Can we make sense out of "Tensor Field Theory"?

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Tensor Field Theory

A new playground for non-perturbative QFT

- **1979**. 't Hooft's series of lectures *Can we make sense out of "Quantum Chromodynamics"?*
  - Proposes a strategy to control QCD via the study of its singularities in the Borel plane.
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- **2013**. First constructive results about Tensor models [Gurau].
- **202?**. First non-perturbative definition of a *just renormalisable* Bosonic field theory [Rivasseau, Vignes-Tourneret].

Outline

T_5^4 and its divergences

Discrete flow from multiscale analysis

Holomorphic RG flow

T_5^4 and its divergences

The T_5^4 field theory

- Tensors:

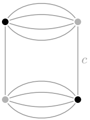
$$T : \mathbb{Z}^5 \to \mathbb{C}, \qquad T_{\boldsymbol{n}}, \overline{T}_{\overline{\boldsymbol{n}}} \text{ with } \boldsymbol{n}, \overline{\boldsymbol{n}} \in \mathbb{Z}^5.$$

- Free action:

$$C_{\boldsymbol{n},\overline{\boldsymbol{n}}} = \frac{\delta_{\boldsymbol{n},\overline{\boldsymbol{n}}}}{Z_b} \frac{\kappa_{j_{\max}}(\boldsymbol{n}^2)}{\boldsymbol{n}^2 + m_b^2}, \quad \boldsymbol{n}^2 := n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2.$$

- Interactions:

$$V(T, \overline{T}) = \tfrac{g_b Z_b^2}{2} \sum_{c=1}^{5} V_c(T, \overline{T}), \qquad V_c(T, \overline{T}) =$$



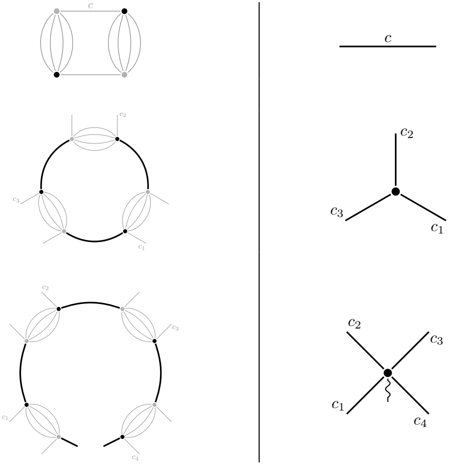
Lemma

T_5^4 is just renormalizable to all orders of perturbation with a power-counting similar to ϕ_4^4 [Avohou-Rivasseau-Tanasa 2015, Rivasseau-V.-T. 2021].

The T_5^4 field theory

Intermediate Field Representation

There is a bijection between quartic melonic coloured graphs and ciliated edge-coloured maps.



The T_5^4 field theory

Divergence degree: $\omega(\mathbb{G}) = 4 - E(\mathbb{G}) - \big(C(\partial\mathbb{G}) - 1\big) - \delta(G)$

Lemma (Superficially divergent graphs)

The superficially divergent graphs all belong to one of the cases listed below. Moreover, in the intermediate field representation,

- *divergent four-point graphs are trees such that the unique path between their two cilia is monochrome,*
- *the closed superficially divergent graphs are*
  - *plane trees if $\omega = 5$,*
  - *unicyclic maps if $\omega = 0$ or $\omega = 2$*

Finally, in the latter case, $\omega(\mathbb{G}) = 2$ if and only if the unique cycle of $\mathfrak{G}$ is monochrome.

<table>
<tr><th>$E(\mathbb{G})$</th><th>$C(\partial\mathbb{G})$</th><th>$\delta(G)$</th><th>$\omega(\mathbb{G})$</th></tr>
<tr><td>4</td><td rowspan="2">1</td><td rowspan="2">0</td><td>0</td></tr>
<tr><td>2</td><td>2</td></tr>
<tr><td rowspan="3">0</td><td rowspan="3">0</td><td>0</td><td>5</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>5</td><td>0</td></tr>
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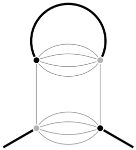
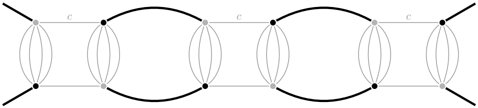
The T_5^4 field theory

The fundamental melons

- Divergent 2-point graphs constructed by recursive insertion of the tadpole into itself.



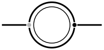
- Divergent 4-point graphs made of a monochrome chain of melonic interaction vertices plus insertions of tadpoles.



