QUANTUM GRAVITY ENTANGLEMENT GRAPHS AS TENSOR NETWORKS:

HOLOGRAPHIC PROPERTIES AND HORIZON-LIKE REGIONS FROM VOLUME ENTANGLEMENT

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Tensor Journal Club
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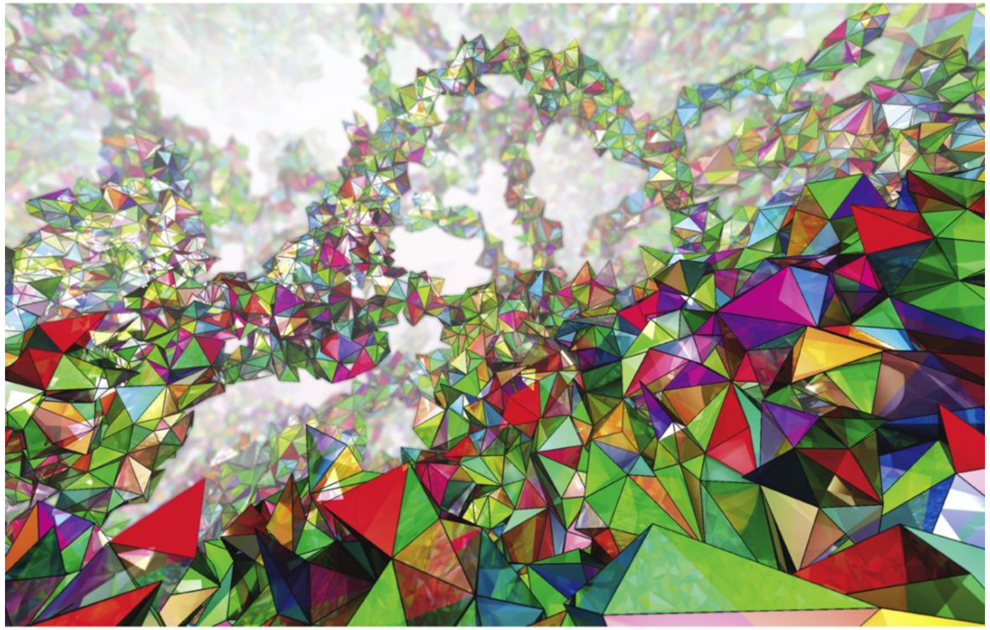
WORKING AT THE INTERFACE OF QUANTUM GRAVITY AND QUANTUM INFORMATION

The **breakdown of the continuum framework** for **quantum spacetime** is suggested by several results in classical and semiclassical gravitational physics:

- **Black hole thermodynamics** finite entropy ⟷ discreteness
- **Spacetime singularities in GR**
- **Challenges to localization in semiclassical gravity**
- …

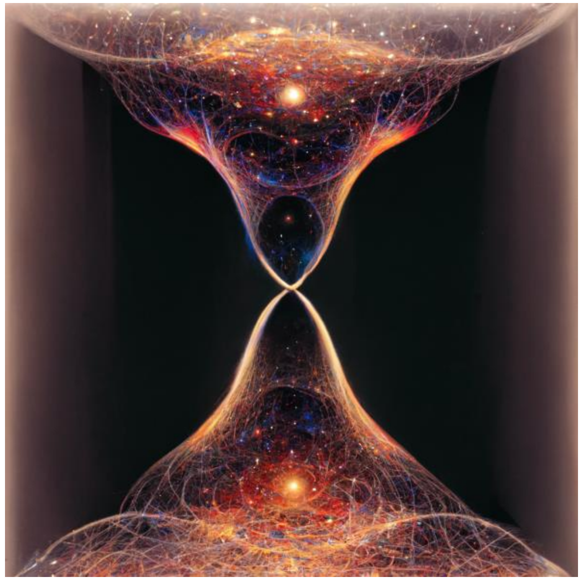
Smooth spacetime geometry description replaced, at the Planck scale, by a more fundamental **atomistic picture**



WORKING AT THE INTERFACE OF QUANTUM GRAVITY AND QUANTUM INFORMATION

Microstructure of spacetime at the Planck scale

geometry from quantum correlations (**entanglement**) of the pre-geometric quantum entities



Brian Swingle
A unification of tensor networks
and quantum spacetime

spacetime as a (background-independent) **quantum many-body system**

Emergent spacetime

continuum classical spacetime should emerge
from the collective behaviour of the fundamental quantum entities

…need of quantum information and condensed matter techniques!

WORKING AT THE INTERFACE OF QUANTUM GRAVITY AND QUANTUM INFORMATION

GRAVITY from ENTANGLEMENT

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GRAVITY from ENTANGLEMENT

Spatial geometries from entanglement of abstract quantum dof
[Cao, Carroll, Michalakis…]

Notion of **distance** from entanglement
[Livine, Terno, Feller…]

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Geometry from entanglement
in Group Field Theory

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GRAVITY from ENTANGLEMENT

Entanglement responsible for **spacetime connectivity**
[Van Raamsdonk, Maldacena, Susskind…]

Einstein's equations from CFT entanglement "first law"
[Lashkari, McDermott…]

Bulk causality from boundary entanglement
[Source, May, Penington]

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HOLOGRAPHIC GRAVITY

- **Bekenstein-Hawking area law** for black hole entropy
- Hamiltonian is a boundary term
- …

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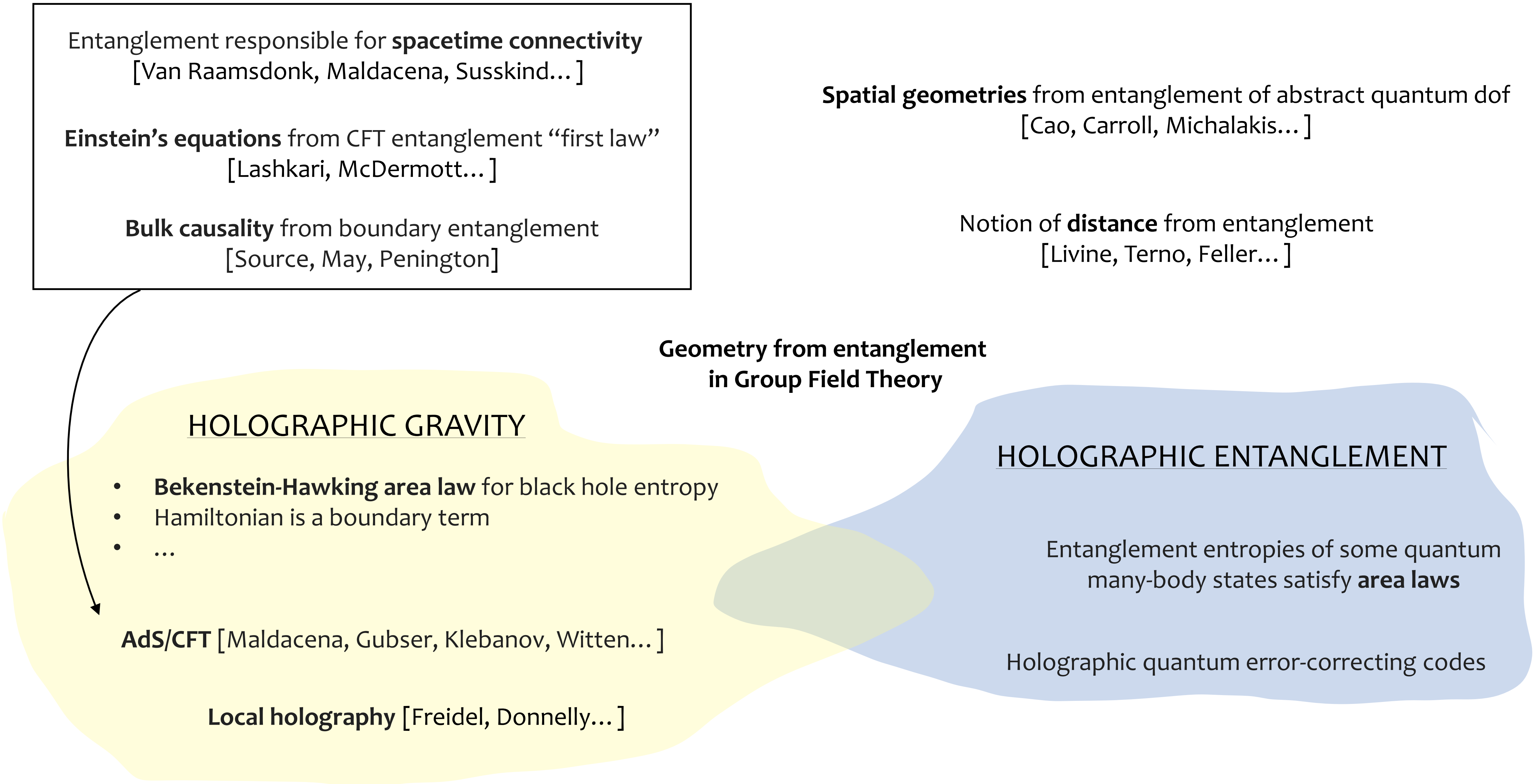

