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The large charge expansion

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based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.08642,1909.02571, 2008.03308, 2010.07942, 2102.12488, 2110.07617, 2110.07616, 2203.12624

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Introduction

Strongly coupled physics is notoriously difficult to access.
We do not have small parameters in which to do a perturbative expansion. Our most basic notions of field theory are of a perturbative nature.

Make use of symmetries, look at special limits/ subsectors where things simplify.

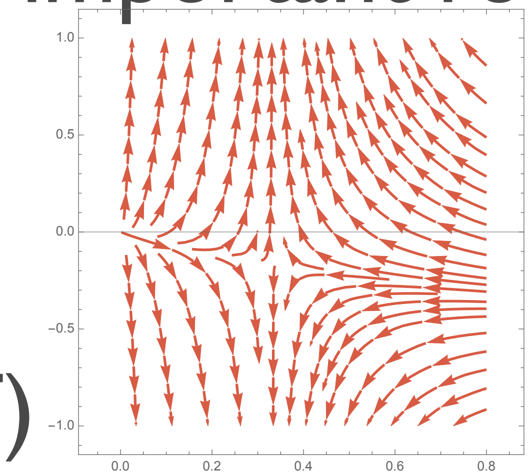
Here: study theories with a global symmetry group. Hilbert space of the theory can be decomposed into sectors of fixed charge Q.
Study subsectors with large charge Q.

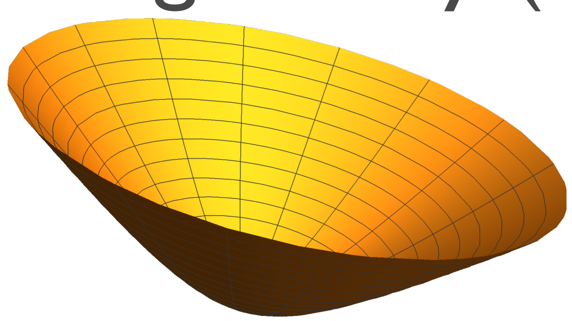
Large charge Q becomes controlling parameter in a perturbative expansion!

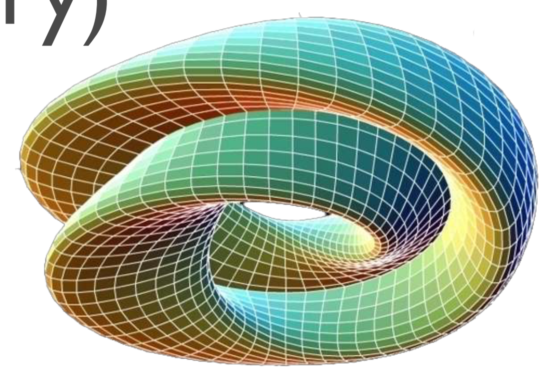
Introduction

Conformal field theories play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity (via AdS/CFT)
- string theory (WS theory)



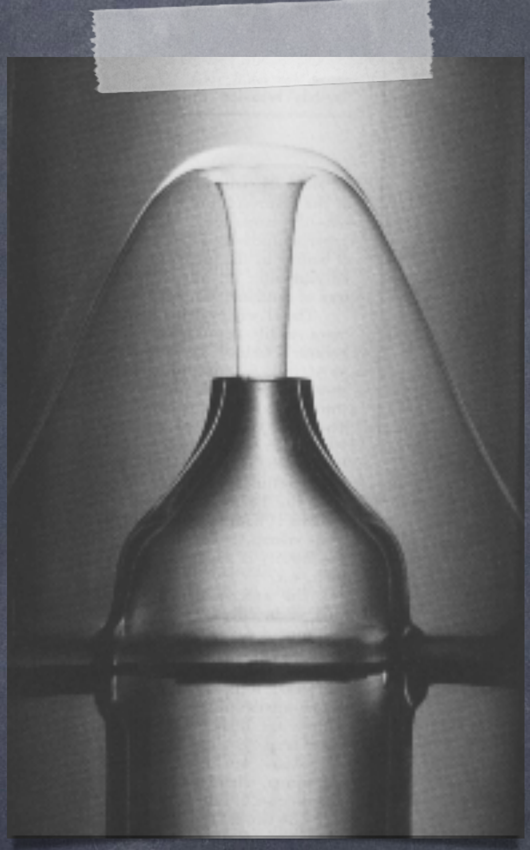




But: CFTs do not have any intrinsic scales, most have by naturalness couplings of O(1).
Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…
Prime candidate for the large-charge approach.

Overview

- Introduction
- The O(2) model
  - semi-classical treatment
  - quantum treatment
  - results and lattice comparison
- O(2N) vector model
  - Finite N results
  - Large N limit + resurgence
  - Leaving the conformal point
- Summary/Outlook



The O(2) model

The O(2) model

Consider simple model: O(2) model in (2+1) dimensions

$$\mathcal{L}_{\mathrm{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

$\varphi_{IR} = a \, e^{ib\chi}$ Global U(1) symmetry: $\chi \to \chi + \text{const.}$

Look at scales: put system in box (2-sphere) of scale R
Second scale given by U(1) charge Q: $\rho^{1/2} \sim Q^{1/2}/R$

Study the CFT at the fixed point in a sector with

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

UV scale (g^2)

cut-off of effective theory (Λ)

Write Wilsonian (linear sigma model) action.

The O(2) model

LSM: Assume large vev for a: $\Lambda \ll a^2 \ll g^2$

scalar curvature (w. conf. coupling)

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

dimensionless constants

ϕ has approximately mass dimension 1/2 and the action has a potential term $\propto |\varphi|^6$

Lagrangian is approximately scale-invariant.

Semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

Solution must be homogeneous in space.

