Melonic large N limit of 5-index irreducible random tensors

Sylvain Carrozza

Radboud University

Tensor Journal Club
February 16, 2022

Results based on:

- arXiv:2104.03665 – Commun. Math. Phys. (2022) with S. Harribey.

but also

- arXiv:1712.00249 – Commun. Math. Phys. (2019) with D. Benedetti, R. Gurau and M. Kolanowski.
- arXiv:1803.02496 – JHEP (2018)

OUTLINE

Introduction

O(N) irreducible random tensors with complete graph interaction

Existence of the large N limit

Melonic dominance at leading order

Outline

RANDOM TENSORS

Space of tensors $T = T_{a_1 \dots a_p}$, $a_i \in \{1, \dots, N\}$, equipped with measure of the form:

$$d\nu(T) = d\mu_{\boldsymbol{P}}(T) e^{-S_N(T)}$$

- $d\mu_{\boldsymbol{P}}$ is Gaussian with covariance $\boldsymbol{P}$:

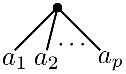
$$\int d\mu_{\boldsymbol{P}}(T) T_{a_1 \dots a_p} T_{b_1 \dots b_p} = \boldsymbol{P}_{a_1 \dots a_p, b_1 \dots b_p}$$

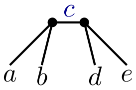
- both $\boldsymbol{P}$ and S_N are invariant under the action of some unitary group: $O(N)$, $U(N)$ or $Sp(N)$.

What type of universal behaviour can we obtain in the asymptotic limit $N \to \infty$?

Tensors and invariants

Symmetric tensor:

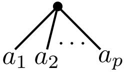
$T_{a_1 a_2 \cdots a_p} =$ 

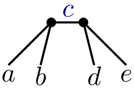
$\sum_{c=1}^{N} T_{abc} T_{cde} =$ 

Connected invariants:

Tensors and invariants

Symmetric tensor:

$T_{a_1 a_2 \cdots a_p} =$ 

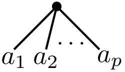
$\sum_{c=1}^{N} T_{abc} T_{cde} =$ 

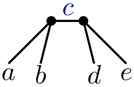
Connected invariants:

$p = 1$  $(\phi_a \phi^a)$

Tensors and invariants

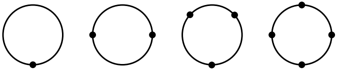
Symmetric tensor:

$T_{a_1 a_2 \cdots a_p} =$ 

$\sum_{c=1}^{N} T_{abc} T_{cde} =$ 

Connected invariants:

$p = 2$

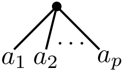


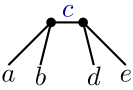
$\cdots$

$(\mathrm{tr}(M^n))$

Tensors and invariants

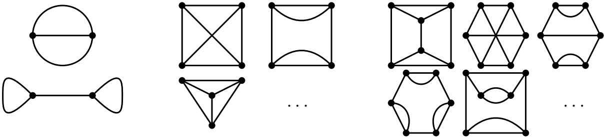
Symmetric tensor:

$T_{a_1 a_2 \cdots a_p} =$ 

$\sum_{c=1}^{N} T_{abc} T_{cde} =$ 

Connected invariants:

$p = 3$

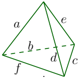


$\#\{\text{invariants of order } 2n\} \sim \left(\frac{3}{2}\right)^n n!$

$\Rightarrow$ Rapid growth of theory space for $p \geq 3$. Universal features at large N?

QG IN $D \geq 3$ AS A TENSOR INTEGRAL?

$$\mathcal{F}(\lambda) = \ln \int \mathrm{d}T \, \exp\left(-T_{abc}T_{abc} + \frac{\lambda}{N^{\alpha}} T_{aeb}T_{bfc}T_{ced}T_{dfa} \right)$$



[Ambjørn, Durhuss, Jónsson '91; Gross '91; Sasakura '91;...]

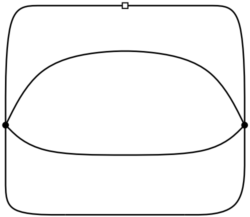
- Challenges:
  - matrix techniques not available (spectral representation)
  - interplay between combinatorics and topology: nice global properties from local Feynman rules?
  - large-N expansion ?

- Path to progress: [Gurau '09; Gurau, Rivasseau, Bonzom,... '10s]
  - more symmetry: $\mathrm{U}(N)^D$ → colored tensor models
  - tractable combinatorics, mapping to sufficiently regular topological spaces.