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HOLOGRAPHIC ENTANGLEMENT

Entanglement entropies of some quantum many-body states satisfy **area laws**

Holographic quantum error-correcting codes

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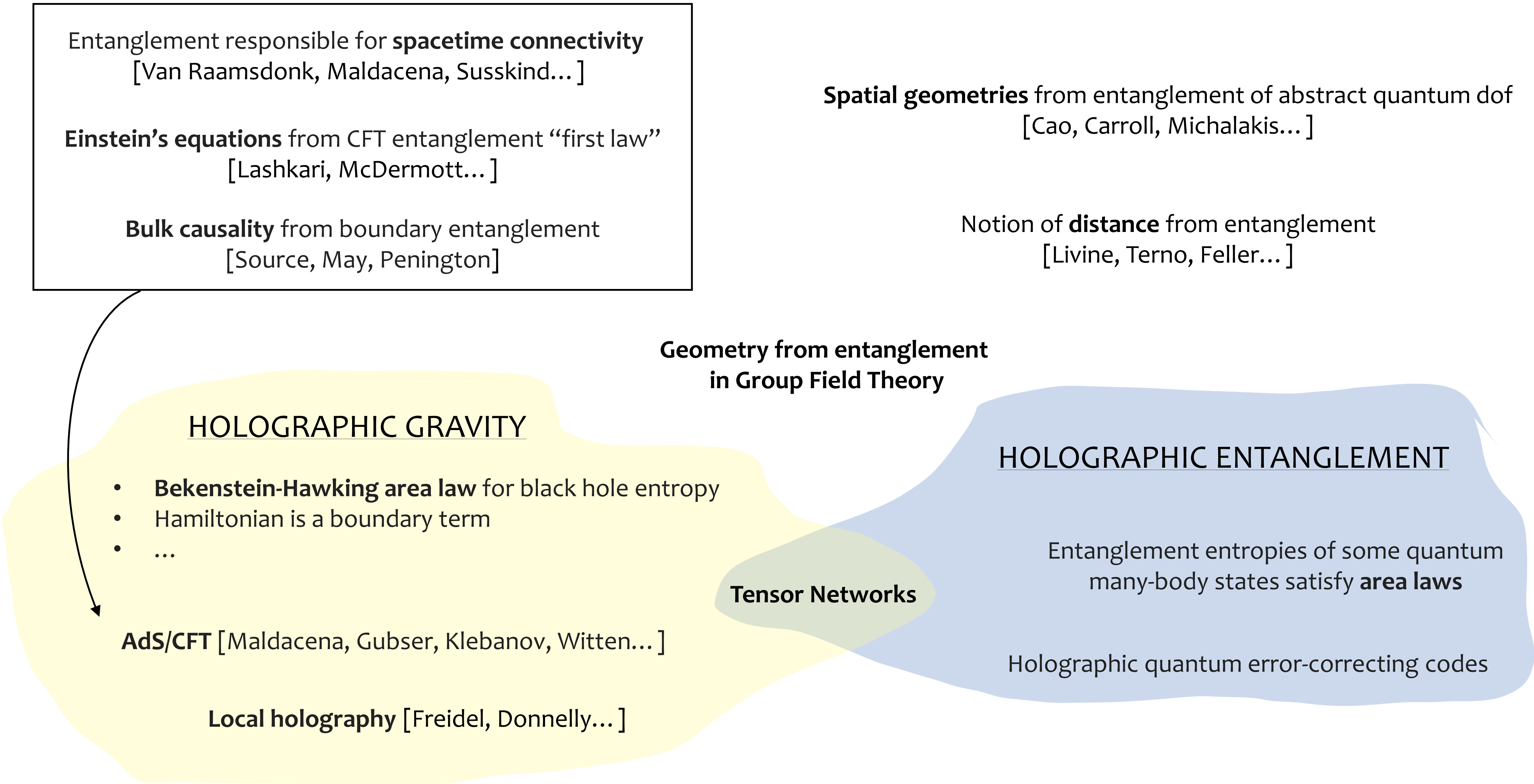

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Tensor Networks

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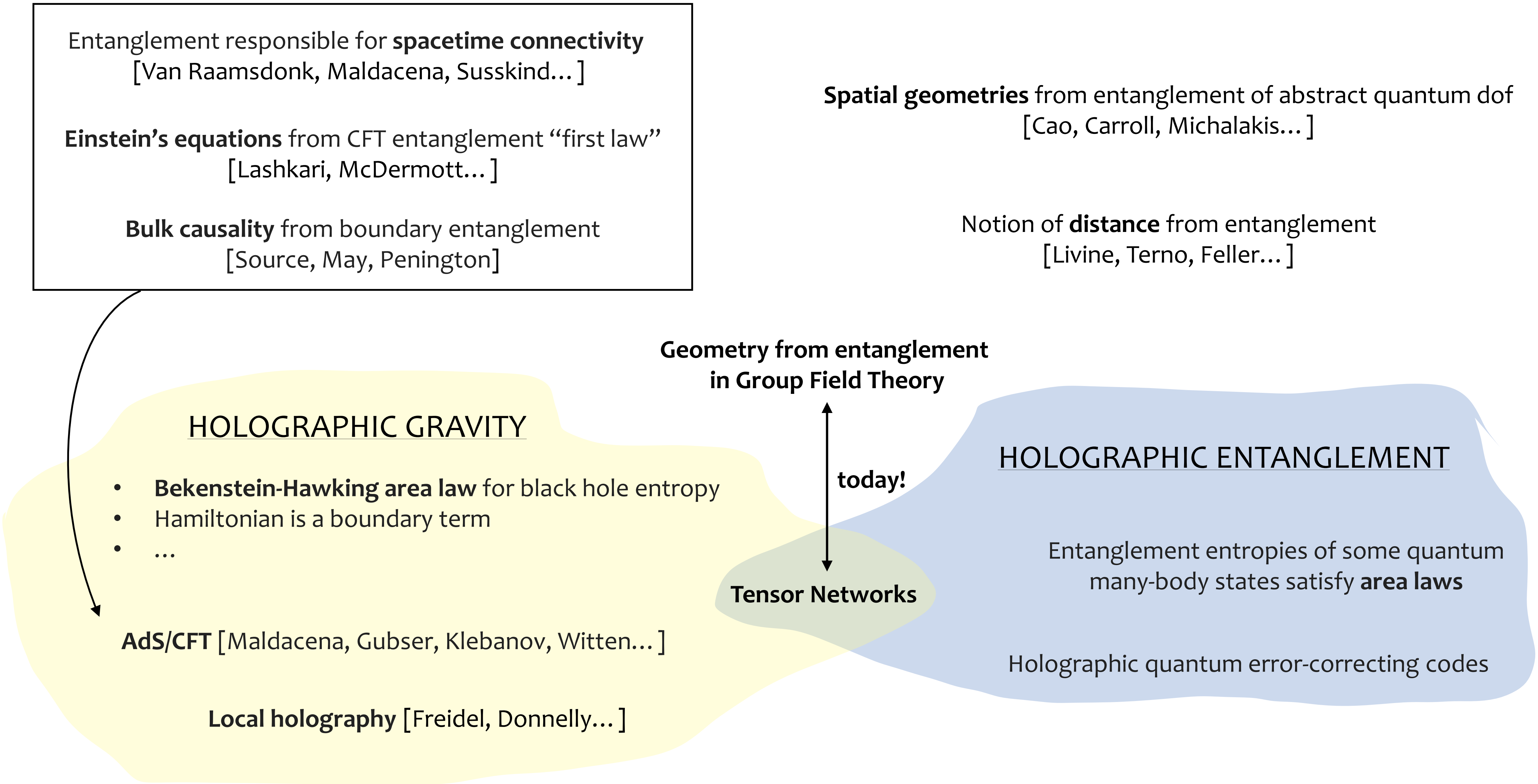

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Geometry from entanglement
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today!