The O(2) model

Charge density: $\rho = b^2 a^2 \dot{\chi},$ $Q = \rho \cdot \mathrm{Vol}(S^2)$

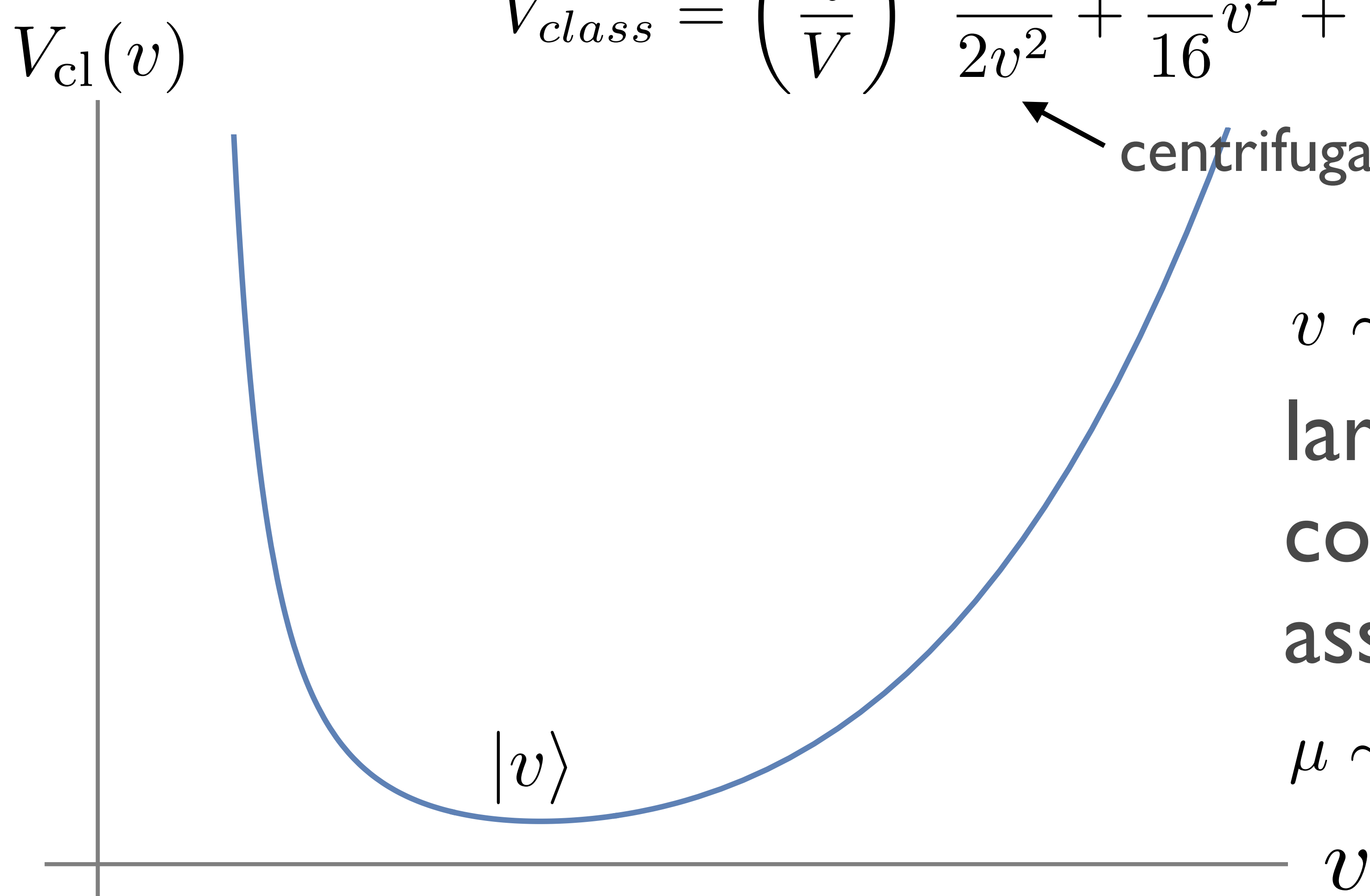
Lowest-energy solution: $a = v = \mathrm{const.}$

$\chi = \mu t,$ ← non-const. vev $\mu = \frac{\rho}{b^2 v^2}$

Determine radial vev v by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16} v^2 + \frac{\lambda}{6} v^6$$

↑ centrifugal term



$v \sim Q^{1/4}$

large condensate is compatible with our assumption $a \gg 1$

$\mu \sim \rho^{1/2}$

The O(2) model

Energy of classical ground state at fixed charge:

2 dimensionless parameters

cannot be calculated within EFT!

$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

Quantum story: study the low-energy spectrum
Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu t + \frac{\hat{\chi}}{v}$$

Goldstone

massive mode, not relevant for low-energy spectrum $m \sim \mathcal{O}(\sqrt{Q})$

The O(2) model

Ground state at fixed charge breaks symmetries:

$$SO(3,2) \times O(2) \to SO(3) \times D \times O(2) \rightsquigarrow SO(3) \times D'$$

$$D' = D - \mu O(2)$$

Go to NLSM: Integrate out a (saddle point for LO).
Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2}$$

can get this purely by dimensional analysis

Beyond LO: use dimensional analysis, parity and scale invariance to determine (tree-level) operators in effective action (Lorentz scalars of scaling dimension 3, including couplings to geometric invariants)

Use ρ-scaling to determine which terms are not suppressed: $\partial\chi \sim \rho^{1/2}, \quad \partial \ldots \partial\chi \sim \rho^{-1/4}$

The O(2) model

Result for NLSM action in d=3:

scale-inv. but NOT conformally inv.

LO Lagrangian

$$\mathcal{L} = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \partial^\mu \chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by inverse powers of Q

To be understood as an expansion around the classical ground state $\mu t + \hat{\chi}$

Expand action around GS to second order in fields:

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2} R\mu + (\partial_t \hat{\chi})^2 - \tfrac{1}{2}(\nabla_{S^2}\hat{\chi})^2 + \dots$$

Compute zeros of inverse propagator for fluctuations and get dispersion relation:

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

dictated by conf. invariance $1/\sqrt{d}$

⇒ χ is indeed a Goldstone (type I)

The O(2) model

Are also the quantum effects controlled?

Yes! All effects except Casimir energy of χ are suppressed (negative ρ-scaling).

Evaluate Casimir energy from Coleman-Weinberg formula:

$$E_{\text{Cas}} = \lim_{T\to\infty} \frac{1}{2T} \log\det\left(-\partial_\tau^2 - \frac{1}{d}\Delta_{S^2}\right) = \frac{1}{2\sqrt{d}} \operatorname{Tr}(-\Delta_{S^2}^{1/2})$$

$$= \frac{1}{2\sqrt{2}}\, \zeta(s|S^2)\big|_{s=-1/2} = -0.0937\ldots$$

Effective theory at large Q:

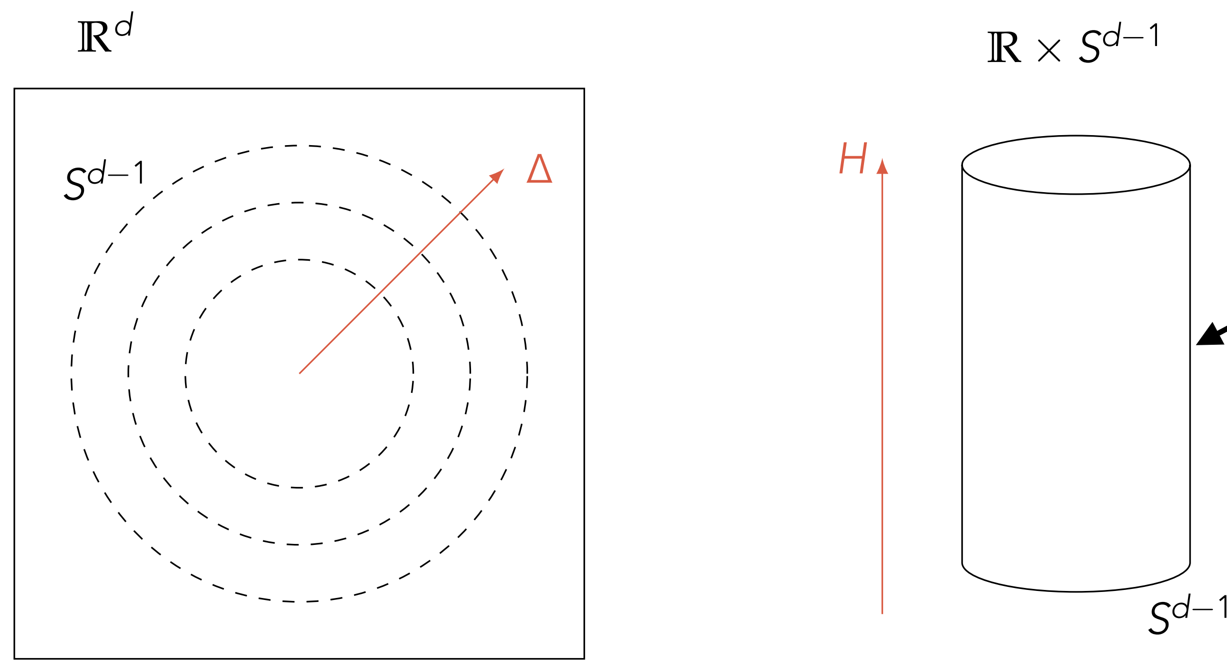
vacuum + Goldstone + 1/Q-suppressed corrections