⇒ universal large-N expansion, in any $D \geq 3$

MELONS

[BONZOM, GURAU, RIELLO, RIVASSEAU '11...]



$$\omega(\Delta) = 0 \qquad \Leftrightarrow \qquad \Delta \text{ is melonic}$$

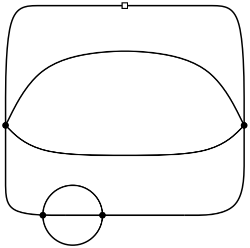
$\rightarrow$ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$\boxed{G(\lambda) = 1 + \lambda G(\lambda)^{D+1}} \qquad \text{(Fuss-Catalan)}$$

MELONS

[BONZOM, GURAU, RIELLO, RIVASSEAU '11...]



$$\omega(\Delta) = 0 \qquad \Leftrightarrow \qquad \Delta \text{ is melonic}$$

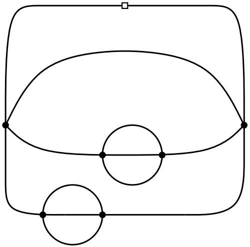
→ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$\boxed{G(\lambda) = 1 + \lambda G(\lambda)^{D+1}} \qquad \text{(Fuss-Catalan)}$$

MELONS

[BONZOM, GURAU, RIELLO, RIVASSEAU '11...]



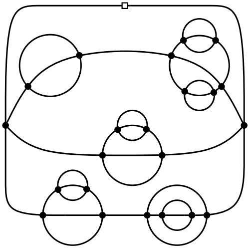
$$\omega(\Delta) = 0 \quad \Leftrightarrow \quad \Delta \text{ is melonic}$$

$\rightarrow$ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$\boxed{G(\lambda) = 1 + \lambda G(\lambda)^{D+1}} \qquad \text{(Fuss-Catalan)}$$

Melons



[Bonzom, Gurau, Riello, Rivasseau '11...]

$$\omega(\Delta) = 0 \quad \Leftrightarrow \quad \Delta \text{ is melonic}$$

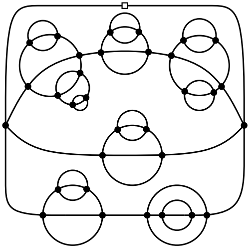
$\rightarrow$ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$G(\lambda) = 1 + \lambda G(\lambda)^{D+1} \qquad \text{(Fuss-Catalan)}$$

MELONS

[BONZOM, GURAU, RIELLO, RIVASSEAU '11...]



$$\omega(\Delta) = 0 \quad \Leftrightarrow \quad \Delta \text{ is melonic}$$

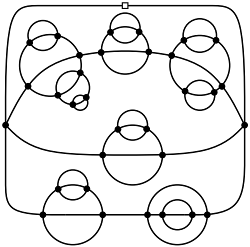
→ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$\boxed{G(\lambda) = 1 + \lambda G(\lambda)^{D+1}} \qquad \text{(Fuss-Catalan)}$$

MELONS

[BONZOM, GURAU, RIELLO, RIVASSEAU '11...]



$$\omega(\Delta) = 0 \quad \Leftrightarrow \quad \Delta \text{ is melonic}$$

→ special triangulations of the D-sphere, with a tree-like combinatorial structure.

Closed equation for their generating function:

$$G(\lambda) = 1 + \lambda G(\lambda)^{D+1} \qquad \text{(Fuss-Catalan)}$$

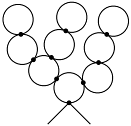
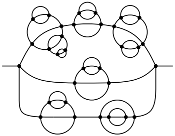
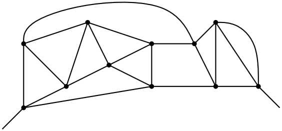
Critical regime / continuum limit:

Melons are branched polymers [Gurau, Ryan '13]
i.e. they converge to the continuous random tree [Aldous '91].

$$d_{\text{spectral}} = 4/3$$

⇒ strong universality: limit independent of D!

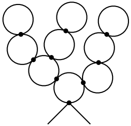
The melonic limit in large-N QFT

Vector field $\phi_a(x)$	Tensor field $T_{abc}(x)$	Matrix field $M_{ab}(x)$
Bubble diagrams	Melon diagrams	Planar diagrams
		
Easy	Tractable	Hard

The melonic limit in large-N QFT

Vector field $\phi_a(x)$

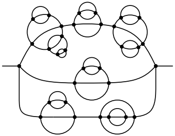
Bubble diagrams



Easy

Tensor field $T_{abc}(x)$

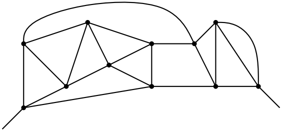
Melon diagrams



Tractable

Matrix field $M_{ab}(x)$

Planar diagrams



Hard

- Strong-coupling regime of the SYK model: disordered model of Majorana fermions [Kitaev; Maldacena, Stanford; Gross, Rosenhaus;...]
- Tensor model realization: fermionic tensor field $\Psi_{abc}(t)$
  - no disorder [Witten '16; Klebanov, Tarnopolsky '16]
  - natural QFT generalizations

[Klebanov et al., Gurau, Benedetti, Harribey, Suzuki, Lettera,...]

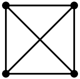
Outline

Models

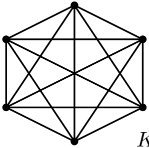
p-index tensor $T_{a_1 \ldots a_p}$, with p odd and measure of the form:

$$d\nu(T) = d\mu_{\boldsymbol{P}}(T) e^{-S_N(T)}$$

- $\boldsymbol{P}$ = orthogonal projector on an irreducible representation of $O(N)$;
- $S_N = -\frac{\lambda}{N^\alpha} \mathrm{Inv}(T)$, where $\mathrm{Inv}(T)$ is a complete-graph invariant (graph K_{p+1}).