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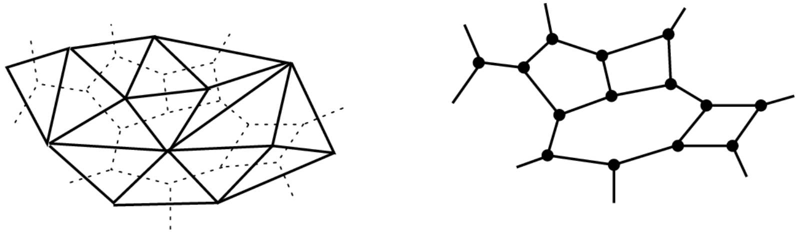
WHICH IS THE ORIGIN OF THE **GRAVITY/HOLOGRAPHY/ENTANGLEMENT** THREEFOLD CONNECTION, AND WHAT DOES IT TELL US ABOUT QUANTUM GRAVITY?

✓ **Characterize a quantum gravity language from a quantum information perspective**

Spin networks for quantum geometries regarded as **patterns of entanglement** among space quanta and put in correspondence with **tensor networks**

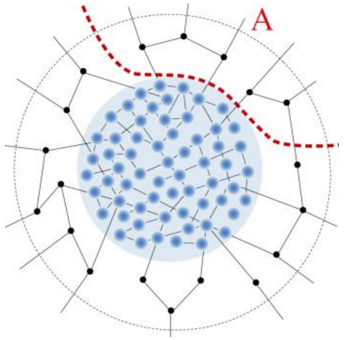
D. ORITI, EC, JHEP (2021) arXiv:2012.12622



✓ **Holographic entanglement**

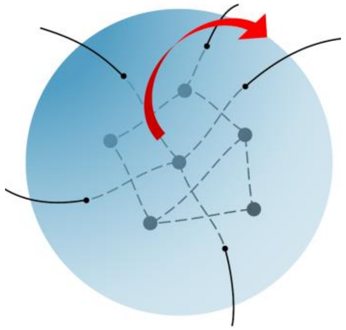
I. **Holographic boundary entropy and black hole like regions**
Investigate how entanglement of bulk degrees of freedom affects the boundary state and its entropy → emergence of **horizon-like regions**

G. CHIRCO, D. ORITI, EC, Phys. Rev. D (2022) arXiv: 2110.15166



II. **Bulk-to-boundary isometries**
Identify properties of the entanglement structure of a spin network which make **bulk** and **boundary holographically related**

G. CHIRCO, D. ORITI, EC, Phys. Rev. D (2022) arXiv:2105.06454
S. LANGENSCHEIDT, D. ORITI, EC, arXiv:2207.07625



GROUP FIELD THEORY APPROACH TO QUANTUM GRAVITY

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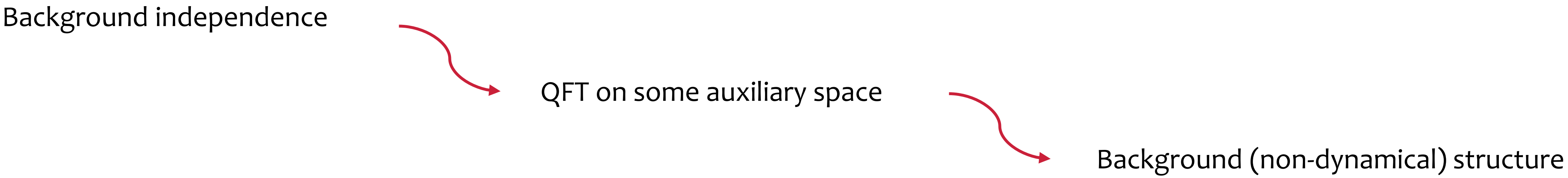

Background independence

QFT on some auxiliary space

Background (non-dynamical) structure

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2. **Group structure:** group manifolds as domain of definition of the field → local symmetry group of the theory

   Lorentz or rotation group

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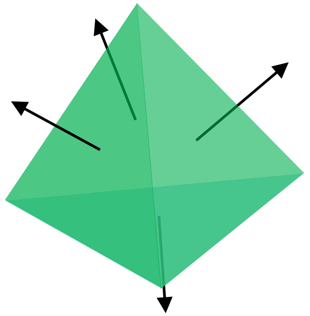
   Lorentz or rotation group

3. **Combinatorial non-locality** of GFT interactions

GROUP STRUCTURE

$\Sigma_i \overrightarrow{L_i}=0$ **closure condition**



$\overrightarrow{L_i}$

$\overrightarrow{L_i} \in su(2)$

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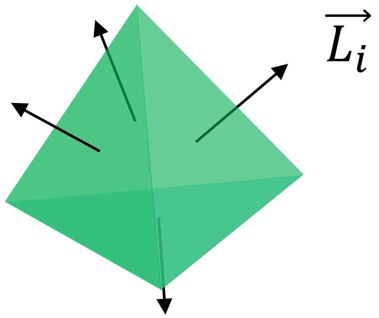
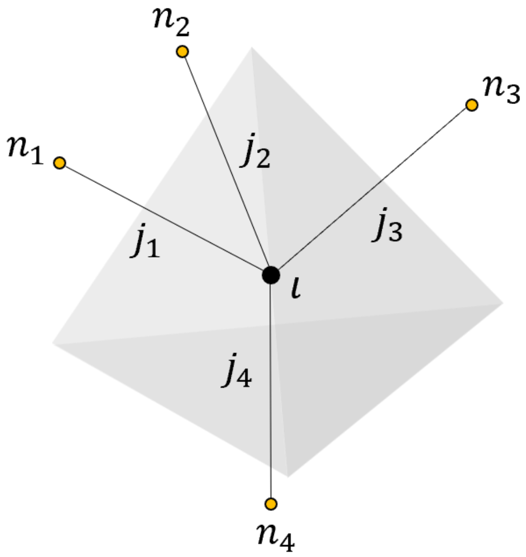


$\overrightarrow{L_i} \in su(2)$

quantization of the phase space of classical geometries of a tetrahedron:

$\widehat{\overrightarrow{L_i}} \in su(2)$

$|j_i n_i\rangle \in V^{j_i}$



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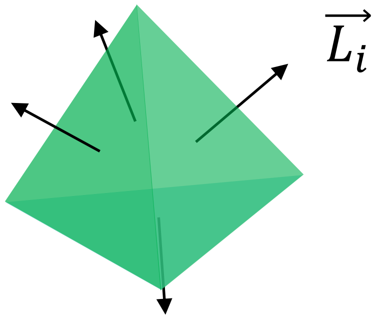

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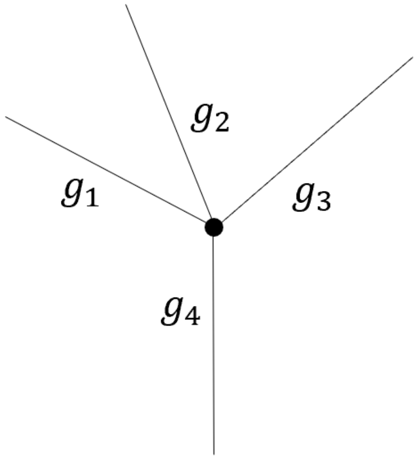
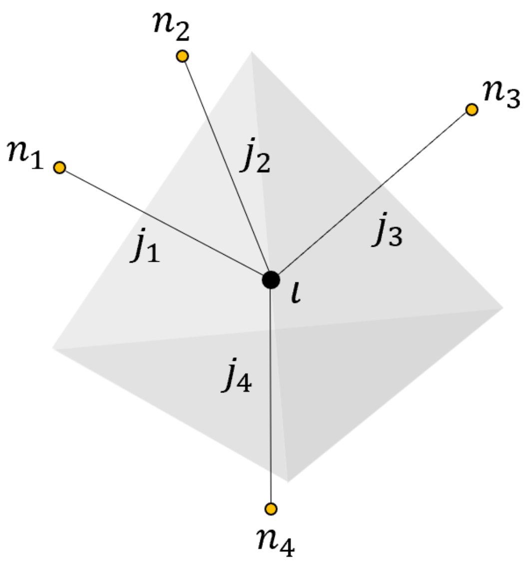


$$\hat{g}_i \in SU(2) \longleftrightarrow \widehat{\overrightarrow{L_i}} \in su(2)$$

$$|j_i n_i\rangle \in V^{j_i}$$



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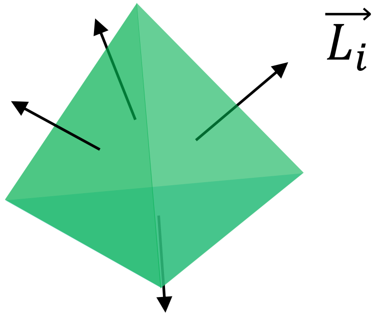