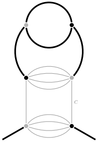
The T_5^4 field theory

1PI bare functions

- $G_{2,b}^{\text{mel}}$:= bare *melonic* connected 2-point function,
  Σ_b^{mel} := bare *melonic* self-energy, $\Sigma_b^{\text{mel}}(\boldsymbol{n}) = \sum_{c=1}^{5} \overline{\Sigma}_b^{\text{mel}}(n_c)$:



$=$



or

$$\overline{\Sigma}_b^{\text{mel}}(n_c) = -g_b Z_b^2 \sum_{\boldsymbol{p} \in \mathbb{Z}^5} \frac{\delta_{p_c, n_c}}{C_b^{-1}(\boldsymbol{p}) - \Sigma_b^{\text{mel}}(\boldsymbol{p})}$$

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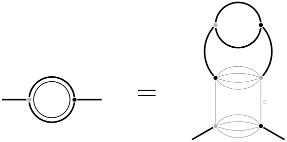

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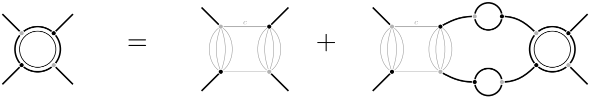
- $\Gamma_{4,b}^{\text{mel}}$:= bare *melonic* 1PI 4-point function,
  $\Gamma_{4,b}^{\text{mel}}(\boldsymbol{n},\overline{\boldsymbol{n}},\boldsymbol{m},\overline{\boldsymbol{m}}) = \sum_{c=1}^{5} \delta_{\boldsymbol{n}_{\hat{c}},\overline{\boldsymbol{n}}_{\hat{c}}} \delta_{\boldsymbol{m}_{\hat{c}},\overline{\boldsymbol{m}}_{\hat{c}}} \delta_{n_c,\overline{m}_c} \delta_{m_c,\overline{n}_c} \overline{\Gamma}_{4,b}^{\text{mel}}(n_c,\overline{n}_c)$:



Discrete flow from multiscale analysis

Multiscale analysis in one slide

Multiscale analysis is a discrete implementation of Wilson's interpretation of renormalization: Physics changes with scale. Integrating out high energy degrees of freedom leads to an effective theory, the parameters of which are related to the initial ones via the RG flow.

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Multiscale analysis is one of the main tools of constructive QFT.

- Let ϕ be a Gaussian random field of covariance C. If $C = \sum_{j=0}^{\infty} C^j$ (where $C^j(p)$ ensures $|p| \simeq M^j$), then

$$\phi \overset{\text{law}}{=} \sum_j \phi^j \text{ with } \phi^j \sim \mu_{C^j}.$$

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- Integrating over $\phi^{j_{\max}}$ down to ϕ^{j+1} (included) gives the effective theory at scale j. Its parameters m_j^2, Z_j, λ_j are defined by the sums of the (local parts of the) Feynman graphs, all edges of which bear propagators $C^k, k \geqslant j+1$.

Mass renormalization

We first perform ***full* mass renormalization**:

$$\Sigma_b^{\text{mel}}(\boldsymbol{n}) = \Sigma_b^{\text{mel}}(\boldsymbol{0}) + \Sigma_{mr}^{\text{mel}}(\boldsymbol{n}).$$

We define the **renormalized mass** as follows:

$$m_r^2 := m_b^2 - \Sigma_b^{\text{mel}}(0)$$

so that

$$G_{2,b}^{\text{mel}}(m_b^2; \boldsymbol{n}) = \frac{\kappa_{j_{\max}}(\boldsymbol{n})}{C_b^{-1}(\boldsymbol{n}) - \Sigma_b^{\text{mel}}(\boldsymbol{n})} = \frac{\kappa_{j_{\max}}(\boldsymbol{n})}{Z_b \boldsymbol{n}^2 + m_r^2 - \Sigma_{mr}^{\text{mel}}(\boldsymbol{n})} =: G_{2,mr}^{\text{mel}}(m_r^2; \boldsymbol{n}).$$

Effective wave-functions

The effective wave-function constant Z_j is

$$Z_j := Z_b - \frac{\partial \overline{\Sigma}^{\text{mel}}_{mr;\geqslant j+1}}{\partial n_c^2}(0)$$

where $\Sigma^{\text{mel}}_{mr;\geqslant j}(\boldsymbol{n}) = \sum_c \overline{\Sigma}^{\text{mel}}_{mr;\geqslant j}(n_c)$ is the sum of mass-renormalized amplitudes of all 1PI melonic 2-point graphs, all internal scales of which are greater than or equal to j, namely

$$\overline{\Sigma}^{\text{mel}}_{mr;\geqslant j}(n_c) := -g_b Z_b^2 \sum_{\boldsymbol{p}\in\mathbb{Z}^5} \eta_{\geqslant j}(\boldsymbol{p}^2) \frac{\delta_{p_c,n_c} - \delta_{p_c,0}}{Z_b \boldsymbol{p}^2 + m_r^2 - \sum_{c'} \overline{\Sigma}^{\text{mel}}_{mr}(p_{c'})}.$$

Note that with these notations, $Z_{j_{\max}} = Z_b$ and $Z_{-1} = Z_r = 1$.

