A quick review of melonic CFTs

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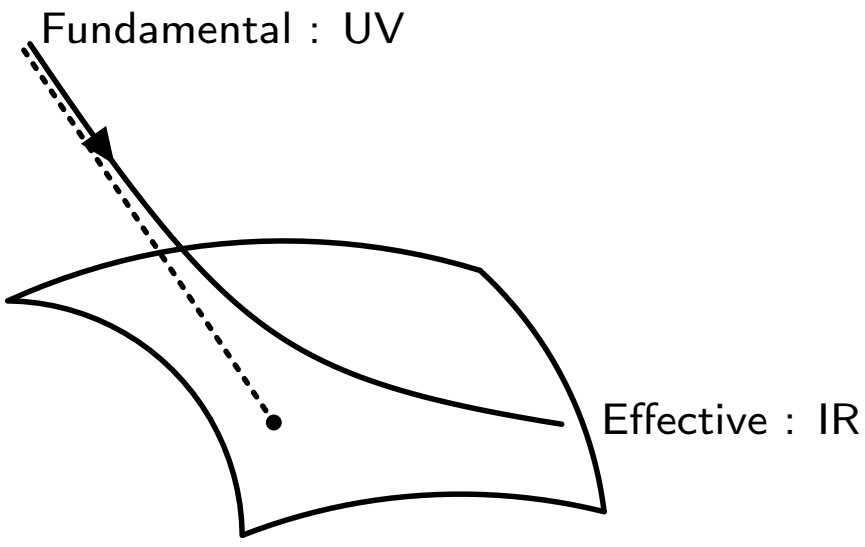
General motivations: renormalization

Physics change with the energy scale: **Renormalization group**



Flow in the space of theories with respect to the energy scale [Wilson 1972, Polchinski 1984, . . .]:

$\rightarrow$ fixed points and trajectories

Weak versus strong coupling

Weak coupling	**Strong coupling**
Perturbation theory	?
Non perturbative ?	?

- Study non-trivial fixed points of the renormalization group [Wilson]
- Gain control over non-perturbative phenomena
- Spontaneous symmetry breaking [Coleman, Jackiw, Politzer]
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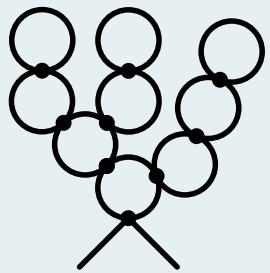
- Dynamical mass generation [Gross, Neveu]
- Large N limit: helpful approximation scheme
- Vast array of applications: statistical mechanics, QCD, quantum gravity

From vector to tensor models

Vector ϕ_a

$\frac{\lambda}{N}(\phi_a\phi_a)^2$

Cactus diagrams

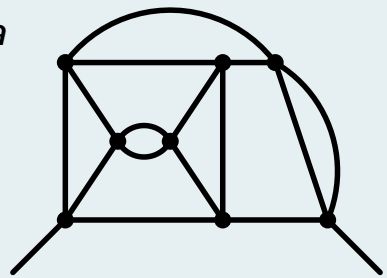


$\rightarrow$ Easy

Matrix M_{ab}

$\frac{\lambda}{N}M_{ab}M_{bc}M_{cd}M_{da}$

Planar diagrams

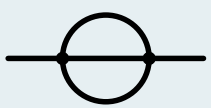


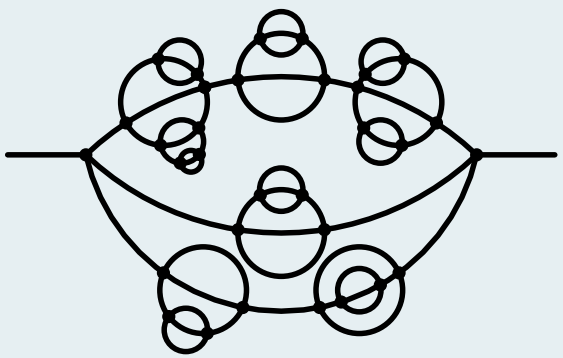
$\rightarrow$ Hard

Tensor T_{abc}

$\frac{\lambda}{N^{3/2}}T_{aeb}T_{cfb}T_{ced}T_{afd}$

Melon diagrams





$\rightarrow$ Tractable

Melonic quantum mechanics

- First introduced in zero dimension: random geometry and quantum gravity [Gurau, Bonzom, Rivasseau, ...]
- Strongly coupled QFTs and holography ($d = 1$): SYK model without disorder [Witten, Klebanov, Tarnopolsky, ...]
- Tensor models in higher dimension: new class of conformal field theories

- Problem: divergences
- Renormalization group: computation of beta functions, stable IR fixed point? Unitarity ?

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  - Closed Schwinger-Dyson equation
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  - Exact marginality for long-range models
- Unitarity range: real scaling dimensions above unitarity bounds and real OPE coefficients

Tensor models in $d = 1$

Model	Fermions	Symmetry	Key features
GW [1]	Real	$O(n)^{\frac{D(D+1)}{2}}$	SYK-like without disorder
KT [2]	Real	$O(N)^3$	Scaling dimensions, spectrum
KT [2]	Complex	$O(N)^3$	Scaling dimensions, SYK-like
$Sp(N)$ [3]	Complex	Irreducible rank-3 $Sp(N)$	Non-zero tetrahedron SYK-like
Higher rank [2,4]	Real	$O(N)^{q-1}$ $q \geq 6$	Spectrum, chaos, bulk Growing number of invariants

[1] [Gurau, Witten,...] [2] [Klebanov,Tarnopolsky,Giombi,Kim,Milekhin,Pallegar,Popov,Zhao,...]

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Further studies:

- Symmetry breaking
- Counting singlet states and operators
- Supersymmetric SYK-like tensor model [Peng,Spradlin,Volovich]
- Supersymmetric bosonic tensor model [Chang, Colin-Ellerin, Rangamani]

Higher dimensions: bosonic models

Model	Sym.	d	FP	Stable	Unitary	NLO
CTKT [1]	$O(N)^3$	$4-\epsilon$	Complex	$\times$	$\times$	stable FP
BGHS [2]	$O(N)^3$	$d<4$	Real	$\surd$	$\surd$	Non-unitary
Prismatic [3]	$O(N)^3$	$3-\epsilon$	Real	$\surd$	$\surd$	$\surd$
Sextic [4]	$U(N)^3$	$3-\epsilon$	Real	$\surd$	$\times$	?
Sextic [4]	$U(N)^3$	$d<3$	Real	$\surd$	?	?
Rank 5 [4]	$O(N)^5$	$3-\epsilon$	Trivial	-	-	-

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Other models:

- Supersymmetric $O(N)^3$ model with $d = 3 - \epsilon$ perturbative expansion, IR stable fixed point [Popov]
- Tensor model with four supercharges: spectrum of bilinears real and above unitarity bounds [Lettera, Vichi]

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- Multi-matrix models [Ferrari, Rivasseau, Tanasa, Toriumi, Valette]

Outlook and long-term goals

- What about fermionic long-range models ? Other symmetry groups ?
- Complete proof of conformal invariance
- Conformal data for higher-order invariants
- Fixed points at sub-leading order
- Test properties of QFTs
- Holographic dual ? [de Mello-Koch, Gossman, Tahiridimbisoa, Mahu, Tribelhorn]