epsilon,$$

$$z_{0,J\geq 4} = -\frac{2\epsilon}{3}\left[1+\frac{6\Gamma(1+J)}{\Gamma(3+J)}\right] + \frac{2\Gamma(1+J)\epsilon^2}{9\Gamma(3+J)}\Big(13-6\gamma_E+3\psi(1+J)-9\psi(3+J)\Big) + \frac{8\Gamma(1+J)^2\epsilon^2}{\Gamma(3+J)^2}\Big(1+\psi(1+J)-\psi(3+J)\Big) + \mathcal{O}(\epsilon^3),$$

$$z_{1,J} = -\frac{2\epsilon}{3}\left[1-\frac{6\Gamma(2+J)}{\Gamma(4+J)}\right] + \frac{2\Gamma(2+J)\epsilon^2}{9\Gamma(4+J)}\Big(-19+6\gamma_E-3\psi(2+J)+9\psi(4+J)\Big) + \frac{8\Gamma(2+J)^2\epsilon^2}{\Gamma(4+J)^2}\Big(-1+\psi(2+J)-\psi(4+J)\Big) + \mathcal{O}(\epsilon^3),$$

$$z_{n,J} = -\frac{2\epsilon}{3} + \frac{4\epsilon^2}{3n(n-1)}\frac{\Gamma(1+n+J)}{\Gamma(3+n+J)} + \mathcal{O}(\epsilon^3), \quad (n\geq 2).$$

$(\zeta = d/6)$

$$k_{d/6}(h,J) = 2g^2 \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(\frac{d}{3})\Gamma\left(\frac{d}{3}-\frac{h-J}{2}\right)\Gamma\left(\frac{h+J}{2}-\frac{d}{6}\right)}{\Gamma(\frac{d}{6})\Gamma\left(\frac{2d}{3}-\frac{h-J}{2}\right)\Gamma\left(\frac{h+J}{2}+\frac{d}{6}\right)}\,.$$

$h = h_{n,J} = 2d/3 + 2n + J + z_{n,J}$

- $d \neq 1, 3$:

$$z_{n,J} = (-1)^{n+1} g^2 A_d \frac{\Gamma(d/6+n+J)}{\Gamma(d/2+n+J)\Gamma(d/3-n)n!}\,, \quad (n, J \geq 0)\,,$$

- $d = 3$:

$$z_{0,J} = -\frac{1}{2\pi^2} g^2 \frac{\Gamma(1/2+J)}{\Gamma(3/2+J)}\,, \quad (J \geq 0)\,,$$

(cf. $d = 2$ case in [Benedetti et al. '19 "Hints of unitarity"])

- for $d = 1$:

$$z_{n,J} = (-1)^{n+1} g^2 A_1 \frac{\Gamma(1/3-n)\Gamma(1/6+J+n)}{\Gamma(1/2+J+n)n!}\,, \quad (n, J \geq 0)\,,$$

$A_d = \frac{\Gamma(d/3)}{2^{d-2}\pi^{d/2}\Gamma(d/6)}\,.$

Unitarity

In unitary CFTs, conformal dimensions satisfy:

$$h_{n,J} \geq \begin{cases} \frac{d-2}{2} & \text{if } J = 0\,, \\ d-2+J & \text{if } J \geq 1\,. \end{cases}$$

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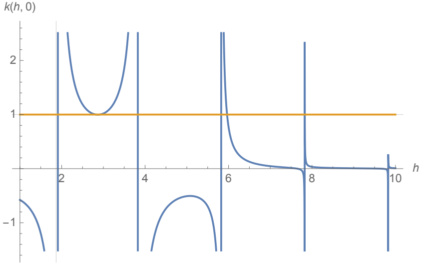

$(\zeta = 1)$ $h_{0,0}$ merges with its shadow $h_{-1,0} = d - h_{0,0}$ at $\epsilon \approx 0.264$:

$$h_{0,0} = d/2 + \mathrm{i}\, f\ , \quad f \in \mathbb{R}\ .$$

For $J > 0$, $h_{n,J}$ obey the bound numerically up to $\epsilon \approx 0.264$.

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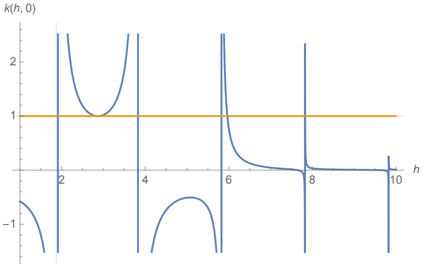

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NB: $[(\phi^2)^2] = 2h_{0,0} = d + \mathrm{i}\, f'$, dangerously irrelevant operator may destabilize the CFT. Spontaneous symmetry breaking? [Kim, Klebanov et al. '19]

Unitarity

$(\zeta = d/6)$ Unitarity bounds imply in $d = 6 - \epsilon$:

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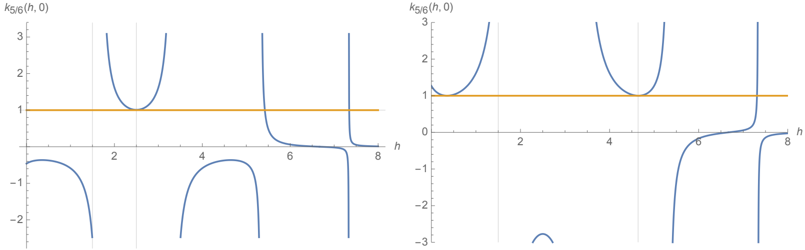

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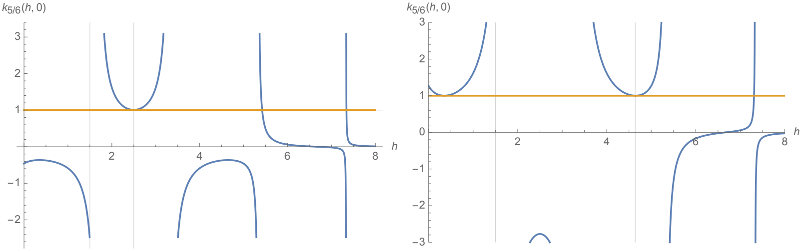

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NB: compared to Ising short-to-long-range transition [e.g. Behan, Rastelli et al '17]: (g vs ζ), (ϕ^3 marginal, ϕ^4 is not), presence of operators of complex dimension.

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Bosonic cubic model, invariant under $SO(3)$.
Melonic large-N limit and no disorder.
Short ($\zeta = 1, d = 6 - \epsilon$) and long-range ($\zeta = d/6, 0 < d < 6$).

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Thank you!