Effective coupling constants

The effective coupling constant $g_j Z_j^2$ is

$$-g_j Z_j^2 := \overline{\Gamma}^{\text{mel}}_{4,b;\geqslant j+1}(0,0)$$

where

$$\overline{\Gamma}^{\text{mel}}_{4,b;\geqslant j}(n_c,\bar{n}_c) := \frac{-g_b Z_b^2}{1 + g_b Z_b^2 \sum_{\boldsymbol{p},\bar{\boldsymbol{q}}} \delta_{\boldsymbol{p}_{\hat{c}},\bar{\boldsymbol{q}}_{\hat{c}}} \delta_{p_c,n_c} \delta_{\bar{q}_c,\bar{n}_c} G^{\text{mel}}_{2,mr;\geqslant j}(\boldsymbol{p}) G^{\text{mel}}_{2,mr;\geqslant j}(\bar{\boldsymbol{q}})}$$

and

$$G^{\text{mel}}_{2,mr;\geqslant j}(\boldsymbol{n}) := \frac{\eta_{\geqslant j}(\boldsymbol{n}^2)}{Z_b \boldsymbol{n}^2 + m_r^2 - \Sigma^{\text{mel}}_{mr;\geqslant j}(\boldsymbol{n})}.$$

With these conventions, $g_{j_{\max}} = g_b$ and $g_{-1} = g_r$.

Analyticity

Theorem

*The effective wave-functions and coupling constants are analytic functions of the bare coupling g_b (*a priori* in a disk of radius going to 0 as $j_{max} \to \infty$).*

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Proof.

$$Z_j = Z_b + \sum_{n=1}^{\infty} (g_b Z_b^2)^n A_n(m_r^2, Z_b, j_{\max}, j) \qquad \text{bivariate analytic}$$

$$Z_b = 1 + \frac{\partial \overline{\Sigma}^{\text{mel}}_{mr}}{\partial n_c^2}(0) \qquad \text{analytic (implicit fct thm)}$$

$Z_j(g_b) = Z_j(g_b, Z_b(g_b))$ is holomorphic around 0. □

Asymptotic freedom

Theorem

For all $j \in \{-1, 0, \ldots, j_{max} - 1\}$,

$$g_{j+1} - g_j = \beta_j g_j^2 + O(g_j^3)$$

where $\beta_j = \beta_2 + O(M^{-j})$, β_2 *is a negative real number and* $O(g_j^3) = g_j^3 f(g_j)$ *where* f *is analytic around the origin (*a priori *in a domain which shrinks to* $\{0\}$ *as* $j_{max} \to \infty$*).*

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Proof.

$$g_{j+1} Z_{j+1}^2(g_b),\ g_j Z_j^2(g_b),\ Z_{j+1}(g_b),\ Z_j(g_b) \implies g_{j+1}\big(g_b(g_j)\big) = g_{j+1}(g_j)$$

⚠ Products and derivatives of cut-off functions. □

Holomorphic RG flow

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Holomorphic RG flow

- The power of holomorphic dynamical system theory
- 7 definitions and a theorem
- Appetizer for quantitative results

From discrete to continuous

For $g_r > 0$ small enough, the discrete RG flow is decreasing and goes to 0 as $j \to \infty$. **But what happens if g_r is complex?**

In order to answer, we invoke the theory of discrete holomorphic dynamical systems. Moreover it will allow us to relate the discrete flow to a *continuous* Cauchy problem and thus get a preciser behaviour.

Remark. From a RG point of view, it would be more natural to study

$$g_j\big((g_k)_{k\geqslant j+1},(Z_k)_{k\geqslant j+1}\big).$$

RG flow goes from UV to IR and g_r is the endpoint of it. But it is easier to go to a one (complex) dimensional continuous initial value problem.

Two simplifying assumptions

$$g_{j+1} = g_j + \beta_j g_j^2 + g_j^3 f(g_j) =: h_{j_{\max},j}(g_j)$$

A priori $\Omega_{j_{\max}} \to \{0\}$ as $j_{\max} \to \infty$. But the first two Taylor coefficients of $h_{j_{\max},j}$ are uniformly bounded in $j_{\max}$.