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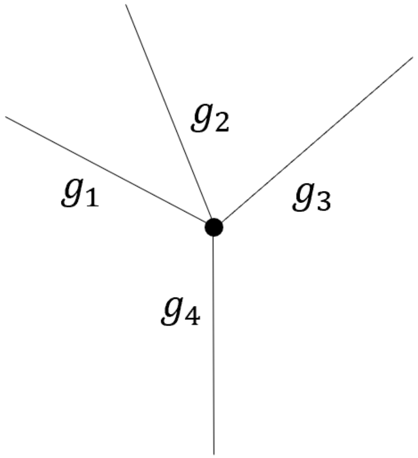

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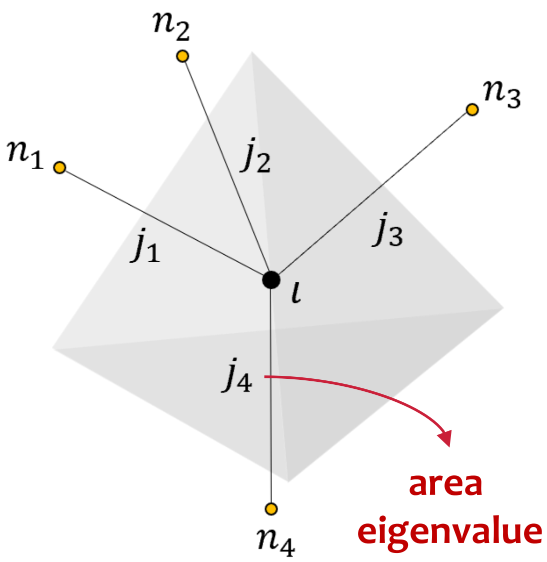


area eigenvalue

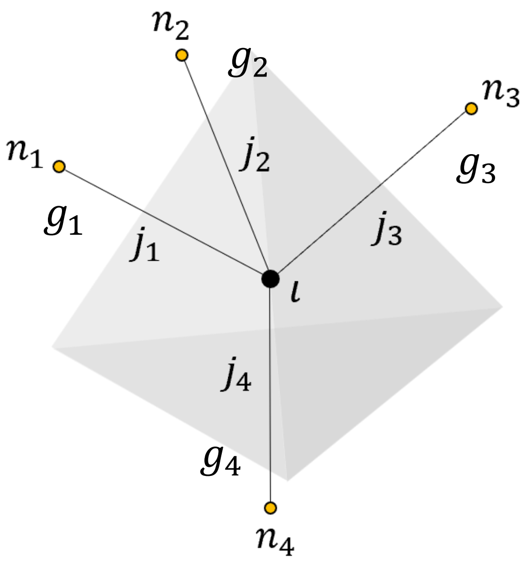
$$\mathcal{H} = L^2(G^4/G) = \bigoplus_{j^1\ldots j^4}\left(\bigotimes_{i=1}^{4} V^{j^i} \otimes \mathcal{I}^{\vec{j}}\right)$$

$$f(hg_1,\ldots,hg_4) = f(g_1,\ldots,g_4) = \quad \forall h \in SU(2)$$

Intertwiner space $\mathcal{I}^{\vec{j}} = \mathrm{Inv}_{SU(2)}\left[V^{j^1} \otimes \ldots \otimes V^{j^4}\right] \ni |\iota\rangle$

volume eigenvalue

GROUP STRUCTURE



$$\begin{array}{cccc} \phi : & G^4 & \to & \mathbb{C} \\ & g^1,\dots,g^4 & & \phi(g^1,\dots,g^4) \end{array}$$

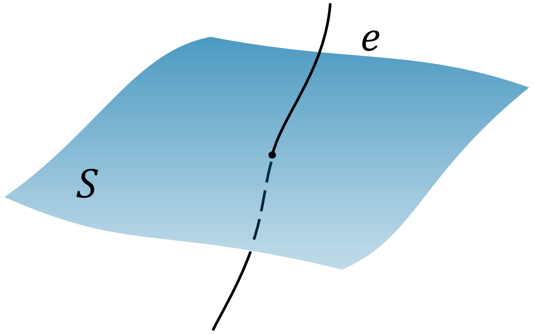
excitation of the field:
3-simplex dual to a 4-valent vertex

$$\phi(hg^1,\dots,hg^4) = \phi(g^1,\dots,g^4) \quad \forall h \in SU(2)$$

From canonical quantization of General Relativity (canonical Loop quantum gravity):

$$g_e[A] = \mathcal{P}\exp - \int_e A$$

Ashtekar-Barbero
connection

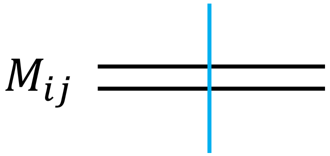


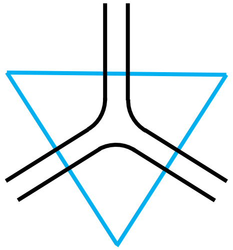
COMBINATORIAL NON-LOCALITY

Matrix models for 2d quantum gravity:

$$S(M) = \frac{1}{2} tr M^2 - \frac{g}{\sqrt{N}} tr M^3$$

$$= \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

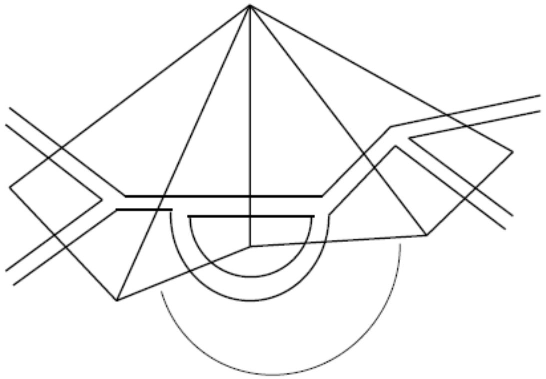




non-local "vertices" of interaction

$$Z = \int \mathcal{D}M_{ij}\, e^{-S(M)} = \sum_{\Gamma} \left(\frac{g}{\sqrt{N}} \right)^{\frac{1}{2}} Z_{\Gamma}$$

2-dimensional simplicial complexes of arbitrary topology



Feynman amplitude = simplicial path integral for 2d GR with Λ, discretized on the associated simplicial complex

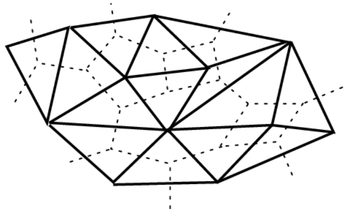
$$Z_{\Gamma} \simeq \int \mathcal{D}g_{\Delta}\, e^{-S_{\Delta}(g)}$$

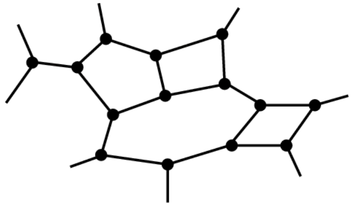
sum over all possible 2d complexes **of all topologies**

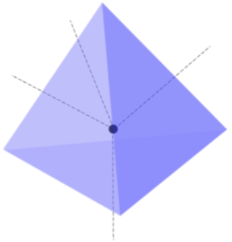
GROUP FIELD THEORY APPROACH TO QUANTUM GRAVITY