Let's calculate observables: CFT data!
Scaling dimensions, fusion coefficients.

The O(2) model

Use state-operator correspondence of CFT:

conformal dimension



energy

Scaling dimension of lowest operator of charge Q:

energy of class. ground state

$$D(Q) = R_0(E_0 + E_{Cas}) = c_{3/2}Q^{3/2} + c_{1/2}Q^{1/2} - 0.0937\cdots + \mathcal{O}(Q^{-1/2})$$

quantum correction from Casimir energy of Goldstone

S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

The O(2) model

What about excited states?

Dispersion relation: $\omega_l = \frac{1}{R_0}\sqrt{\frac{E_{\Delta_{S^d}}}{d}} = \frac{1}{R_0}\sqrt{\frac{l(l+d-1)}{d}}$

Energy of an excited state:

$$E_Q^{(n_1,n_2,\dots)} = \frac{D(Q)}{R_0} + \frac{1}{R_0}\sum_l n_l \omega_l$$

Take l=1: lowest excited state, D=D(Q)+1. Descendant state! $n_1 = 0$: conformal primary

Lowest spin=1 state: one mode with l=2.

Lowest excited scalar: tensor product of two l=2 oscillators:

$$\square\square \otimes \square\square = \square\square\square\square \oplus \square\square \oplus \mathbf{1}$$

Lowest vector: product of one l=2 and one l=3 oscillators:

$$\square\square\square \otimes \square\square = \square\square\square\square\square \oplus \square\square\square \oplus \square$$

States with large charge and arbitrary spin (below unitarity bound): use particle-vortex duality.

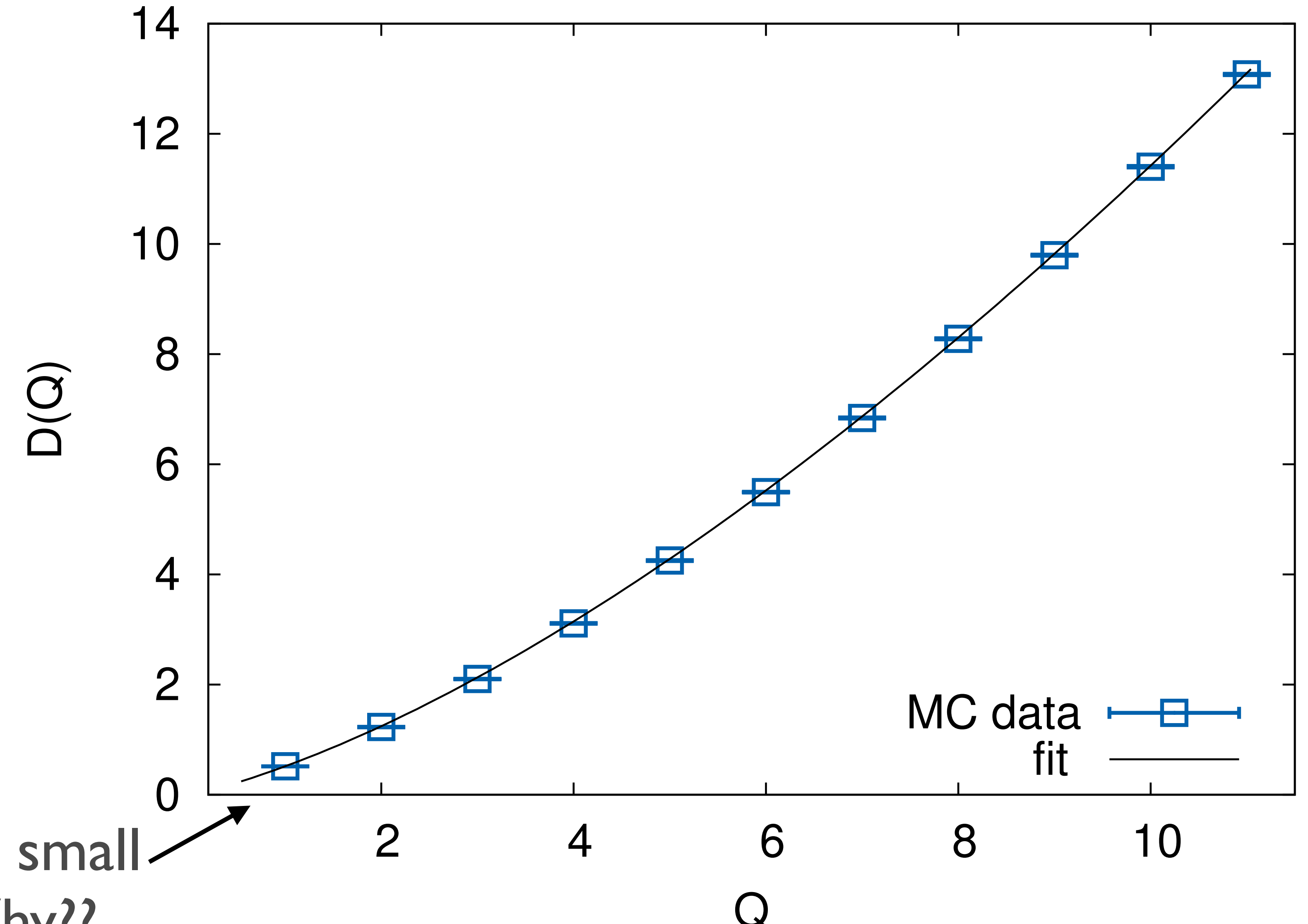
G. Cuomo, A. de la Fuente, A. Monin, D. Pirtskhalava, R. Rattazzi, arXiv:1711.02108 [hep-th]

The O(2) model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent calculation on the lattice:



works for small charge. Why??

Excellent agreement!!

$$c_{3/2} = 1.195(10)$$
$$c_{1/2} = 0.075(10)$$

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Large-charge expansion works extremely well for O(2). Where else?

