Trifundamental quartic model.

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Based on Phys. Rev. D 103, 046018 with Dario Benedetti and Razvan Gurau

April 21st 2021, vTJC

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Multi-scalar models

Interactions of the type:

$$\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d$$

- Indices from 1 to $\mathcal{N}$.
- Broad class of field theories such as $O(\mathcal{N})$ model
- Important universality classes: Ising, Heisenberg, ...

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- Important universality classes: Ising, Heisenberg, ...

Challenge: Full classification of all possible universality classes

- Gradually breaking the maximal symmetry group: $O(N_1) \times O(N_2)$, ...
- Here: trifundamental model $O(N_1) \times O(N_2) \times O(N_3)$

Further motivation: $1/N$ corrections in tensor models

- Homogeneous case: $O(N^3)$ tensor model
- Long-range: line of infrared stable fixed points
- Unitary large N CFT
- What about subleading corrections ?

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$\Longrightarrow$ Use multi-scalar results to compute $1/N$ corrections

Outline

The short-range quartic multi-scalar model

$$S[\phi] = \int d^d x \left[\frac{1}{2} \partial_\mu \phi_{\mathbf{a}}(x) \partial_\mu \phi_{\mathbf{a}}(x) + \frac{1}{4!} \lambda_{\mathbf{abcd}} \phi_{\mathbf{a}}(x) \phi_{\mathbf{b}}(x) \phi_{\mathbf{c}}(x) \phi_{\mathbf{d}}(x) \right],$$

- $d = 4 - \epsilon$
- Minimal subtraction scheme
- Beta functions up to two loops

$$\beta_{\mathbf{abcd}} = -\epsilon \tilde{g}_{\mathbf{abcd}} + \left(\tilde{g}_{\mathbf{abef}} \tilde{g}_{\mathbf{efcd}} + 2 \text{ terms}\right) - \left(\tilde{g}_{\mathbf{abef}} \tilde{g}_{\mathbf{eghc}} \tilde{g}_{\mathbf{fghd}} + 5 \text{ terms}\right) + \frac{1}{12} \left(\tilde{g}_{\mathbf{abce}} \tilde{g}_{\mathbf{efgh}} \tilde{g}_{\mathbf{fghd}} + 3 \text{ terms}\right) + \mathcal{O}(\tilde{g}^4),$$

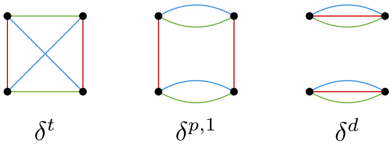
with rescaled coupling $\tilde{g}_{\mathbf{abcd}} = g_{\mathbf{abcd}} (4\pi)^{-d/2} / \Gamma(d/2)$

The short-range quartic trifundamental model

- Fields: rank 3 tensors transforming in the tri-fundamental representation of $O(N_1) \times O(N_2) \times O(N_3)$.
- Couplings:

$$\tilde{g}_{\mathbf{abcd}} = \tilde{g}\left(\delta^{t}_{\mathbf{abcd}} + 5 \text{ terms}\right) + \sum_{i=1,2,3} \tilde{g}_{p,i}\left(\delta^{p,i}_{\mathbf{ab;cd}} + 5 \text{ terms}\right) + 2\tilde{g}_{d}\left(\delta^{d}_{\mathbf{abcd}} + 2 \text{ terms}\right)$$

where:



Beta functions

- Easily obtained by substitution
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- Numerical solutions
- Vector, Matrix and Tensor-like limits

Numerical solutions for small N_i's

- Search for fixed points at one loop
- Real critical couplings
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- Search for fixed points at one loop
- Real critical couplings
- Non zero tetrahedral coupling

Results:

No real fixed point with non zero tetrahedral coupling stable in all five directions in the range $2 \leq N_i \leq 50$

Vector-like limit

- Send $N_1 \to \infty$ and keep N_2, N_3 fixed.
- New orthogonal couplings:

$$\tilde{g}_S = \tilde{g} + \tilde{g}_{p,1}\,, \quad \tilde{g}_D = \tilde{g} - \tilde{g}_{p,1}\,, \quad \tilde{g}_2 = \tilde{g}_d + \frac{\tilde{g}_{p,2}}{N_2} + \frac{\tilde{g}_{p,3}}{N_3}\,,$$

- Rescaling to obtain a large N_1 limit: $\tilde{g}_S = \bar{g}_S / N_1$ and so on.