A **quantum field theory** of fundamental **building blocks of quantum space**, whose combinations build up **spatial manifolds of arbitrary topology**, and whose **dynamics and interaction processes** generate **arbitrary spacetime topologies**.

$$S_d[\phi,\phi^*] = \int d\vec{g}\,d\vec{q}\,\phi(\vec{g})\mathcal{K}(g^i\,(q^i)^{-1})\,\phi(\vec{q}) + \frac{\lambda}{d+1}\int \prod_{i\neq j=1}^{d+1} dg_i^j\,\mathcal{V}(g_i^j\,(g_j^i)^{-1})\phi(\vec{g}_1)\dots\phi(\vec{g}_{d+1}) + c.c.$$







“combinatorial non-locality”
in pairing of field arguments

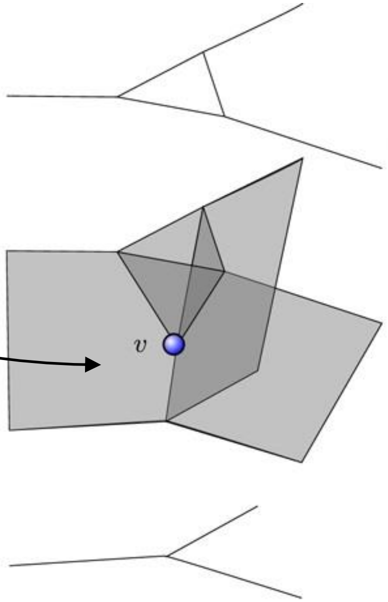
$$Z = \int D\phi D\phi^* e^{-S_d[\phi,\phi^*]} = \sum_{\Gamma} \frac{\lambda^{N(\Gamma)}}{\mathrm{sym}(\Gamma)} Z(\Gamma)$$

Dynamical Triangulations

Quantum Regge calculus

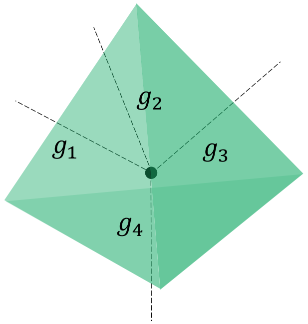
Feynman amplitudes:

- **Lattice path integrals**
- **Spin foam models,** i.e. sum over “histories” of spin networks



SPIN NETWORKS AS ENTANGLEMENT GRAPHS

THE GFT FOCK SPACE



$$\mathcal{H} = L^2(G^4/G) = \bigoplus_{j^1\dots j^4}\left(\bigotimes_{i=1}^{4} V^{j^i} \otimes \mathcal{I}^{\vec{j}}\right)$$

one-particle sector

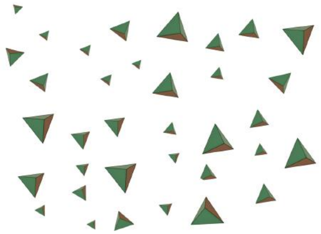
$$\mathcal{F}(\mathcal{H}) = \bigoplus_{N=1}^{\infty} \mathrm{sym}\left(\underbrace{\mathcal{H}\otimes\cdots\otimes\mathcal{H}}_{N}\right)$$

Fock space

⟒

$$\psi(\vec{g}_1, \dots, \vec{g}_N)$$

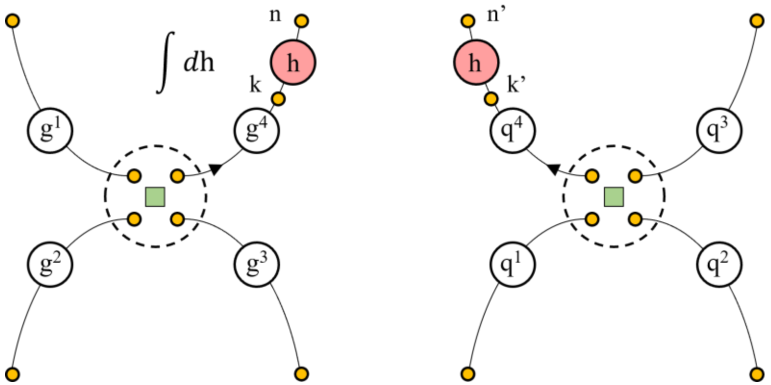
many- body wave-function

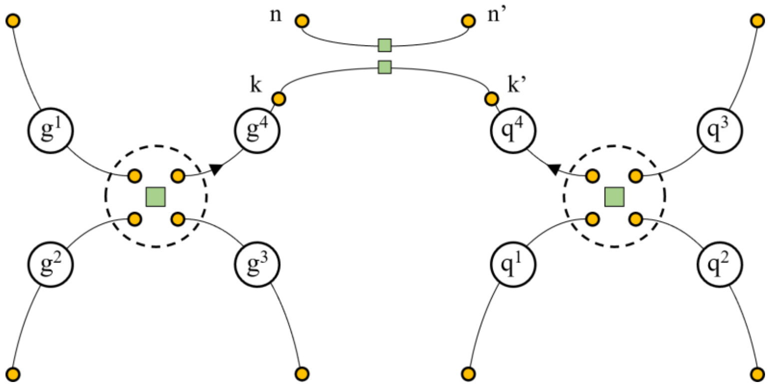


Gluing from entanglement:

$$\int dh\psi(\dots, g_v^i h, \dots, g_w^i h, \dots) = \psi_\ell(\dots, g_v^i (g_w^i)^{-1}, \dots) \longleftrightarrow \psi^{\vec{j}\vec{j}'}_{\vec{n}\vec{n}',\iota\iota'}\delta_{jj'}I_{nn'}$$

where $I_{kk'} := \dfrac{(-1)^{j+k}}{\sqrt{2j+1}}\delta_{k,-k'}$ bivalent intertwiner





↕

singlet state

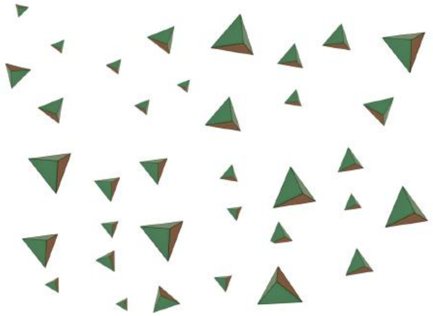
SPIN NETWORKS AS ENTANGLEMENT GRAPHS

Spin network states arising from the entanglement of individual vertices:

Hilbert space
for V open vertices

$$\mathcal{H}_V = \bigotimes_v \mathcal{H}_v \longrightarrow \mathcal{H}_\gamma = \bigoplus_J \left(\bigotimes_v \mathcal{I}^{\vec{j}_v} \otimes \bigotimes_{e\in\partial\gamma}^{d} V^{j_e} \right)$$

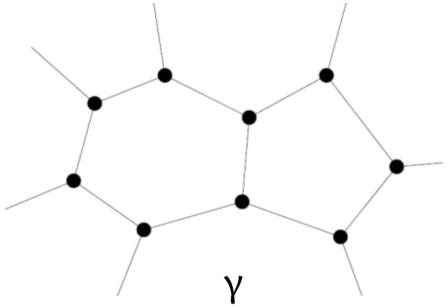
Hilbert space
for graph γ



$$|\psi\rangle$$

gluing
(entangling edge dof)

$$|\psi\rangle \longrightarrow |\psi_\gamma\rangle = \left(\bigotimes_{e\in\gamma} \langle e| \right) |\psi\rangle$$



many- body wave-function for
V open vertices

spin network wave-function
for graph γ

$$|e\rangle = \bigoplus_j \sum_n \frac{(-1)^{j+n}}{\sqrt{2j+1}} |j,n\rangle \otimes |j,-n\rangle$$