The O(2) model

Let's look at states with a one-phonon (type I Goldstone) excitation:

$$|{}^{Q}_{\ell m}\rangle = a^{\dagger}_{\ell m}|Q\rangle,$$

To leading order, the 2-pt function on the cylinder of two such states is

time-evolution operator on cylinder

$$\langle {}^{Q}_{\ell_2 m_2} \,|\, {}^{Q}_{\ell_1 m_1} \rangle = \langle Q \,|\, a_{\ell_2 m_2}\, e^{-(\tau_2-\tau_1)D/R} a^{\dagger}_{\ell_1 m_1} \,|\, Q\rangle = R^{\Delta} e^{-(E_0+\omega_\ell)(\tau_2-\tau_1)/R} \delta_{\ell_1 \ell_2} \delta_{m_1 m_2}$$

$$\Delta = \frac{E_0+\omega_\ell}{R}$$

Next, we look at the 3-pt function of two such states with a current insertion:

tree-level contribution (homogeneous)

$$\langle \mathcal{O}^{-Q}_{\ell_2 m_2} J_\tau(\tau,\mathbf{n}) \mathcal{O}^{Q}_{\ell_1 m_1}\rangle = -i \frac{Q}{\Omega_D R^{D-1}} \Bigg\{ \mathcal{A}_{\Delta_Q + R\omega_{\ell_2}}(\tau_1,\tau_2)\, \delta_{\ell_1 \ell_2} \delta_{m_1 m_2}$$
$$+ \mathcal{A}^{\Delta_Q+R\omega_{\ell_2}}_{\Delta_Q+R\omega_{\ell_1}}(\tau_1,\tau_2 \mid \tau)\, (D-1)(D-2)\Omega_D \frac{R\sqrt{\omega_{\ell_2}\omega_{\ell_1}}}{2D\Delta_0} \left[Y^*_{\ell_2 m_2}(\mathbf{n}) Y_{\ell_1 m_1}(\mathbf{n}) - \frac{\partial_i Y^*_{\ell_2 m_2}(\mathbf{n})\, \partial_i Y_{\ell_1 m_1}(\mathbf{n})}{R^2 (D-1)\omega_{\ell_2}\omega_{\ell_1}} \right] \Bigg\}, \tag{4.6}$$

from fluctuations

$$\langle \mathcal{O}^{-Q}_{\ell_2 m_2} J_i(\tau,\mathbf{n}) \mathcal{O}^{Q}_{\ell_1 m_1}\rangle = i \frac{Q(D-2)}{2\Delta_0 R^{D-1} D} \mathcal{A}^{\Delta_Q+R\omega_{\ell_2}}_{\Delta_Q+R\omega_{\ell_1}}(\tau_1,\tau_2 \mid \tau) \left[\sqrt{\frac{\omega_{\ell_2}}{\omega_{\ell_1}}} Y^*_{\ell_2 m_2}(\mathbf{n})\, \partial_i Y_{\ell_1 m_1}(\mathbf{n}) - (1 \leftrightarrow 2)^* \right].$$

quantum piece only since class. part of insertion is 0

$$\mathcal{A}_{\Delta}(\tau_1,\tau_2) := e^{-\Delta(\tau_2-\tau_1)}, \qquad \mathcal{A}^{\Delta_2}_{\Delta_1}(\tau_1,\tau_2 \mid \tau) := e^{-\Delta_2(\tau_2-\tau)/R - \Delta_1(\tau-\tau_1)/R}.$$

The O(2) model

The 3-pt coefficient is

$$C_{\mathcal{O}^{Q}_{\ell m} J_\tau}{}^{\mathcal{O}^{Q}_{\ell m}} = \frac{\langle \mathcal{O}^{-Q}_{\ell m} J_\tau(\tau, n) \mathcal{O}^{Q}_{\ell m} \rangle}{\langle \mathcal{O}^{-Q}_{\ell m} \mathcal{O}^{Q}_{\ell m} \rangle} = -i \frac{Q}{\Omega_D R^{D-1}}.$$

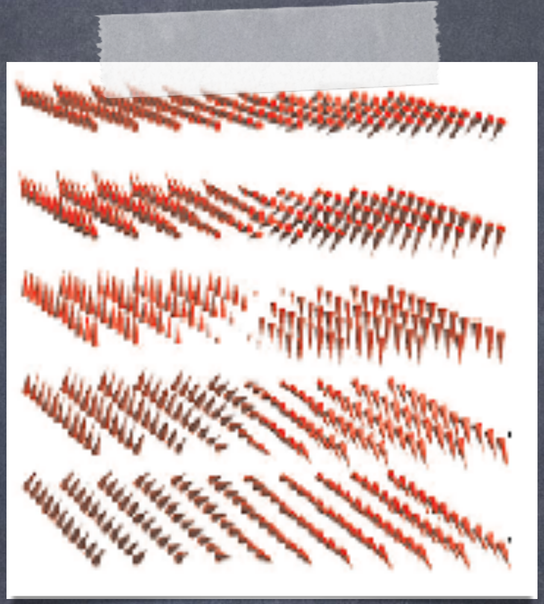
These results can be easily generalized to states with more phonon excitations, insertions of T, JJ, TT, TJ etc.

When studying loop corrections to the scaling dimension, we find in even D contributions of the form

$$\Delta_l \supset \frac{1}{Q^{(l-1)D/(D-1)}} \left(\alpha_0 + \alpha_1 \log Q + \ldots + \alpha_l (\log Q)^l \right).$$

N. Dondi, R. Moser, I. Kalogerakis, D. Orlando, S. R., arXiv:2203.12624 [hep-th]

Other 3- and 4 pt correlators (mostly involving just the scalar ground state), both with current insertions and of the for HLH have appeared in the literature (Rattazzi et al, Cuomo, Komargodski et all, Jafferis et al.)



Beyond O(2):
3d O(2N) vector model

The O(2n) vector model

Generalize to O(2n). LSM action (2+1 dim):

$$L = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{2}\sum_{i=1}^{2n}\left(\frac{1}{8}R\phi^a\phi^a + \frac{\lambda}{12}(\phi^a\phi^a)^3\right), \quad a = 1, \dots, 2n \qquad \mathbb{R}_t \times \mathbb{R}^2$$

Conserved current is matrix-valued and transforms under O(2n):

$$(j^\mu)^{ab} = (\phi^a\, \partial^\mu \phi^b - \phi^b\, \partial^\mu \phi^a)$$

Charge can be decomposed on generators of so(2n):

$$Q = \int \mathrm{d}x j^0 = \sum_A q_A T^A$$

can fix only the n Cartan generators! (T^A)

fix these coefficients (q_A)

Introduce complex coordinates: $U(n) \subset O(2n)$

$$\varphi^I = \phi^{2I-1} + i\phi^{2I}, \quad I = 1, \dots, n$$

Find classical ground state (ansatz: homogeneous):

$$\varphi = (A^1, \dots, A^n)e^{i\mu t}, \qquad \|\varphi\|^2 = v^2$$

only one μ

$$v^2 = A_1^2 + \dots + A_n^2 = \frac{1}{\mathrm{Vol}(S^2)}\frac{\sum_I q_I}{\sqrt{V'(v^2)}}$$

$$\mu^2 = V'(v^2)$$

The O(2n) vector model

Fixing n charges explicitly breaks O(2n) to U(n).