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- Rescaling to obtain a large N_1 limit: $\tilde{g}_S = \bar{g}_S / N_1$ and so on.
- Decoupled beta functions at leading-order

$$\begin{aligned}
\beta_S &= -\epsilon \bar{g}_S + 2\bar{g}_S^2 \\
\beta_D &= -\epsilon \bar{g}_D - 2\bar{g}_D^2 \\
\beta_{p,2} &= -\epsilon \bar{g}_{p,2} + 4\bar{g}_S \bar{g}_{p,2} + 2N_3 \bar{g}_{p,2}^2 \\
\beta_{p,3} &= -\epsilon \bar{g}_{p,3} + 4\bar{g}_S \bar{g}_{p,3} + 2N_2 \bar{g}_{p,3}^2 \\
\beta_2 &= -\epsilon \bar{g}_2 + 4\bar{g}_S \bar{g}_2 + 2N_2 N_3 \bar{g}_2^2\,.
\end{aligned}$$

Fixed points and stability

32 fixed points:

$$\bar{g}_S^\star = \{0, \frac{\epsilon}{2}\}\,, \quad \bar{g}_D^\star = \{0, -\frac{\epsilon}{2}\}\,,$$

$$\bar{g}_{p,2} = \{0, \pm\frac{\epsilon}{2N_3}\}\,, \quad \bar{g}_{p,3} = \{0, \pm\frac{\epsilon}{2N_2}\}\,, \quad \bar{g}_2 = \{0, \pm\frac{\epsilon}{2N_2 N_3}\}\,,$$

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- One stable fixed point: $(\bar{g}_S^\star, \bar{g}_D^\star, \bar{g}_{p,2}^\star, \bar{g}_{p,3}^\star, \bar{g}_2^\star) = (\frac{\epsilon}{2}, -\frac{\epsilon}{2}, 0, 0, 0)$
- Corresponds to $\bar{g}_{p,1} = \frac{\epsilon}{2}$ and $\bar{g}^\star = \bar{g}_{p,2}^\star = \bar{g}_{p,3}^\star = \bar{g}_2^\star = 0$
- Chiral fixed point with symmetry $O(N_1) \times O(N_2 N_3)$ similar to bi-fundamental models

Matrix-like limit

- Double-scaling limit: $N_1 = cN$, $N_2 = N$, $N \to \infty$ and N_3, c fixed.
- Redefinition of double-trace coupling: $\tilde{g}_{dp} = \tilde{g}_d + \frac{\tilde{g}_{p,3}}{N_3}$
- Rescaling:

$$\tilde{g} = \frac{\bar{g}}{N}\,,\ \tilde{g}_{p,1} = \frac{\bar{g}_{p,1}}{N}\,,\ \tilde{g}_{p,2} = \frac{\bar{g}_{p,2}}{N}\,,\ \tilde{g}_{p,3} = \frac{\bar{g}_{p,3}}{N^2}\,,\ \tilde{g}_{dp} = \frac{\bar{g}_{dp}}{N^2}\,.$$

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- Standard scaling of quartic matrix invariants
  - Single-trace: tetrahedron and first two pillows
  - Double-trace: third pillow and double-trace

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Fixed points and stability

- 32 fixed points
- Only 16 with non zero tetrahedral coupling
- Lengthy but straightforward study of the signs of the critical exponents
- No real stable fixed point
- For $N_3 > \frac{c^2+1}{c}$: complex infrared fixed point stable in all five directions

Tensor-like limit

- Homogeneous large N limit: $N_1 = N_2 = N_3 = N$, $N \to \infty$
- Only one pillow: $\tilde{g}_p/3 = \tilde{g}_{p,1} = \tilde{g}_{p,2} = \tilde{g}_{p,3}$
- Usual rescaling:

$$\tilde{g} = \frac{\bar{g}}{N^{3/2}}\,, \qquad \tilde{g}_p = \frac{\bar{g}_p}{N^2}\,, \qquad \tilde{g}_d = \frac{\bar{g}_d}{N^3}\,,$$