$K_4 \quad (p = 3)$



$K_6 \quad (p = 5)$

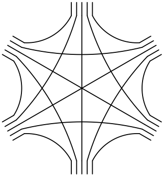
Does this model admit a large N expansion? Is it melonic?

Models

p-index tensor $T_{a_1 \dots a_p}$, with p odd and measure of the form:

$$d\nu(T) = d\mu_{\boldsymbol{P}}(T) e^{-S_N(T)}$$

- $\boldsymbol{P}$ = orthogonal projector on an irreducible representation of $O(N)$;
- $S_N = -\frac{\lambda}{N^\alpha} \mathrm{Inv}(T)$, where $\mathrm{Inv}(T)$ is a complete-graph invariant (graph K_{p+1}).

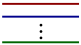


Does this model admit a large N expansion? Is it melonic?

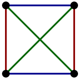
PRECURSORS: COLORED O(N) MODELS

$T_{a_1 a_2 \dots a_p}$, in fundamental representation of $O(N) \times O(N) \times \cdots \times O(N)$:

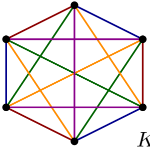
- $\boldsymbol{P}_{a_1 a_2 \dots a_p, b_1 b_2 \dots b_p} = \delta_{a_1 b_1} \delta_{a_2, b_2} \cdots \delta_{a_p, b_p}$



- The complete-graph K_{p+1} is p-edge-colorable



$K_4 \quad (p = 3)$



$K_6 \quad (p = 5)$

Theorem: (Ferrari, Rivasseau, Valette '17)
A melonic large N limit exists for prime p.

($p = 3$: [Tanasa, SC '15])

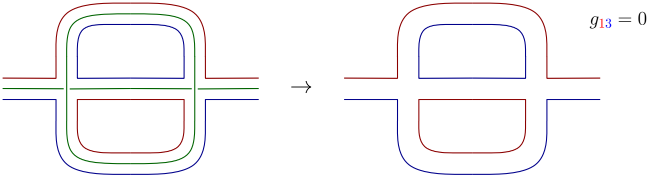
PRECURSORS: COLORED $O(N)$ MODELS

$(p = 3)$ $\qquad \frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$

- $A(G) \sim N^{-\omega}$ with $\omega = 3 + \frac{3}{2}V - F \geq 0$
- G leading order $\Leftrightarrow$ $\omega = 0$ $\Leftrightarrow$ G is a melon diagram

Idea of proof:

- Euler relation: $\omega := g_{13} + g_{12} + g_{23} \in \frac{\mathbb{N}}{2}$, where g_{ij} = genus of the jacket (ij).
- melons are "super-planar" i.e. they have planar jackets



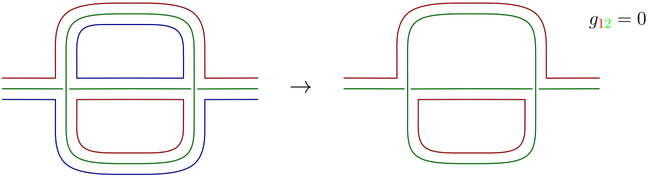
PRECURSORS: COLORED $O(N)$ MODELS

$(p = 3)$ $\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$

- $A(G) \sim N^{-\omega}$ with $\omega = 3 + \frac{3}{2}V - F \geq 0$
- G leading order $\Leftrightarrow$ $\omega = 0$ $\Leftrightarrow$ G is a melon diagram

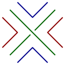
Idea of proof:

- Euler relation: $\omega := g_{13} + g_{12} + g_{23} \in \frac{\mathbb{N}}{2}$, where g_{ij} = genus of the jacket (ij).
- melons are "super-planar" i.e. they have planar jackets



PRECURSORS: COLORED O(N) MODELS

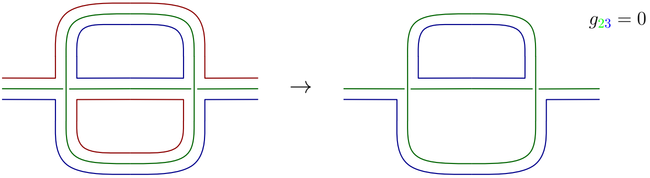
$(p = 3)$ $\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$



- $A(G) \sim N^{-\omega}$ with $\omega = 3 + \frac{3}{2}V - F \geq 0$
- G leading order $\Leftrightarrow$ $\omega = 0$ $\Leftrightarrow$ G is a melon diagram

Idea of proof:

- Euler relation: $\omega := g_{13} + g_{12} + g_{23} \in \frac{\mathbb{N}}{2}$, where g_{ij} = genus of the jacket (ij).
- melons are "super-planar" i.e. they have planar jackets

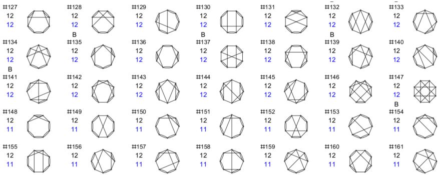


Conjecture (Klebanov–Tarnopolsky '17)

For $p = 3$, $\exists$ melonic large N limit for O(N) symmetric **traceless** tensors.