Can do a basis transformation (U(n) rotation) in which the ground state has the form

$$\varphi = (v, 0, \ldots, 0) e^{i\mu t}$$

In this basis, the charge matrix has the form

$$Q^{ab} = \sum q_I H^1 = \begin{pmatrix} 0 & q & & \\ -q & 0 & & \\ & & 0 & \\ & & & \ddots \end{pmatrix}$$

Vacuum breaks symmetry spontaneously to U(n-1).

We see that all homogeneous states of minimal energy with fixed total charge q are related by a U(n) transformation and have the same energies (and conformal dimensions).

The O(2n) vector model

What happens if instead, we choose a configuration with n different chemical potentials that cannot be rotated into the state $\varphi = (v, 0, \ldots, 0)e^{i\mu t}$?

Ground state must be inhomogeneous!

For quantum description, study fluctuations around the ground state:

$$\varphi^I(t,x) = e^{i\mu t}\left(\frac{1}{\sqrt{2}}A^I + \pi^I(t,x)\right)$$

Expand action to quadratic order in fluctuations.
In the rotated basis, the fluctuation in the 0-direction decouples from the others:

$$\mathcal{L}_\pi = \mathcal{L}[\pi^0] + \sum_i \mathcal{L}[\pi^i]$$

same as for conformal Goldstone in O(2)!

$$\mathcal{L}[\pi^0] = \overline{D_0\pi^0}D^0\pi^0 - \overline{\nabla_j\pi^0}\nabla^j\pi^0 - \mu^2\bar{\pi}^0\pi^0 + \frac{v^2}{8}V''(v^2/2)(\pi^0+\bar{\pi}^0)^2,$$

$$\mathcal{L}[\pi^i] = \overline{D_0\pi^i}D^0\pi^i - \overline{\nabla_j\pi^i}\nabla^j\pi^i - \mu^2\bar{\pi}^i\pi^i$$

new!

$$D_0\pi^I = (\partial_0 + i\mu)\pi^I$$

The O(2n) vector model

New sector contains n-1 massive modes with m=2μ and n-1 massless fields with dispersion relation

$$\omega = \frac{p^2}{2\mu} + \mathcal{O}\mu^{-3}$$

We find all in all:

- a universal sector already present for n = 1 governed by a conformal type I Goldstone
- a sector of n-1 massless modes with quadratic dispersion relation typical of non-relativistic type II Goldstones. They are paired with n-1 massive modes. The non-relativistic Goldstones count double.

Nielsen and Chadha; Murayama and Watanabe

The symmetry-breaking pattern is

$$SO(3,2) \times O(2n) \to SO(3) \times D \times U(n) \rightsquigarrow SO(3) \times D' \times U(n-1)$$

We expect thus dim[U(n)/U(n-1)] = n-1 Goldstone d.o.f.

The O(2n) vector model

Counting type I and type II modes, indeed,

$$1 + 2(n-1) = 2n - 1 = \dim(U(n)/U(n-1))$$

Non-relativistic Goldstones have no zero-point energy in flat space and contribute to the conformal dimensions only at higher order.
The ground-state energy is again determined by a single relativistic Goldstone!

Same formula for anomalous dimensions as for O(2):

n-dependent

universal for O(2n)

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

verified at large n for CP(n-1) model

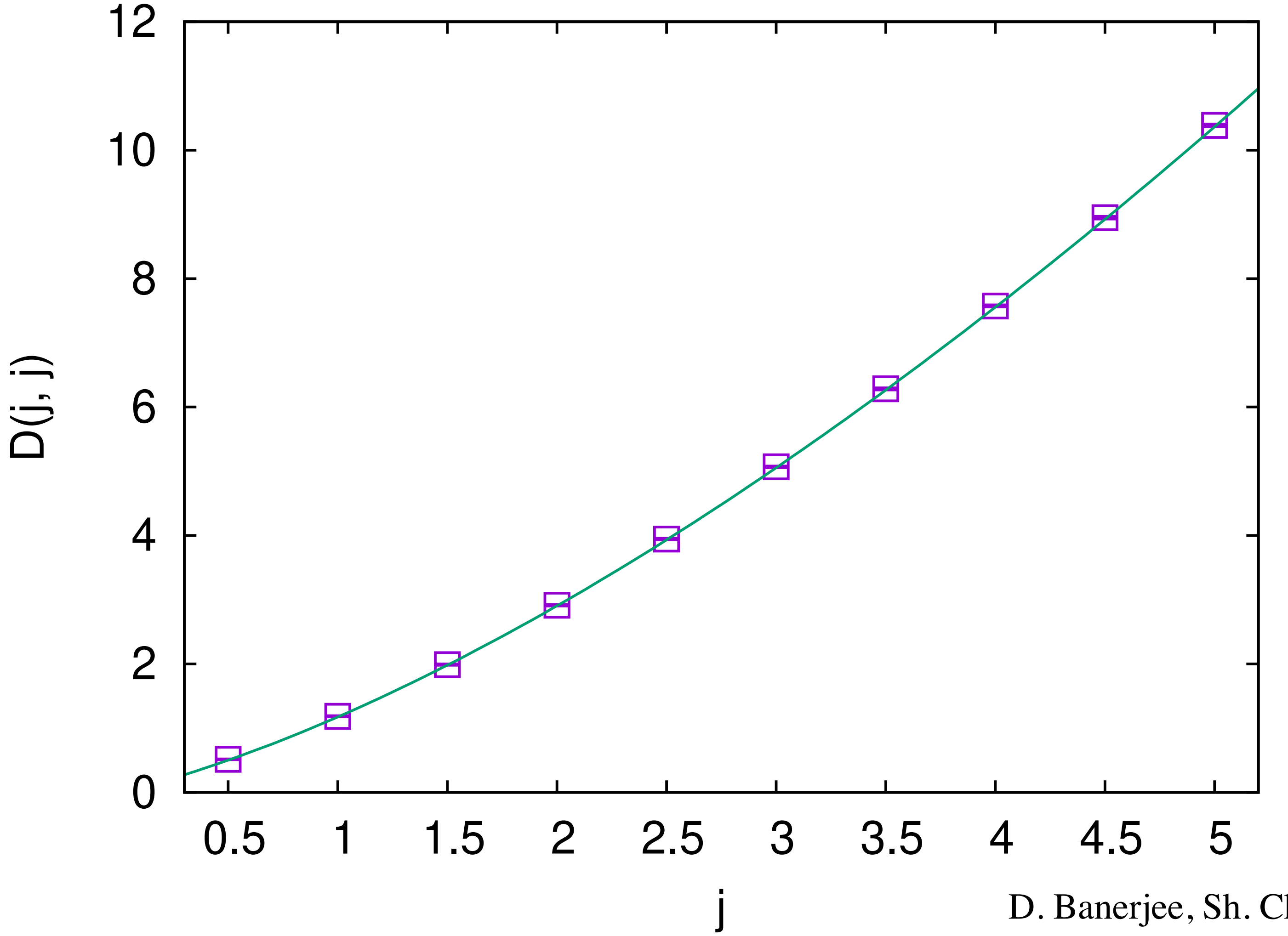
de la Fuente

The O(2N) vector model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

New lattice data for O(4) model:



$c_{3/2} = 1.068(4)$

$c_{1/2} = 0.083(3)$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

Again excellent agreement with large-Q prediction!