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- Two-loop beta functions up to order $\mathcal{O}(N^{-3/2})$: reproduces previous results at leading order

Hierarchy between N and ϵ

- Naive expansion: non-perturbative sub-leading order
- Fictitious single coupling beta function: $-\epsilon g + g^3 + \frac{2a}{N} g^2$
- Fixed points:

$$g_{\star,\pm} = -\frac{a}{N} \pm \sqrt{\epsilon + \frac{a^2}{N^2}}\,.$$

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- Fixed points behavior governed by ϵN^2
- Demand that the fixed point from the leading order remains dominant in the beta functions
- Here we assume : $\epsilon N^2 \gg 1$ and we set $N = \tilde{N}/\sqrt{\epsilon}$

Fixed points

- Parametrize the couplings as $\bar{g}^\star = \bar{g}^\star_{(0)} + \tilde{N}^{-\frac{1}{2}}\,\bar{g}^\star_{(1)} + \tilde{N}^{-1}\bar{g}^\star_{(2)} + \mathcal{O}(\tilde{N}^{-3/2})$ and so on.

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- Leading-order

$$\bar{g}^\star_{(0)} = \pm\sqrt{\frac{\epsilon}{2}}\,, \qquad \bar{g}^\star_{p,(0)} = \pm\,3i\sqrt{\frac{\epsilon}{2}} + \frac{3\epsilon}{2} + \mathcal{O}(\epsilon^{3/2})\,,$$

$$\bar{g}^\star_{d,(0)} = \mp\, i\sqrt{\frac{\epsilon}{2}}(3 \pm \sqrt{3}) + \mathcal{O}(\epsilon^{3/2})\,.$$

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- Leading-order

$$\bar{g}^\star_{(0)} = \pm\sqrt{\frac{\epsilon}{2}}\,, \qquad \bar{g}^\star_{p,(0)} = \pm 3{\color{red}i}\sqrt{\frac{\epsilon}{2}} + \frac{3\epsilon}{2} + \mathcal{O}(\epsilon^{3/2})\,,$$
$$\bar{g}^\star_{d,(0)} = \mp {\color{red}i}\sqrt{\frac{\epsilon}{2}}(3 \pm \sqrt{3}) + \mathcal{O}(\epsilon^{3/2})\,.$$

- Order $\tilde{N}^{-1/2}$

$$\bar{g}^\star_{(1)} = 0\,, \qquad \bar{g}^\star_{p,(1)} = \mp 3\sqrt{2}\epsilon^{3/4}\,, \qquad \bar{g}^\star_{d,(1)} = \pm 3\frac{\epsilon^{3/4}}{\sqrt{2}}\,.$$

- Can also compute order $\tilde{N}^{-1}$: starts at $\sqrt{\epsilon}$.

Stability

Critical exponents up to order $\tilde{N}^{-1}$:

$$\omega_t = 2\epsilon \mp \frac{6i\sqrt{2}\epsilon}{\tilde{N}} + \mathcal{O}(\epsilon^{3/2}, \tilde{N}^{-3/2})\,,$$

$$\omega_p = \pm 2i\sqrt{2\epsilon} + 12\frac{\sqrt{\epsilon} \mp i\sqrt{2}\epsilon}{\tilde{N}} + \mathcal{O}(\epsilon^{3/2}, \tilde{N}^{-3/2})\,,$$

$$\omega_d = \pm 2i\sqrt{6\epsilon} \mp 12\sqrt{3}\frac{\sqrt{\epsilon} \mp i\sqrt{2}\epsilon}{\tilde{N}} + \mathcal{O}(\epsilon^{3/2}, \tilde{N}^{-3/2})\,.$$

- Choice of lower sign in $\bar{g}^{\star}_{d,(0)}$: positive real part
- Complex fixed point of [Giombi,Klebanov,Tarnopolski] subsists at subleading orders
- Order $\tilde{N}^{-1}$ gives real part to the three critical exponents: IR stable

Reminder: Long-range models

- Kinetic term of the form $\phi(\partial^2)^{\zeta}\phi$ with $0 < \zeta < 1$
- Vast array of applications [Campa, Dauxois, Ruffo, 2009]
- Admit phase transition [Dyson]
- One-parameter families of universality classes: ζ
- Study transition between short-range and long-range universality classes [Angelini et al., Brezin et al.,...]
- Rigorous renormalization group in $d = 3$ [Brydges et al., Abdesselam,...]