Evidence. Explicit numerical check of all diagrams up to order λ^8.

[Klebanov, Tarnopolsky, JHEP '17]



Proof and further generalizations.

1. O(N) irreducible, $p = 3$
   [Benedetti, SC, Gurau, Kolanowski, Commun. Math. Phys. '19; SC, JHEP '18]
2. Sp(N) irreducible, $p = 3$ [SC, Pozsgay, Nucl. Phys. B '19]
3. O(N) irreducible, $p = 5$ [SC, Harribey '21]

Much more involved and subtle constructions than in the colored case.

Irreducible tensors – propagator

P = orthogonal projector on one of the irreducible tensor spaces.

example: for traceless tensors with symmetry $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}$

Irreducible tensors – propagator

$\mathbf{P}$ = orthogonal projector on one of the irreducible tensor spaces.

example: for traceless tensors with symmetry

1	2
3	

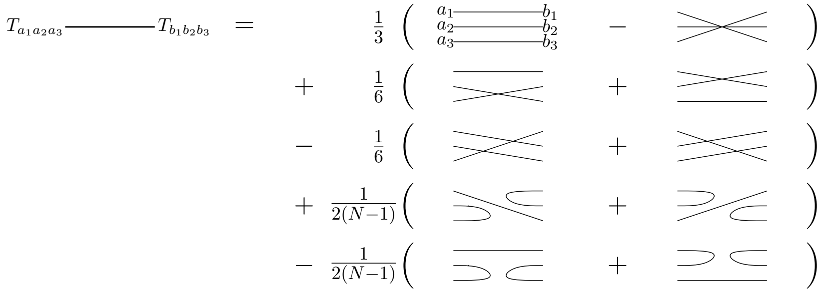
$$\int d\mu_{\mathbf{P}}(T) T_{a_1 a_2 a_3} T_{b_1 b_2 b_3} = \mathbf{P}_{a_1 a_2 a_3, b_1 b_2 b_3} = \frac{1}{3}\left(\delta_{a_1 b_1}\delta_{a_2 b_2}\delta_{a_3 b_3} - \delta_{a_1 b_3}\delta_{a_2 b_2}\delta_{a_3 b_1}\right)$$
$$+ \frac{1}{6}\left(\delta_{a_1 b_2}\delta_{a_2 b_1}\delta_{a_3 b_3} + \delta_{a_1 b_1}\delta_{a_2 b_3}\delta_{a_3 b_2}\right)$$
$$- \frac{1}{6}\left(\delta_{a_1 b_2}\delta_{a_2 b_3}\delta_{a_3 b_1} + \delta_{a_1 b_3}\delta_{a_2 b_1}\delta_{a_3 b_2}\right)$$
$$+ \frac{1}{2(N-1)}\left(\delta_{a_1 b_3}\delta_{a_2 a_3}\delta_{b_1 b_2} + \delta_{a_1 a_2}\delta_{a_3 b_1}\delta_{b_2 b_3}\right)$$
$$- \frac{1}{2(N-1)}\left(\delta_{a_1 b_1}\delta_{a_2 a_3}\delta_{b_2 b_3} + \delta_{a_1 a_2}\delta_{a_3 b_3}\delta_{b_1 b_2}\right)$$

Irreducible tensors – propagator

P = orthogonal projector on one of the irreducible tensor spaces.

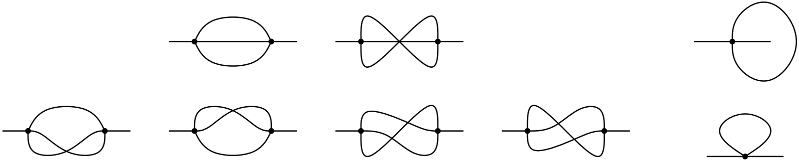
example: for traceless tensors with symmetry

1	2
3	

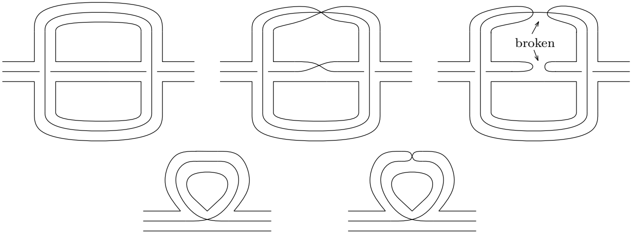


IRREDUCIBLE TENSORS – MAPS AND STRANDED GRAPHS

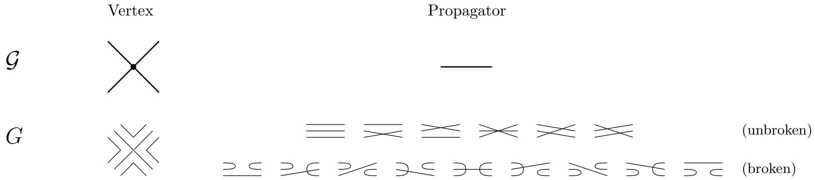
- Feynman expansion $\rightarrow$ combinatorial maps $\mathcal{G}$:



- Decomposition of propagators $\rightarrow$ stranded graphs G:



Irreducible tensors – Feynman amplitudes



$$\mathcal{F}_N(\lambda) = \sum_{\text{connected maps } \mathcal{G}} \frac{\lambda^{V(\mathcal{G})}}{s(\mathcal{G})} A(\mathcal{G})$$

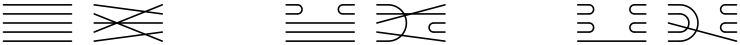
$\mathcal{G}$ decomposes into up to $15^{E(\mathcal{G})}$ stranded graphs G:

$$A(\mathcal{G}) = \sum_G A(G)\,, \qquad A(G) \sim N^{-\omega(G)}$$

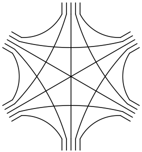
$$\omega(G) = 3 + \frac{3}{2}V(G) + B(G) - F(G)$$

$V = \#\{\text{vertices}\}$, $B = \#\{\text{broken edges}\}$, $F = \#\{\text{faces}\}$

Irreducible tensors – 5-index tensors



Unbroken *Broken* *Doubly-broken*



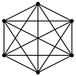
Map $\mathcal{G}$ decomposes into up to $945^{E(\mathcal{G})}$ stranded graphs G:

$$A(\mathcal{G}) = \sum_{G} A(G)\,, \qquad A(G) \sim N^{-\omega(G)}$$

$$\omega(G) = 5 + 5V(G) + B_1(G) + 2B_2(G) - F(G)$$

$B_1 = \#\{\text{broken edges}\}$, $B_2 = \#\{\text{doubly} - \text{broken edges}\}$

MAIN THEOREMS

$$Z_{\boldsymbol{P}}(\lambda, N) = \int \mathrm{d}\mu_{\boldsymbol{P}} \exp\left(\frac{\lambda}{6N^5} \right.$$  $$\left. \vphantom{\frac{\lambda}{6N^5}} \right) \qquad F_{\boldsymbol{P}}(\lambda, N) = \frac{6}{N^5} \lambda \partial_\lambda \ln Z_{\boldsymbol{P}}(\lambda, N)$$

Theorem 1 (SC, Harribey '21) In the sense of formal power series:

$$F_{\boldsymbol{P}}(\lambda, N) = \sum_{\omega \in \mathbb{N}} N^{-\omega} F_{\boldsymbol{P}}^{(\omega)}(\lambda)$$

Theorem 2 (SC, Harribey '21) For sufficiently small λ, $F_{\boldsymbol{P}}^{(0)}(\lambda)$ is the unique continuous solution of the polynomial equation

$$1 - X + m_{\boldsymbol{P}} \lambda^2 X^6 = 0$$

such that $F_{\boldsymbol{P}}^{(0)}(0) = 1$, and where $m_{\boldsymbol{P}}$ is a model-specific real constant.

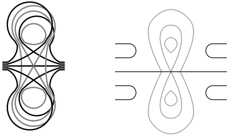
Example. For the symmetric traceless and antisymmetric reps, $m_{\boldsymbol{P}} = \left(\frac{1}{5!}\right)^4$.

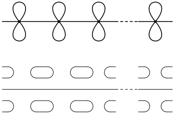
Outline

MAIN DIFFICULTIES

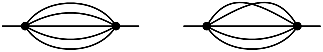
Natural conjecture: For any stranded graph G, $\omega(G) \geq 0$.

× Incorrect ! × Counter-example: chain of "bad double-tadpoles"





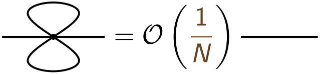
- Subclass of graphs with good scaling properties: stranded graphs containing **no** melon or double-tadpole subgraphs.

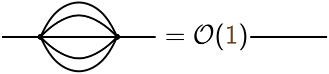


- No global constraint such as Euler's relation available $\Rightarrow$ analysis of local combinatorial structure of G.

PROOF STRATEGY

1. Eliminate melon and double-tadpole 2-point functions at the Feynman map level:





This is where the irreducibility assumption plays a crucial role.

2. Obtain $\mathcal{G}$ with no melon and no double-tadpole.

<u>Proposition:</u> For any stranded configuration G of $\mathcal{G}$, $\omega(G) \geq 0$.

Proof. Induction on $V = \#\{\text{vertices}\}$. Conceptually straightforward but challenging by its complexity.

IDEA OF PROOF – COMBINATORIAL MOVES

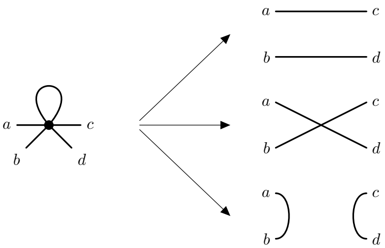
Find local combinatorial moves that:

- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.