The O(2N) vector model

Let's take the large N limit!

Standard large-N methods (Stratonovich transformation, integrating out fields + charge fixing)

Start from first principles, expand path integral around saddle point (no EFT!)

Leading order: theory is solvable and we find the same powers in the large-Q expansion of the scaling dimension.

NLO in N: reproduce dispersion relations of Goldstones.

Since we have an extra control parameter at large N, we can go beyond simply verifying known results!

The O(2N) vector model

Find coefficients of the expansion (leading order in N):

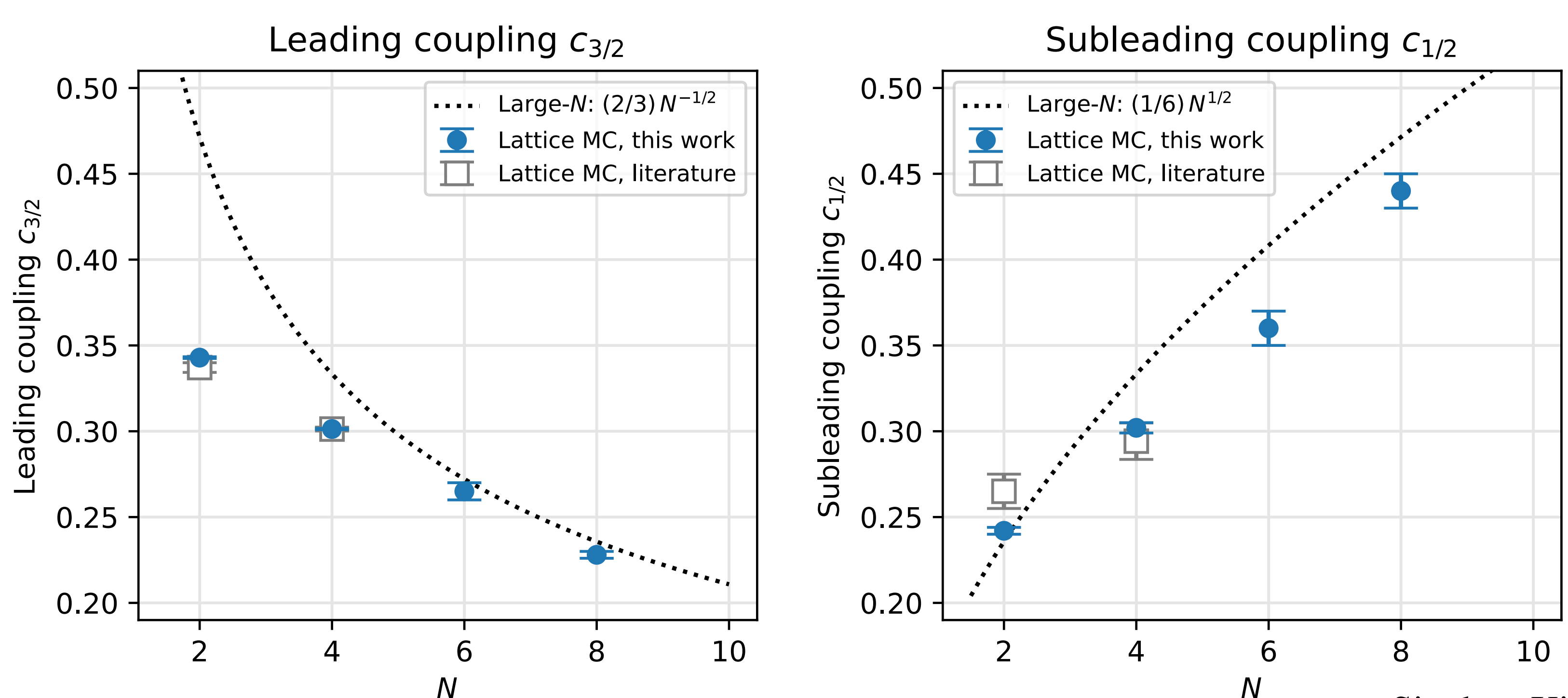
$$c_{3/2} = 4/3\sqrt{\pi/n}$$
$$c_{1/2} = 1/12\sqrt{n/\pi}$$

L. Alvarez-Gaume, D. Orlando, S.R. 1909.02571

Within 10% of the lattice measurements for O(4):

$$c_{3/2}^{n=4} = 1.18 \qquad c_{3/2} = 1.068(4)$$
$$c_{1/2}^{n=4} = 0.094 \qquad c_{1/2} = 0.083(3)$$

New lattice data:



Singh, arXiv:2203.00059 [hep-lat]

The O(2N) vector model

Small charge limit: At large N, we now have more control and can also take the limit of $Q/N \ll 1$.

In this limit, the operator of charge Q whose dimension we are calculating is φ^Q.

engineering dimension of ϕ

$$\frac{\Delta(Q)}{Q} = \frac{1}{2} + \frac{4}{\pi^2}\frac{Q}{N} + \mathcal{O}\left(\frac{Q}{2N}\right)^2$$

tree-level

one-loop

Jack, Jones; Antipin et al.

Can be verified by a perturbative (loop) calculation around the zero-charge vacuum (Benvenuti, unpublished)!

The O(2N) vector model

We can interpolate between the large-Q and small Q limits of the O(2N) vector model using resurgence.

The large-Q expansion is an asymptotic series which diverges as (2L)!

The optimal truncation is $\mathcal{O}(\sqrt{Q})$ terms. This explains why the comparison to the lattice calculation works so well.

We can write the transseries and the non-perturbative corrections go like

$$e^{-2\pi k\sqrt{Q/2n}}$$

The result for the scaling dimension in the small-Q expansion and the resurgence result agree at least to 10 digits.

A. Dondi, I. Kalogerakis, D.Orlando, S.R, arXiv: 2102.12488 [hep-th]

The O(2N) vector model

Leaving the conformal point: start in the UV with

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (\partial^i_\mu \phi_i)^\dagger (\partial^i_\nu \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N} (\phi_i^\dagger \phi_i)^2 \right]$$

For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

Instead: keep u finite.

Perform Stratonovich transform and add a chemical potential (= introduce covariant derivative) $D_0 = (\partial_0 + m)$

$$S[\phi_i, \lambda] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (D^i_\mu \phi_i)^\dagger (D^i_\nu \phi_i) + (r + \lambda)(\phi_i^\dagger \phi_i) - \frac{\lambda^2}{2u} \right].$$

Can integrate out ϕ_i. Because of the chemical potential, λ gets a vev m^2.

Adding the chemical potential gives us more structure to work with!