The long-range quartic multi-scalar model

$$S[\phi] = \int d^d x \left[\frac{1}{2}\phi_{\mathbf{a}}(x)(-\partial^2)^{\zeta}\phi_{\mathbf{a}}(x) + \frac{1}{2}\,\kappa_{\mathbf{ab}}\phi_{\mathbf{a}}(x)\phi_{\mathbf{b}}(x) \right.$$
$$\left. + \frac{1}{4!}\,\lambda_{\mathbf{abcd}}\phi_{\mathbf{a}}(x)\phi_{\mathbf{b}}(x)\phi_{\mathbf{c}}(x)\phi_{\mathbf{d}}(x) \right]$$

- Indices take values from 1 to $\mathcal{N}$
- Mass parameter κ treated as a perturbation
- $d < 4$ fixed

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- Indices take values from 1 to $\mathcal{N}$
- Mass parameter κ treated as a perturbation
- $d < 4$ fixed
- Canonical dimension of the field: $\Delta_\phi = \frac{d-2\zeta}{2}$
- Weakly relevant case: $\zeta = \frac{d+\epsilon}{4}$ with small ϵ
- UV dimension of the field $\Delta_\phi = \frac{d-\epsilon}{4}$

Two-loop beta functions

- Renormalization scheme and detailed computations: *Long-range multi scalar model at three loops* [Benedetti,Gurau,SH,Suzuki]
- Here only need two loops

$$\beta_{\mathbf{abcd}} = -\epsilon\tilde{g}_{\mathbf{abcd}} + \alpha_D\left(\tilde{g}_{\mathbf{abef}}\tilde{g}_{\mathbf{efcd}} + 2\ \text{terms}\right) \\ + \alpha_S\left(\tilde{g}_{\mathbf{abef}}\tilde{g}_{\mathbf{eghc}}\tilde{g}_{\mathbf{fghd}} + 5\ \text{terms}\right)\ ,$$

$$\beta^{(2)}_{\mathbf{cd}} = -(d - 2\Delta_\phi)\tilde{r}_{\mathbf{cd}} + \alpha_D\left(\tilde{r}_{\mathbf{ef}}\tilde{g}_{\mathbf{efcd}}\right) + \alpha_S\left(\tilde{r}_{\mathbf{ef}}\tilde{g}_{\mathbf{eghc}}\tilde{g}_{\mathbf{fghd}}\right).$$

with α_D and α_S explicit constants in terms of polygamma functions and an indefinite sum J_0.

Long-range $O(N)^3$ tensor model

- Set $N_1 = N_2 = N_3 = N$
- Choice of couplings:

$$\tilde{g}_{\mathbf{abcd}} = \tilde{g}\left(\delta^t_{\mathbf{abcd}} + 5 \text{ terms}\right) + \tilde{g}_p\left(\delta^p_{\mathbf{ab;cd}} + 5 \text{ terms}\right) + 2\tilde{g}_d\left(\delta^d_{\mathbf{abcd}} + 2 \text{ terms}\right) ,$$

- $\mathbf{a} = (a_1, a_2, a_3)$.
- $\delta^t_{\mathbf{abcd}}$ and $\delta^d_{\mathbf{abcd}}$ are defined as in the short-range case, and

$$\delta^p_{\mathbf{ab;cd}} = \frac{1}{3}\sum_{i=1}^{3} \delta^{p,i}_{\mathbf{ab;cd}} \,.$$

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$$\delta^p_{\mathbf{ab;cd}} = \frac{1}{3}\sum_{i=1}^{3}\delta^{p,i}_{\mathbf{ab;cd}}\,.$$