Assumption 1. The series $g_{j+1}(g_j)$ is holomorphic in a domain uniform in $j_{\max}$.

The dynamics defined by $h_{j_{\max},j}$ is not autonomous, its Taylor coefficients depend on j. Nevertheless, far from the infrared cutoff, the behaviour of β_j suggests that the dynamics becomes autonomous.

Assumption 2. The discrete RG flow $g_{j+1} = h(g_j)$ is defined by the iteration of a (unique) holomorphic map h, tangent to the identity, and such that

$$h(z) = z + \beta_2 z^2 + O(z^3), \quad \beta_2 < 0.$$

Definition (Holomorphic dynamical system)

Let M be a complex manifold, and $p \in M$. A (discrete) *holomorphic local dynamical system* at p is a holomorphic map $f : U \to M$ such that $f(p) = p$, where $U \subseteq M$ is an open neighbourhood of p; we shall assume that $f \neq \mathrm{id}_U$. We shall denote by $\mathrm{End}(M, p)$ the set of holomorphic local dynamical systems at p.

$M = \mathbb{C}$, $p = 0$.

Discrete holomorphic dynamical systems

Some concepts

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The stable set K_f of f is the set of all points $z \in U$ such that the orbit $\{f^{\circ k}(z) : k \in \mathbb{N}\}$ is well-defined.

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Definition (Conjugation)

We say that $f, g \in \mathrm{End}(\mathbb{C}, 0)$ are holomorphically conjugated if there exists a holomorphic map h such that $h \circ f = g \circ h$.

Definition (Multiplicity)

Let $f \in \mathrm{End}(\mathbb{C}, 0)$ be a holomorphic local dynamical system with a parabolic fixed point at the origin. Then we can write:

$$f(z) = e^{2i\pi p/q} z + a_{r+1} z^{r+1} + O(z^{r+2}),$$

with $a_{r+1} \neq 0$. $r+1$ is called the multiplicity of f.

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Definition (Directions)

Let $f \in \mathrm{End}(\mathbb{C}, 0)$ be tangent to the identity of multiplicity $r + 1 \geqslant 2$. Then a unit vector $v \in \mathbb{S}^1$ is an attracting (resp. repelling) direction for f at the origin if $a_{r+1} v^r$ is real negative (resp. real positive).

r attracting and r repelling directions

Definition (Basins)

Let $v \in \mathbb{S}^1$ be an attracting direction for an $f \in \mathrm{End}(\mathbb{C}, 0)$ tangent to the identity. The basin centerd at v is the set of points $z \in K_f \setminus \{0\}$ such that $\lim_{k\to\infty} f^{\circ k}(z) = 0$ and $\lim_{k\to\infty} f^{\circ k}(z)/|f^{\circ k}(z)| = v$.

Definition (Petals)

An attracting petal centered at an attracting direction v of an $f \in \mathrm{End}(\mathbb{C}, 0)$ tangent to the identity is an open simply connected f-invariant set $P \subseteq K_f \setminus \{0\}$ such that a point $z \in K_f \setminus \{0\}$ belongs to the basin centered at v if and only if its orbit intersects P. In other words, the orbit of a point tends to 0 tangent to v if and only if it is eventually contained in P. A repelling petal (centered at a repelling direction) is an attracting petal for the inverse of f.

The flower theorem

Theorem (Leau-Fatou)

Let $f \in \mathrm{End}(\mathbb{C}, 0)$ *be a holomorphic local dynamical system tangent to the identity with multiplicity* $r + 1 \geqslant 2$ *at the fixed point. Let* $v_1^{\pm}, \dots, v_r^{\pm} \in \mathbb{S}^1$ *be the attracting (resp. repelling) directions of* f *at the origin. Then,*

1. *for each attracting (resp. repelling) direction* $v_j^{\pm}$ *there exists an attracting (resp. repelling petal)* $P_j^{\pm}$, *so that the union of these* $2r$ *petals together with the origin forms a neighbourhood of the origin. Furthemore, the* $2r$ *petals are arranged cyclically so that two petals intersects if and only if the angle between their central directions is* π/r.
2. *If* P *is a petal centered at one of the attracting directions, then there is a biholomorphism* $\varphi : P \to \mathbb{C}$ *such that* $\varphi \circ f(z) = \varphi(z) + 1$ *for all* $z \in P$.