Idea of proof – combinatorial moves

Find local combinatorial moves that:

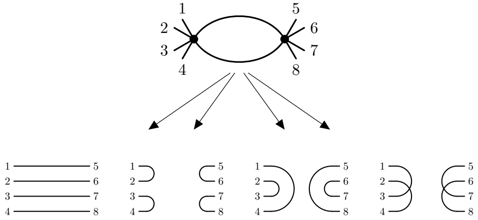
- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.



Idea of proof – combinatorial moves

Find local combinatorial moves that:

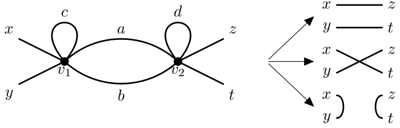
- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.



Idea of proof – combinatorial moves

Find local combinatorial moves that:

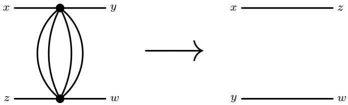
- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.



Idea of proof – combinatorial moves

Find local combinatorial moves that:

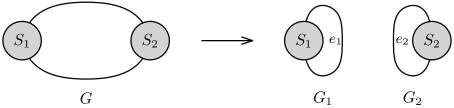
- decrease V;
- decrease ω;
- preserve constraints: connectedness, ∅ melon, ∅ double-tadpole.



Idea of proof – combinatorial moves

Find local combinatorial moves that:

- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.

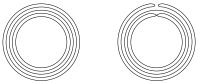


Idea of proof – combinatorial moves

Find local combinatorial moves that:

- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.

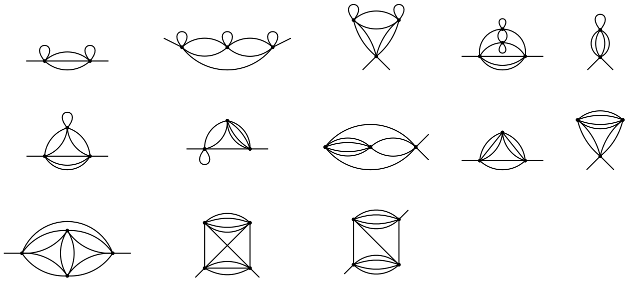
End graphs

- Ring graphs ($V = 0$):  $\cdots$
- G with no face of length 1 or 2 $\Rightarrow \omega(G) > 0$.
- Special cases that need to be treated separately.

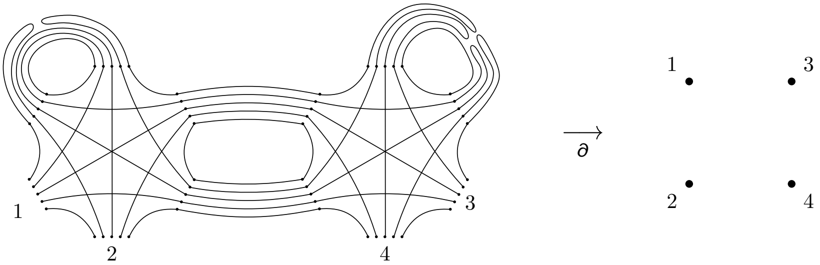
IDEA OF PROOF – COMBINATORIAL MOVES

Find local combinatorial moves that:

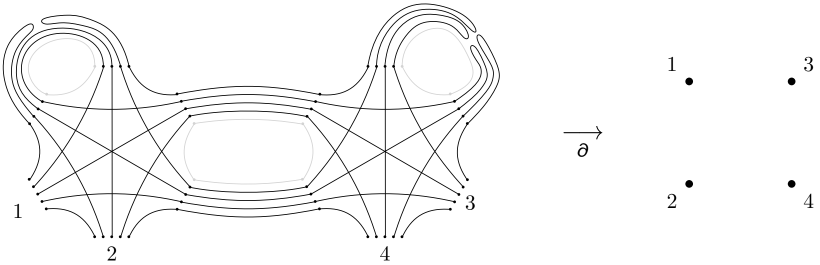
- decrease V;
- decrease ω;
- preserve constraints: connectedness, $\emptyset$ melon, $\emptyset$ double-tadpole.