- Rescaling for the large N limit:

$$\tilde{g} = \frac{\bar{g}}{N^{3/2}}\,,\ \tilde{g}_p = \frac{\bar{g}_p}{N^2}\,,\ \tilde{g}_d = \frac{\bar{g}}{N^3}\,,$$

Scaling limit

- $\epsilon = 0$: exactly marginal tetrahedral coupling in the large N limit
- Next-to-leading order: line of fixed points collapses to the trivial fixed point

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- Next-to-leading order: line of fixed points collapses to the trivial fixed point
- $\epsilon \neq 0$: vanishing tetrahedron coupling at leading-order
- Fictitious beta function: $-\epsilon g + g^2/N$
- Fixed points: $g^\star = 0$ or $g^\star = N\epsilon$
- We need $N\epsilon \ll 1$: set $\epsilon = \frac{\tilde{\epsilon}}{N}$
- Expand in $1/N$ first, then in $\tilde{\epsilon}$.

Beta functions

- Define two new independent couplings:

$$\bar{g}_1 = \frac{\bar{g}_p}{3}\,,\ \bar{g}_2 = \bar{g}_d + \bar{g}_p\,.$$

- Parametrize the alpha coefficients:

$$\begin{aligned}\alpha_D &= 1 + \alpha_{D,1}\,\epsilon + \alpha_{D,2}\,\epsilon^2 + \mathcal{O}(\epsilon^3)\,,\\ \alpha_S &= \alpha_{S,0} + \alpha_{S,1}\,\epsilon + \mathcal{O}(\epsilon^2)\end{aligned}$$

- Two-loop beta functions at order N^{-1}:

$$\beta_t = \frac{\bar{g}}{N}\left[12\bar{g}_1\,(1 + \alpha_{S,0}\bar{g}_1) - \tilde{\epsilon}\right] + \mathcal{O}(N^{-3/2})\,.$$

Fixed points: leading order

- Parametrize the critical couplings as:
  $\bar{g}^\star \;=\; \bar{g}^\star_{(0)} \,+\, \bar{g}^\star_{(1)} N^{-1/2} + \mathcal{O}(N^{-1})$ and so on
- Solve order by order
- Leading order: line of fixed points

$$\bar{g}^\star_{1,(0)} = \pm\sqrt{-\bar{g}^{\star}_{(0)}{}^2} - \bar{g}^\star_{(0)}{}^2 \alpha_{S,0} + \mathcal{O}(\bar{g}^\star_{(0)}{}^3)\,,$$
$$\bar{g}^\star_{2,(0)} = \pm\sqrt{3}\sqrt{-\bar{g}^{\star}_{(0)}{}^2} - 3\bar{g}^\star_{(0)}{}^2 \alpha_{S,0} + \mathcal{O}(\bar{g}^\star_{(0)}{}^3)\,.$$

- Complex for real tetrahedral coupling
- Real for purely imaginary tetrahedral coupling

Fixed points: Next-to-leading order

- Two free parameters: $\bar{g}^\star_{(0)}$ and $\bar{g}^\star_{(1)}$

$$\bar{g}^\star_{1,(1)} = -2\bar{g}^\star_{(0)} - 2\bar{g}^\star_{(0)}\bar{g}^\star_{(1)}\alpha_{S,0} \mp \frac{\bar{g}^\star_{(0)}\bar{g}^\star_{(1)}}{\sqrt{-\bar{g}^{\star\,2}_{(0)}}} + \mathcal{O}(\bar{g}^{\star\,3}_{(0)})\,,$$

$$\bar{g}^\star_{2,(1)} = -3\bar{g}^\star_{(0)} - 6\bar{g}^\star_{(0)}\bar{g}^\star_{(1)}\alpha_{S,0} \mp \frac{\sqrt{3}\bar{g}^\star_{(0)}\bar{g}^\star_{(1)}}{\sqrt{-\bar{g}^{\star\,2}_{(0)}}} + \mathcal{O}(\bar{g}^{\star\,3}_{(0)})\,.$$