The O(2N) vector model

Leading order in N:

in flat space

$$\frac{\mathcal{L}}{2N} = -\frac{1}{2V}\zeta(-\frac{1}{2}|\Sigma, m) + \frac{(m^2-r)^2}{4u} = \left[\frac{m^3}{12\pi} + \frac{(m^2-r)^2}{4u}\right]$$

This is exactly the NLSM for $m^2 = \partial_\mu \chi \partial^\mu \chi$

This expression contains the full information about the model. More transparent, if we extract the effective potential. The LSM has the form

vev of radial mode

$$\mathcal{L}_{\text{LSM}} = \Phi^2 m^2 - V(\Phi)$$

vev of angular mode

E.o.m. for radial mode:

$$m^2 - \frac{\mathrm{d}}{\mathrm{d}(\Phi^2)}V = 0$$

Plugging the solution back in, we must recover

$$\Phi^2 m^2 - V(\Phi)\Big|_{\Phi=\Phi(m^2)} = \mathcal{L}(m)$$

L is the Legendre transform of V in Φ^2

The O(2N) vector model

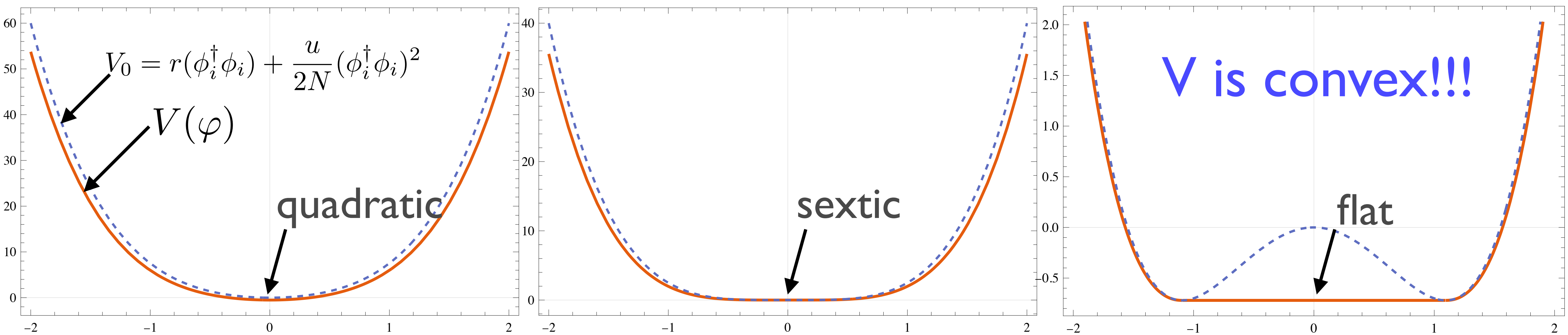
Pay attention to convexity! Use more general definition of Legendre transform: $f^*(y) = \sup_x (xy - f(x))$

$$V(\Phi^2) = \sup_{m^2}(m^2\Phi^2 - \mathcal{L}(m))$$

$$V(\varphi) = \frac{u^3}{3 \cdot 2^{10}\pi^4}\left(1 + \frac{3}{2}\eta + \frac{3}{4}\eta^2 - (1+\eta)^{3/2}\right) \qquad \eta = 64\pi^2\left(\frac{|\varphi|^2}{Nu} + \frac{r}{u^2}\right)$$

There are three cases, depending on the value of r:

r>0 | r=0 | r<0



unbroken phase | critical point | broken phase

$$V(|\varphi|) = \begin{cases} -\frac{Nr^2}{2u} & \text{for } 0 < |\varphi| < \sqrt{-\frac{rN}{u}} \\ \hat{V}(|\varphi|) & \text{for } |\varphi| > \sqrt{-\frac{rN}{u}} \end{cases}$$

Orlando, Reffert, Schmidt 2110.07616

The O(2N) vector model

In the critical case on the cylinder, if the take the Legendre transform of L w.r.t. m, we get the free energy which corresponds to the scaling dimension of the lowest operator of a given charge.

$$\Delta(Q) = \sup_m (mQ - \mathcal{L}(m))$$

$$\frac{\Delta(Q)}{2N} = \frac{2}{3}\left(\frac{Q}{2N}\right)^{3/2} + \frac{1}{6}\left(\frac{Q}{2N}\right)^{1/2} - \frac{7}{720}\left(\frac{Q}{2N}\right)^{-1/2} - \frac{71}{181440}\left(\frac{Q}{2N}\right)^{-3/2} + \dots$$

L. Alvarez-Gaume, D. Orlando, S.R. 1909.02571

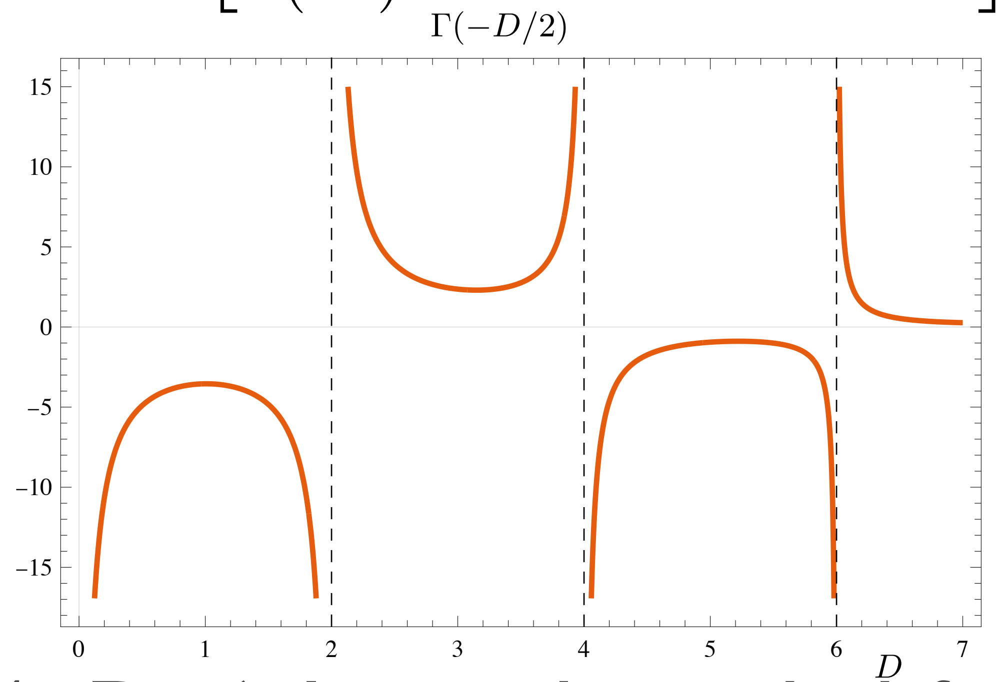
This is always convex thanks to the supremum.

All these results are straightforwardly obtained thanks to the interplay between large Q and large N - no Feynman diagrams needed!

The O(2N) vector model

Repeat the above analysis for general dimension.

$$\mathcal{L} = (2N)\left[\frac{\Gamma(-D/2)}{2(4\pi)^{D/2}} m^D + \frac{(m^2 - r)^2}{4u}\right]$$



We see that for 4<D<6, L is unbounded from below. Instability!