The flower theorem

$f(g) = g - g^4$

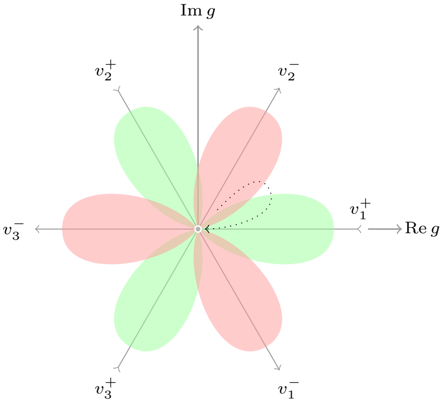

Figure: Attracting (green) and repelling (red) petals of a dynamics of multiplicity 4, and a typical trajectory.

Remark. Petals can be optimized so that their opening angle is $2\pi/r$.

The flower theorem

Consequence for the RG flow

Any holomorphic dynamical system of multiplicity 2 tangent to the identity has a cardioid-like invariant domain.

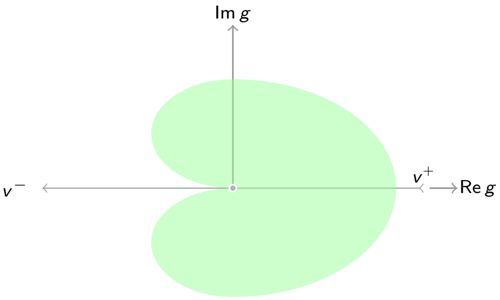

Figure: A unique attracting petal of a multiplicity 2 parabolic dynamics.

Appetizer

$$g_{j+1} = g_j + \beta_2 g_j^2 + g_j^3 h(g_j)$$

1. Does it exist a continuous Cauchy problem such that its unique solution $f(t)$ is such that

$$\forall j \in \{-1, 0, \dots\}, \; g_j = f(j+1)?$$

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2. There exists a simply connected domain D_ϵ $(0 \in \partial D_\epsilon)$ containing a Nevanlinna-Sokal disk such that

$$g_r \in D_\epsilon \Longrightarrow |f(t)| < \epsilon, \; \forall t \geqslant 0.$$

▸ Theorem

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3. If $g_r \in \mathbb{R}_+$ then we have a pretty good control on its large-time behaviour.

▸ Theorem

Conclusion

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Its flow can be controlled with a very good precision (some assumptions must be turned into theorems nevertheless).

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Its flow can be controlled with a very good precision (some assumptions must be turned into theorems nevertheless).

> *The path towards the non-perturbative definition of T_5^4 is yet to be discovered and will very probably involve all currently known tools for constructive tensors.*

Thank you for your attention

Uniform boundedness

Theorem

Let ϵ be a sufficiently small positive real number. There exists a simply connected domain D_ϵ of $\mathbb{C}$ such that $D_\epsilon \subset U$, $0 \in \partial D_\epsilon$, and D_ϵ contains a Nevanlinna-Sokal disk $\mathbb{S}_\delta$, $\delta = \frac{1}{6}\frac{\epsilon}{1+\frac{3\pi}{2}|\beta_{3,2}|\epsilon}$, such that if $g_r \in D_\epsilon$ then, for all $t \geqslant 0$, the unique maximal solution on $\mathbb{R}_+$ of the Cauchy problem belongs to $\mathbb{D}_\epsilon$.

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Large time behaviour

If the initial value g_r is real and h real-valued, we have:

Theorem

There exists $g_c \in \mathbb{R}_+^*$ such that for all g_r real in $(0, g_c)$, the Cauchy problem has a unique decreasing solution f defined on $\mathbb{R}_+$. Moreover let ϵ be a positive real number smaller than 1. Then there exists a positive real number $\alpha(\epsilon)$ (smaller than 1) such that if $g_r \in (0, \alpha g_c)$, f satisfies

$$\frac{g_r}{1 - \beta_2 g_r t + \frac{\beta_3^-}{\beta_2} g_r \log(1 - \beta_2 g_r t) + \frac{\beta_3^-}{\beta_2} g_r \phi_-(t)} < f(t) < \frac{g_r}{1 - \beta_2 g_r t + \frac{\beta_3^+}{\beta_2} g_r \log(1 - \beta_2 g_r t) + \frac{\beta_3^+}{\beta_2} g_r \phi_+(t)},$$

with $\beta_3^- := (1 - \mathrm{sgn}(\beta_3)\epsilon)\beta_3$, $\beta_3^+ := (1 + \mathrm{sgn}(\beta_3)\epsilon)\beta_3$ and ϕ_-, ϕ_+ two bounded functions on $\mathbb{R}_+$.