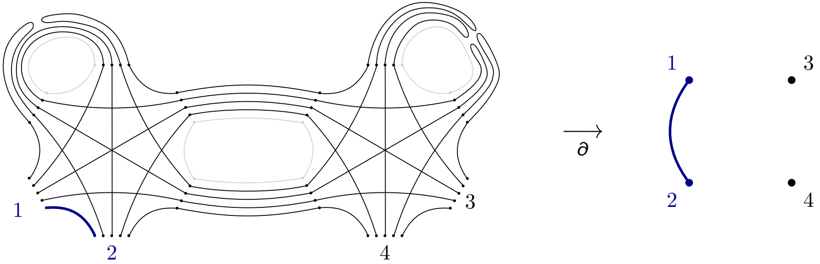
Idea of proof – boundary graphs



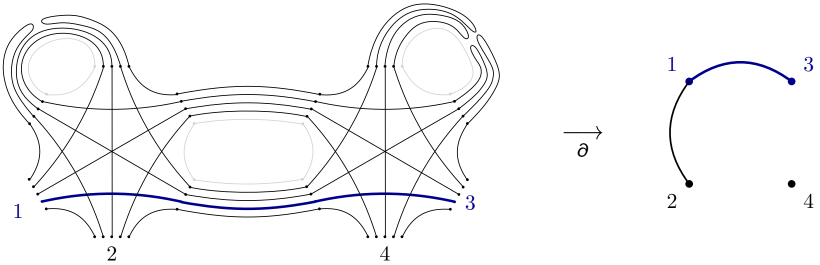
Idea of proof – boundary graphs



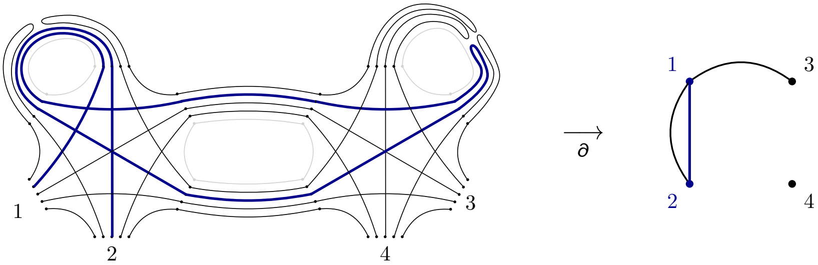
Idea of proof – boundary graphs



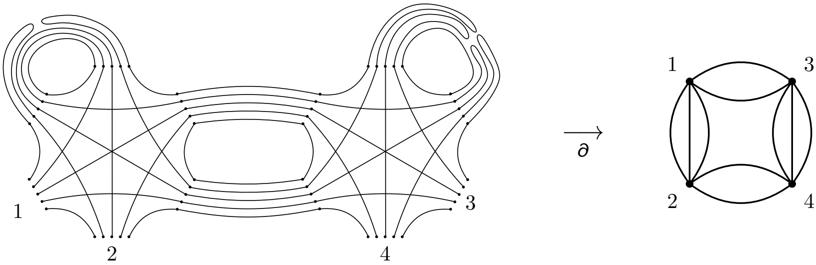
Idea of proof – boundary graphs



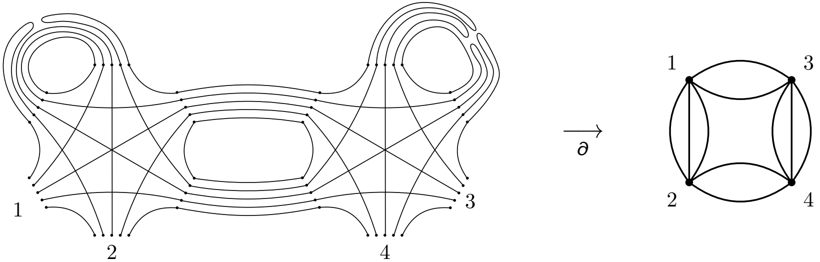
Idea of proof – boundary graphs



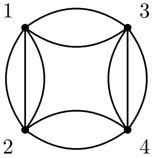
Idea of proof – boundary graphs

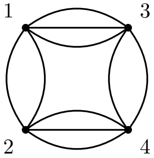


Idea of proof – boundary graphs

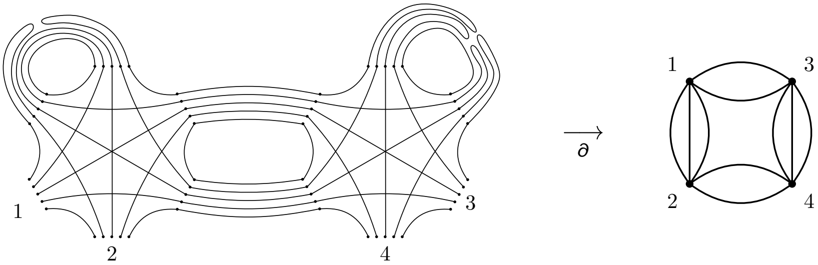


One can recast recursive bounds on ω into bounds on **flip distance** between boundary graphs:

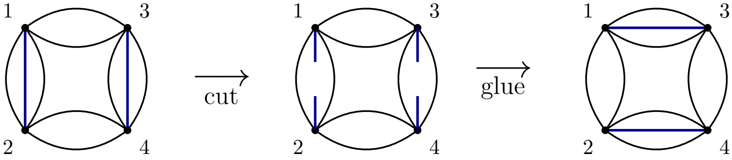




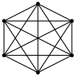
Idea of proof – boundary graphs



One can recast recursive bounds on ω into bounds on **flip distance** between boundary graphs:



MAIN THEOREMS

$$Z_{\boldsymbol{P}}(\lambda, N) = \int \mathrm{d}\mu_{\boldsymbol{P}} \exp\left(\frac{\lambda}{6N^5}\right.$$  $$\left.\right) \qquad F_{\boldsymbol{P}}(\lambda, N) = \frac{6}{N^5}\lambda\partial_\lambda \ln Z_{\boldsymbol{P}}(\lambda, N)$$