Gluing aotms of space
=
Entangling edge degrees of freedom

link state

maximally entangled state
of edge degrees of freedom

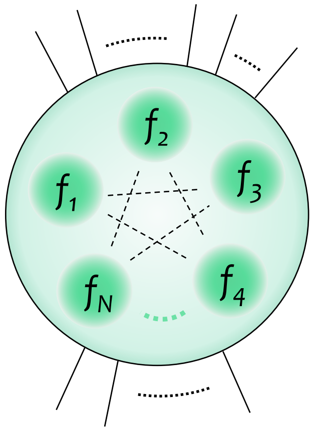
SPIN NETWORKS AS TENSOR NETWORKS

$$|\psi_\gamma\rangle = \left(\bigotimes_{e\in\gamma}\langle e|\right)\bigotimes_{v}|f_v\rangle$$

many- body wave-function
which factorizes per vertex

$$= \bigoplus_{\vec{j}_\gamma}\sum_{\vec{n}_{\partial\gamma}\ \vec{\iota}_\gamma}\underbrace{\left((f_1)^{\vec{j}_1}_{\vec{n}_1\iota_1}\cdots(f_N)^{\vec{j}_N}_{\vec{n}_N\iota_N}\prod_{\gamma}\delta_{n^i_v n^i_w}\right)}_{\text{tensor network}}|\vec{j}_\gamma\ \vec{n}_{\partial\gamma}\ \vec{\iota}_\gamma\rangle$$

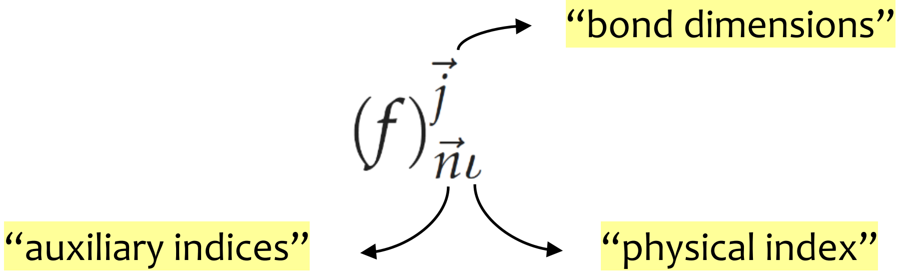
PEPS



individual **vertex tensors**
contracted according
to the pattern γ

Spin networks as generalised **tensor networks:**

- Dynamical (and potentially infinite) bond dimensions
- Intertwiner dimension dependent on spins
- Superposition of combinatorial structures
- 2nd quantized picture (symmetry over vertex re-labelling)
- Distinguishability of vertices recovered via relational approach



$$(f)^{\vec{j}}_{\vec{n}\iota}$$

“bond dimensions”

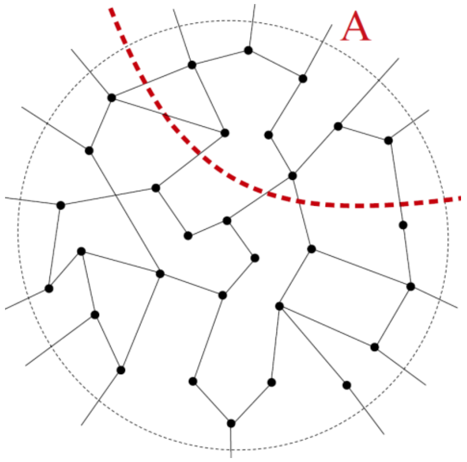
“auxiliary indices”

“physical index”

D. ORITI, EC, JHEP (2021) arXiv:2012.12622

HORIZON-LIKE REGIONS FROM VOLUME CORRELATIONS

BOUNDARY ENTROPY FROM ISING PARTITION FUNCTION

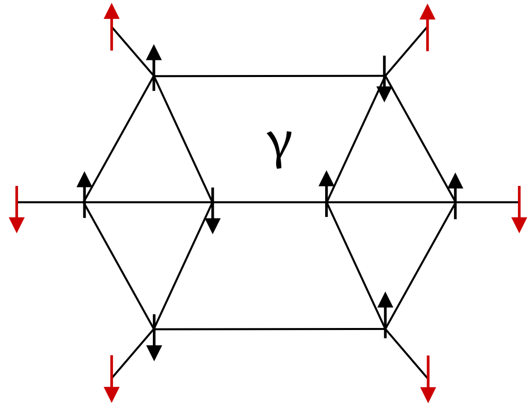


$$|\eta\rangle = \underbrace{\langle\zeta|}_{\text{bulk state}} \left(\bigotimes_{e\in L}\langle e|\right)_{\gamma} \bigotimes_{v} \underbrace{|f_v\rangle}_{\text{random}}$$

boundary state

$$\overline{S_2(\eta_A)} = -\log \mathrm{Tr}\left[\eta_A^2\right] \approx -\log \overline{Z}$$

average Rényi-2 entropy from replica trick:



Ising action

$$\overline{Z} = \mathrm{Tr}\left[(|\zeta\rangle\langle\zeta|)^{\otimes 2} \bigotimes_{e\in\gamma} (|e\rangle\langle e|)^{\otimes 2} \bigotimes_{v} \underbrace{\overline{(|f_v\rangle\langle f_v|)^{\otimes 2}}}_{\mathcal{S}_v+\mathbb{I}} \mathcal{S}_A \right] = \sum_{\vec{\sigma}} e^{-\mathcal{A}(\vec{\sigma})}$$

Ising spin σ_v

$$\mathcal{A}(\vec{\sigma}) = -\frac{1}{2}\sum_{e\in L}(\sigma_v\sigma_w - 1)\log d_{j_e} - \frac{1}{2}\sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)\log d_{j_e} + \underbrace{S_2(\zeta;\downarrow)}_{\text{bulk entropy in }\downarrow\text{-region}}$$

pinning fields

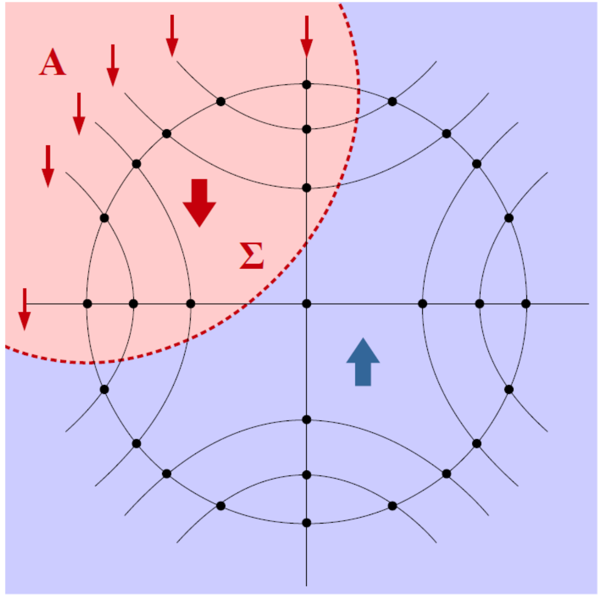
HOMOGENEOUS GRAPHS

$\beta = \log d_j$ inverse Ising temperature $\quad \overline{Z} = \sum_{\vec{\sigma}} e^{-\beta H(\vec{\sigma})}$

$$H(\vec{\sigma}) = \underbrace{-\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1) + \sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)\right]}_{\text{area of domain wall}} + \beta^{-1}S_2(\zeta;\downarrow)$$

$|\Sigma(\vec{\sigma})|$

+1
for every misaligned
pair of spins



large spins regime $\quad \beta \gg 1 : \ \overline{Z} \approx e^{-\beta \min_{\vec{\sigma}} H(\vec{\sigma})}$

Small bulk entropy contribution:

$$\overline{S_2(\eta_A)} \approx \underbrace{\beta \min_{\vec{\sigma}} |\Sigma(\vec{\sigma})|}_{\text{area law}} + \underbrace{S_2(\zeta;\downarrow)}_{\text{bulk correction}}$$

Bulk **area law** for boundary entropy with bulk induced corrections

Large bulk entropy contribution:

$$\overline{S_2(\eta_A)} \approx \beta \min_{\vec{\sigma}} \left\{ |\Sigma(\vec{\sigma})| + \beta^{-1} S_2(\zeta;\downarrow) \right\}$$

Bulk **area+volume** law for boundary entropy

EXAMPLE – HOMOGENEOUS GRAPH WITH BULK ENTANGLEMENT RESTRICTED TO A SUBREGION

Bulk entanglement only in a region Ω :