Fixing the tetrahedral coupling

- Keep the same number of non trivial order for each beta function
- Order $N^{-3/2}$ for the tetrahedral coupling
- Allows us to fix $\bar{g}^{\star}_{(0)}$ and $\bar{g}^{\star}_{(1)}$
- Expanding in $\tilde{\epsilon}$

$$\bar{g}^{\star}_{(0)} = \pm \frac{i}{12} \left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{6} \tilde{\epsilon}^2 \right) + \mathcal{O}(\tilde{\epsilon}^3) \,.$$

$$\bar{g}^{\star}_{(1)} = \begin{cases} \frac{1}{6} \left(-\tilde{\epsilon} + \frac{\alpha_{S,0}}{2} \tilde{\epsilon}^2 \right) + \mathcal{O}(\tilde{\epsilon}^3) & \text{for the upper choice of sign in } \bar{g}^{\star}_{1,(0)} \\ \frac{1}{18} \left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{18} \tilde{\epsilon}^2 \right) + \mathcal{O}(\tilde{\epsilon}^3) & \text{for the lower choice of sign.} \end{cases}$$

Stability

- Two stable fixed points at leading order

$$\bar{g}^\star = \pm\frac{i}{12}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{6}\tilde{\epsilon}^2\right) + \frac{1}{6N^{1/2}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{3}\tilde{\epsilon}^2\right) + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

$$\bar{g}_1^\star = \frac{1}{12}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{12}\tilde{\epsilon}^2\right) \mp \frac{i\alpha_{S,0}}{36N^{1/2}}\tilde{\epsilon}^2 + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

$$\bar{g}_2^\star = \frac{1}{4\sqrt{3}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{12}(2-\sqrt{3})\tilde{\epsilon}^2\right) \pm \frac{i(-3+2\sqrt{3})}{12N^{1/2}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{2}\tilde{\epsilon}^2\right) + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

- Expand in $\tilde{\epsilon}$ the critical exponents at next-to-leading order

$1/N$ corrections to the critical exponents

$$\partial\beta^{(2)}(\bar{g}^\star) = -\nu^{-1} = -\frac{d}{2} + \frac{1}{2\sqrt{3}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{6}\tilde{\epsilon}^2\right) \pm \frac{i}{\sqrt{3}N^{1/2}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{2}\tilde{\epsilon}^2\right) + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

$$\partial\beta_1(\bar{g}^\star) = \frac{1}{3}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{6}\tilde{\epsilon}^2\right) \pm \frac{2i}{3N^{1/2}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{2}\tilde{\epsilon}^2\right) + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

$$\partial\beta_2(\bar{g}^\star) = \frac{1}{\sqrt{3}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{6}\tilde{\epsilon}^2\right) \pm \frac{2i}{\sqrt{3}N^{1/2}}\left(\tilde{\epsilon} - \frac{\alpha_{S,0}}{2}\tilde{\epsilon}^2\right) + \mathcal{O}(\tilde{\epsilon}^3, N^{-1}),$$

$$\omega_t = \frac{\tilde{\epsilon}}{N}\left(1 + \frac{\alpha_{S,0}}{6}\tilde{\epsilon}\right) + \frac{2i\alpha_{S,0}\tilde{\epsilon}^2}{3N^{3/2}} + \mathcal{O}(\tilde{\epsilon}^3, N^{-2}).$$

- From four lines of fixed points to eight isolated fixed points
- Two stable ones
- What was real at LO gets an imaginary part

Conclusion

- Tri-fundamental model $O(N_1) \times O(N_2) \times O(N_3)$ both in short and long-range setting
- In general: NO stable fixed points with non zero tetrahedral coupling
- Consider complex fixed points: unitary CFT ?

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- Similar results for short-range but real part suppressed in $1/N$
- Similar behaviors at finite N

Real unitary CFT at strictly large N only for the long-range model

Perspectives

- General proof of the non-existence of stable real fixed points with non zero tetrahedral coupling
- Group theoretical arguments, gradient flow equations, ...
- Real stable fixed points with rank p symmetry with higher p ?
- Sextic interactions with $p = 3$: real fixed points, what happens at sub-leading orders ?

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Thank you !