If we formally compute the conformal dimension for D=5:

$$\Delta(Q) = r_0 F_{S^4}(Q) = 2N\left[f_1 \frac{4\sqrt{3}}{5}\left(\frac{Q}{2N}\right)^{\frac{5}{4}} - \frac{f_2}{\sqrt{3}}\left(\frac{Q}{2N}\right)^{\frac{3}{4}}\right],$$

	branch 1	branch 2	branch 3	branch 4
f_1	$e^{i\pi/4}$	$e^{-i\pi/4}$	$e^{3\pi i/4}$	$e^{-3\pi i/4}$
f_2	$e^{3i\pi/4}$	$e^{-3i\pi/4}$	$e^{\pi i/4}$	$e^{-\pi i/4}$

Interpretation as non-unitary CFT.

Giombi, Hyman;
Moser, Orlando, Reffert 2110.07617

Summary

Summary

We studied some CFTs in sectors of large global charge

Concrete examples where a (strongly-coupled) CFT simplifies in a special sector.

O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

- Excellent agreement with lattice results for O(2), O(4)
- large Q and large N: path integral at saddle pt., more control than in EFT, can calculate coefficients
- can follow the flow away from conformal point
- find the full effective potential

Summary

Q&A:

- Does the large-Q expansion work?
  - For all the examples, we tried, yes! Confirmation from lattice data (O(2) and O(4))
- For what kinds of theories does it work?
  - (S)CFTs and non-relativistic CFTs
- In how many space-time dimensions?
  - d>=1 space dimensions
- For what kinds of global symmetries does it work?
  - we checked U(1), O(2n) vector models, SU(N) matrix models

Summary

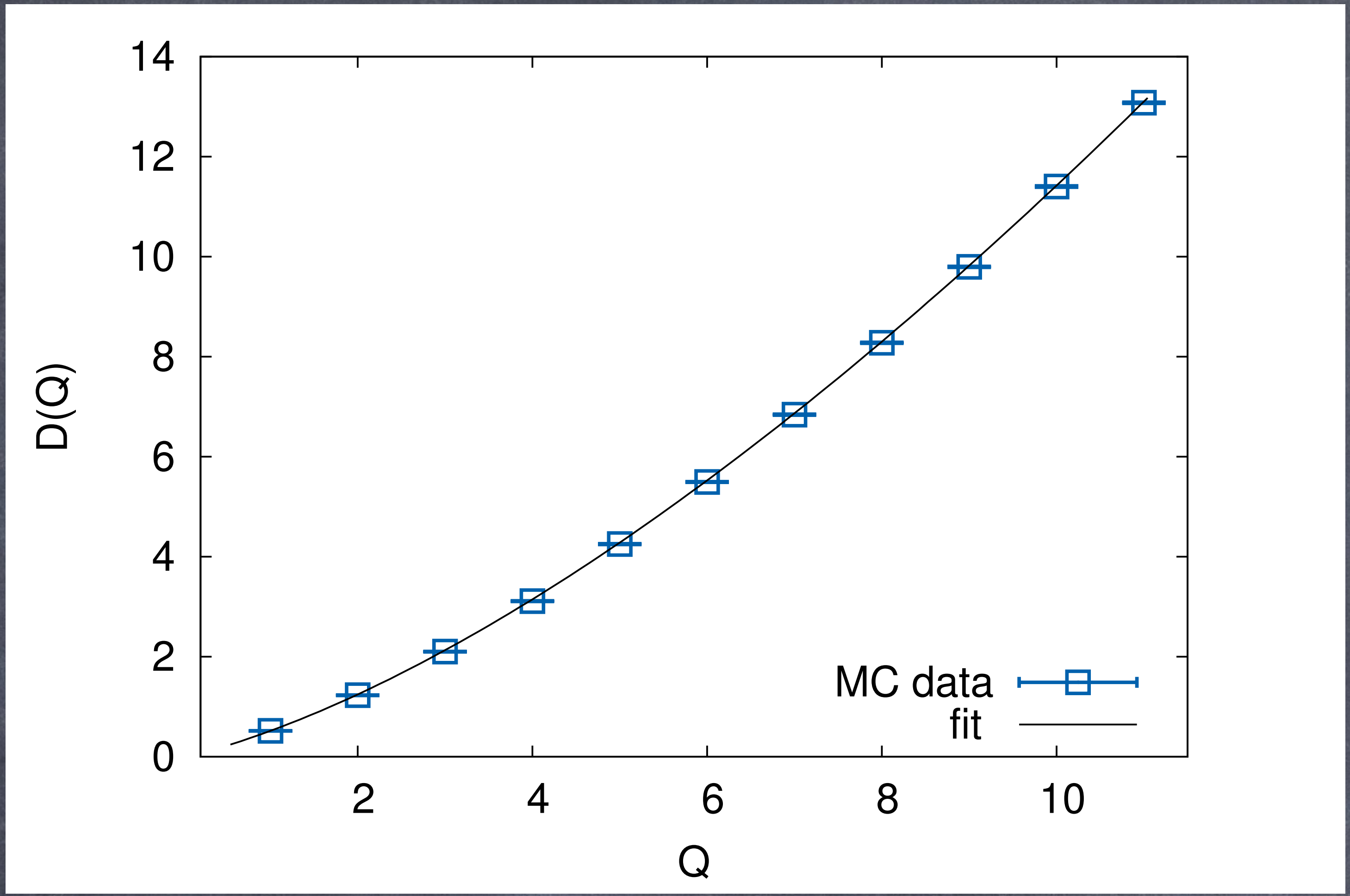
- What happens if we fix several charges?
  - k charges with same chemical potential: homogeneous solution with type I and type II Goldstones.
  - different chemical potentials: inhomogeneous solutions
- What can we learn via this approach?
  - calculate CFT data of <u>strongly coupled</u> CFTs at large charge!
  - in conjunction with large N we can follow RG flow, calculate eff. potential exactly (1st order in N) away from conformal point.

Further directions

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces) Hellerman, Maeda, Orlando, Reffert, Watanabe; Argyres et al.
- Connection to holography (gravity duals) Loukas, Orlando, Reffert, Sarkar; De la Fuente, Zosso; Giombi, Komatsu, Offertaler.
- Operators with spin; connection to large-spin results Cuomo, de la Fuente, Monin, Pirtskhalava, Rattazzi; Cuomo
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap Jafferis and Zhiboedov
- Further lattice simulations: inhomogeneous sector, general O(N) Chandrasekharan et al.
- CFTs in other dimensions (2, 5, 6) Komargodski, Mezei, Pal, Raviv-Moshe; Araujo, Celikbas, Reffert, Orlando; Moser, Orlando, Reffert

Further directions

- Chern-Simons matter theories @large charge Watanabe
- 4-ε expansion @large charge Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe; Antipin et al.
- going away from the conformal point Orlando, Reffert, Sannino; Orlando, Pellizzani, Reffert
- non-relativistic CFTs Favrod, Orlando, Reffert; Kravec, Pal; Orlando, Pellizzani, Reffert; Hellerman, Swanson; Pellizzani
- Boundary CFTs at large Q Cuomo, Mezei, Raviv-Moshe
- Weak gravity conjecture Aharony, Palti; Antipin et al.
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)? Komargodski, Mezei, Pal, Raviv-Moshe



Thank you for your attention!