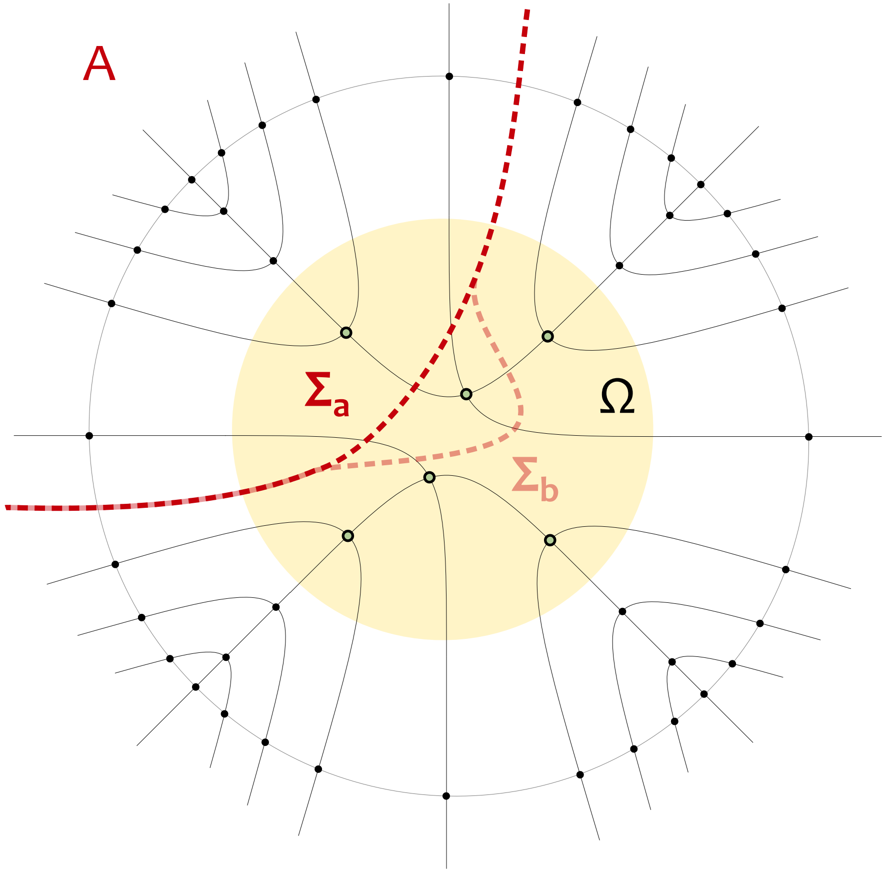
$$|\zeta\rangle = \underbrace{|\zeta_\Omega\rangle}_{\text{random pure state}} \otimes \bigotimes_{v \notin \Omega} |\phi_v\rangle$$

number of vertices inside Ω

$$S_2(\zeta;\downarrow) = \log \frac{d_j^{|\Omega|} + 1}{d_j^{|\Omega_\downarrow|} + d_j^{|\Omega_\uparrow|}}$$

The boundary Rényi-2 entropy follows an area+volume law:

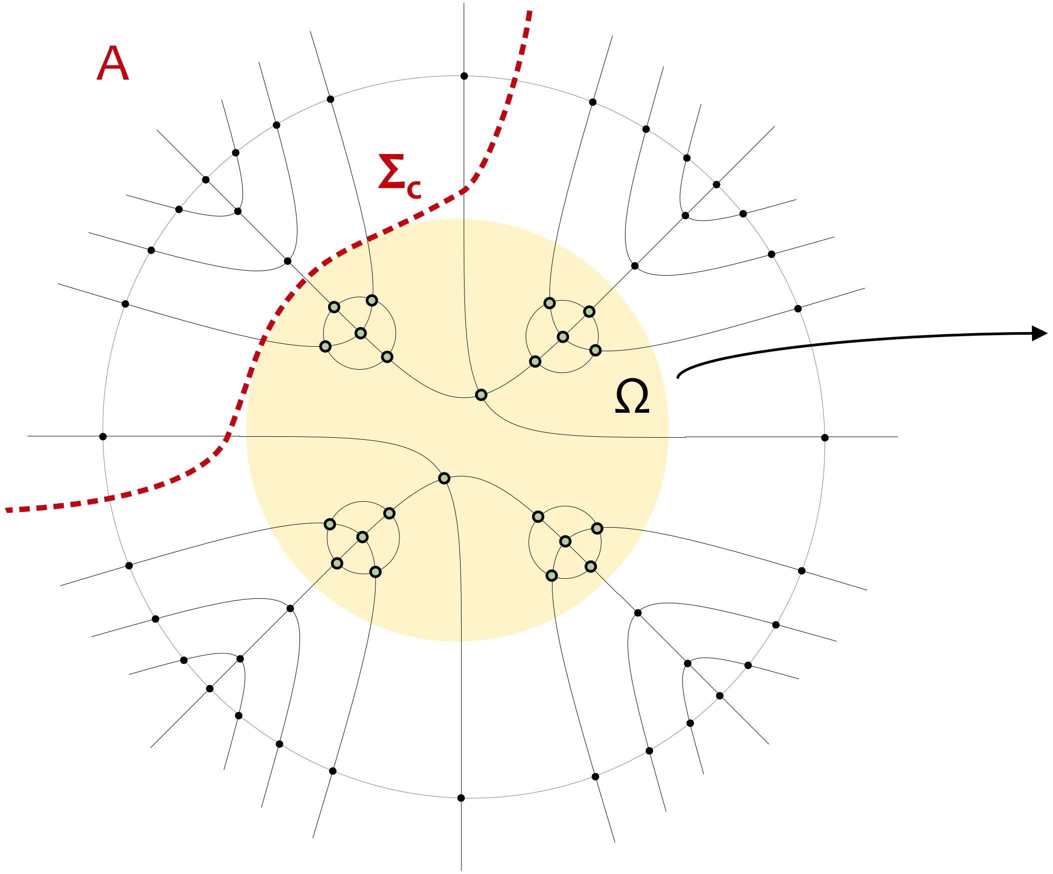
$$\overline{S_2(\eta_A)} \approx \beta \min_{\vec{\sigma}} \left\{ |\Sigma(\vec{\sigma})| + \min\{|\Omega_\uparrow|, |\Omega_\downarrow|\} \right\}$$



When **large intertwiner entanglement** is present in the region Ω, the **degeneracy** of the minimal energy is **removed.**

HOMOGENEOUS GRAPHS - EXAMPLE

By **increasing further the dimension** of the bulk disk Ω via refinement of vertices, the minimal-energy surface is prevented from entering it:



bulk dimension controlled
by the **number of vertices**

emergence of a **black hole-like region** in the bulk!

INHOMOGENEOUS GRAPHS - EXAMPLE

$$\beta = \log d$$

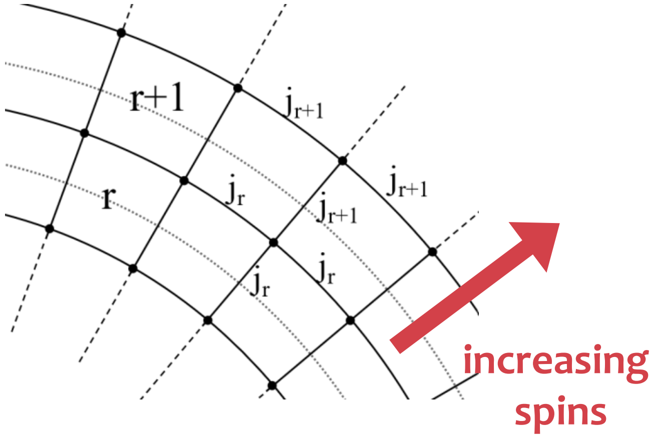
average link dimension

$$H(\vec{\sigma}) = \underbrace{-\frac{1}{2}\left[\sum_{e\in L}(\sigma_v\sigma_w - 1)J_e + \sum_{e\in\partial\gamma}(\sigma_v\mu_e - 1)J_e\right]}_{\text{area of domain wall}} + \beta^{-1}S_2(\zeta;\downarrow) \qquad J_e = \frac{\log d_{j_e}}{\beta}$$

link-dependent coupling

$$|\Sigma(\vec{\sigma})|$$

+ J_e
for every misaligned
pair of spins



Rotationally symmetric geometry with radial gradient of edge spins: $j_{r+1} > j_r$