Theorem 1 (SC, Harribey '21) In the sense of formal power series:

$$\checkmark \qquad F_{\boldsymbol{P}}(\lambda, N) = \sum_{\omega \in \mathbb{N}} N^{-\omega} F_{\boldsymbol{P}}^{(\omega)}(\lambda)$$

Theorem 2 (SC, Harribey '21) For sufficiently small λ, $F_{\boldsymbol{P}}^{(0)}(\lambda)$ is the unique continuous solution of the polynomial equation

$$1 - X + m_{\boldsymbol{P}}\lambda^2 X^6 = 0$$

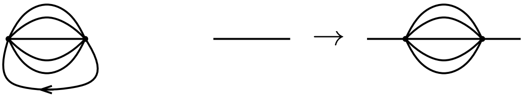
such that $F_{\boldsymbol{P}}^{(0)}(0) = 1$, and where $m_{\boldsymbol{P}}$ is a model-specific real constant.

Example. For the symmetric traceless and antisymmetric reps, $m_{\boldsymbol{P}} = \left(\frac{1}{5!}\right)^4$.

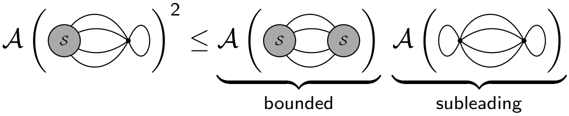
Outline

Melonic dominance

Proposition: $\mathcal{G}$ is leading order $\Leftrightarrow$ $\mathcal{G}$ is melonic.

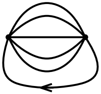


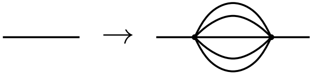
Proof. Cauchy-Schwarz inequalities on maps for which we do not already have strict bounds e.g.



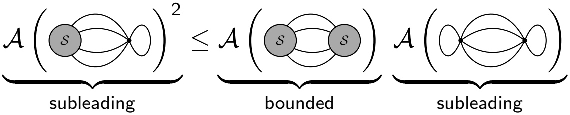
Melonic dominance

Proposition: $\mathcal{G}$ is leading order $\Leftrightarrow$ $\mathcal{G}$ is melonic.



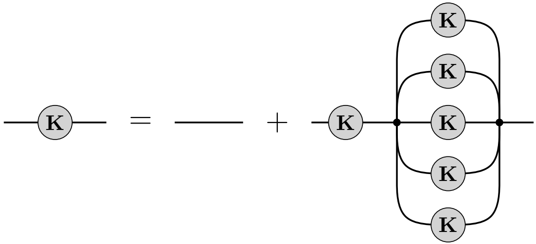


Proof. Cauchy-Schwarz inequalities on maps for which we do not already have strict bounds e.g.



Schwinger–Dyson equation

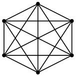
Hallmark of melonic limit: the 2-point function verifies a closed SDE



$\Rightarrow F_{\boldsymbol{P}}^{(0)}$ is a solution of the polynomial equation

$$1 - X + m_{\boldsymbol{P}} \lambda^2 X^6 = 0$$

Main theorems

$$Z_{\boldsymbol{P}}(\lambda, N) = \int \mathrm{d}\mu_{\boldsymbol{P}} \exp\left(\frac{\lambda}{6N^5}\right.$$  $$\left.\right) \qquad F_{\boldsymbol{P}}(\lambda, N) = \frac{6}{N^5}\lambda\partial_\lambda \ln Z_{\boldsymbol{P}}(\lambda, N)$$

Theorem 1 (SC, Harribey '21) In the sense of formal power series:

$$\checkmark \qquad F_{\boldsymbol{P}}(\lambda, N) = \sum_{\omega \in \mathbb{N}} N^{-\omega} F_{\boldsymbol{P}}^{(\omega)}(\lambda)$$

Theorem 2 (SC, Harribey '21) For sufficiently small λ, $F_{\boldsymbol{P}}^{(0)}(\lambda)$ is the unique continuous solution of the polynomial equation

$$\checkmark \qquad 1 - X + m_{\boldsymbol{P}}\lambda^2 X^6 = 0$$

such that $F_{\boldsymbol{P}}^{(0)}(0) = 1$, and where $m_{\boldsymbol{P}}$ is a model-specific real constant.

Example. For the symmetric traceless and antisymmetric reps, $m_{\boldsymbol{P}} = \left(\frac{1}{5!}\right)^4$.

Conclusion and outlook

Large N limit of p-index irreducible random tensors with $p = 5$:

Complete graph interaction + O(N) symmetry $\Rightarrow$ melonic limit.

- In contrast to the matrix case, the irreducibility condition is essential.
- Other interactions, other symmetry groups (e.g. Sp(N)), as well as fermionic tensors can be analyzed with the same method.
- Subleading orders can in principle be characterized as well.

Open questions:

- Generalization to arbitrary (prime) p?
- Useful applications to strongly-coupled QFT?
- Towards a general theory of random tensors?