Bulk entanglement only in a disk Ω of radius R:

$$|\zeta\rangle = |\zeta_\Omega\rangle \otimes \bigotimes_{v\notin\Omega} |\phi_v\rangle$$

random
pure state

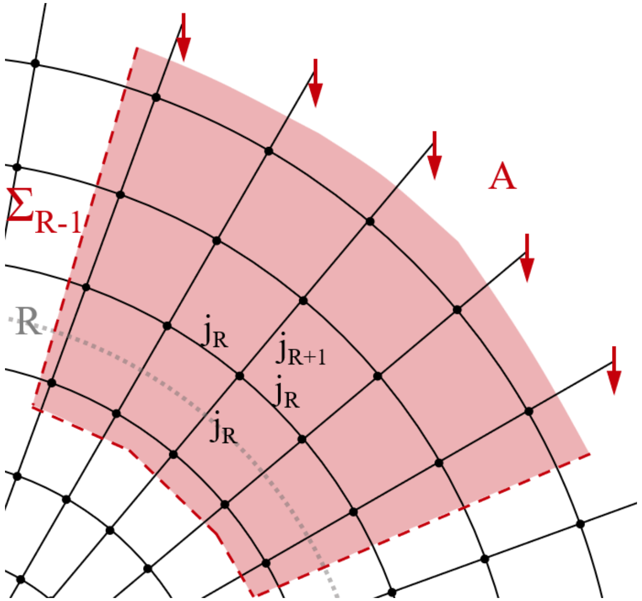
$$S_2(\zeta;\downarrow) = \log \frac{\prod_{v\in\Omega} D_{\vec{j}_v} + 1}{\prod_{v\in\Omega_\downarrow} D_{\vec{j}_v} + \prod_{v\in\Omega_\uparrow} D_{\vec{j}_v}}$$

INHOMOGENEOUS GRAPHS - EXAMPLE

$H(r)$ = Ising-like Hamiltonian of a configuration whose domain wall Σ_r lies between shell *r* and shell *r - 1*

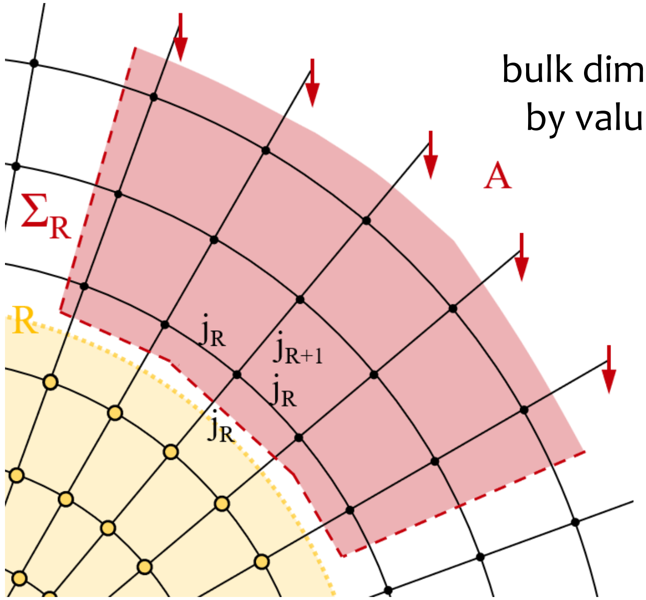
Bulk entanglement not present

The condition for the minimal-energy surface to drop from shell *r + 1* to shell *r* is **satisfied** for $|A| \gg 1$



Bulk entanglement present for $r \leq R$

The condition for the minimal-energy surface to enter the disk Ω of radius R is **violated**



bulk dimension controlled
by values of **edge spins**

emergence of a **black hole-like region**!

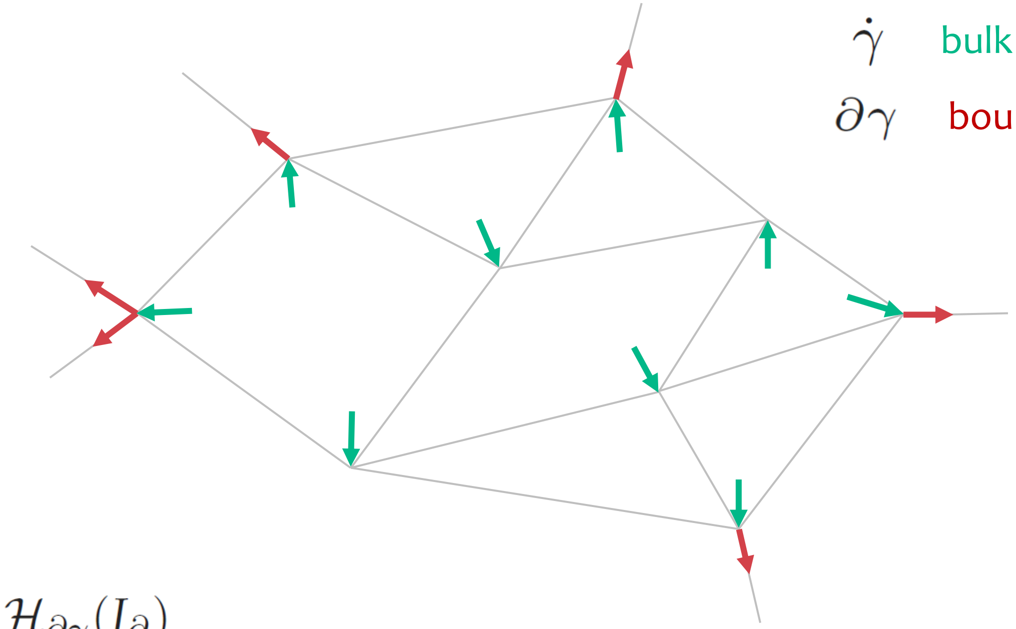
BULK-TO-BOUNDARY ISOMETRIES

OPEN SPIN NETWORK STATES AS BULK-TO-BOUNDARY MAPS

State

$$|\psi_\gamma\rangle \in \mathcal{H}_\gamma(J) = \overset{\text{bulk}}{\underbrace{\bigotimes_v \mathcal{I}^{\vec{j}_v}}_{\mathcal{H}_{\dot\gamma}(J)}} \otimes \overset{\text{boundary}}{\underbrace{\bigotimes_{e\in\partial\gamma} V^{j_e}}_{\mathcal{H}_{\partial\gamma}(J_\partial)}}$$

input space output space



$\dot\gamma$ bulk intertwiners

$\partial\gamma$ boundary edges

Bulk-to-boundary map

$$M[\psi_\gamma] : \mathcal{H}_{\dot\gamma}(J) \rightarrow \mathcal{H}_{\partial\gamma}(J_\partial)$$

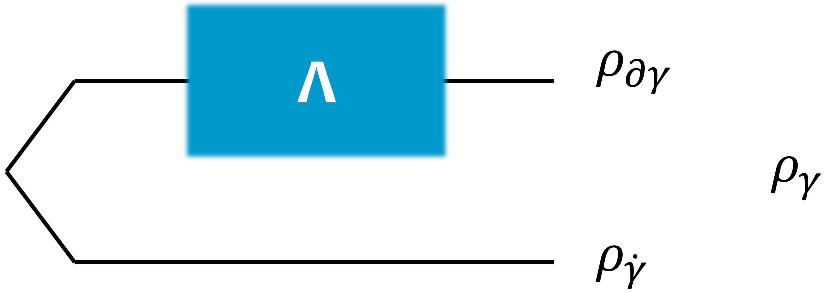
$$|\zeta\rangle \qquad \langle\zeta|\psi_\gamma\rangle = |\psi_{\partial\gamma}(\zeta)\rangle$$

bulk
input state $\displaystyle\sum_{\iota_1,\dots,\iota_N} \zeta_{\iota_1,\dots,\iota_N} \bigotimes_v |\iota_v\rangle$

$\displaystyle\sum_{\{n_{e\in\partial\gamma}\}} (\psi_{\partial\gamma}(\zeta))_{\{n_{e\in\partial\gamma}\}} \bigotimes_{e\in\partial\gamma} |j_e n_e\rangle$ boundary
output state

OPEN SPIN NETWORK STATES AS BULK-TO-BOUNDARY MAPS

Study of the holographic properties of the bulk-to-boundary flow of information via the channel/state duality of quantum information theory:



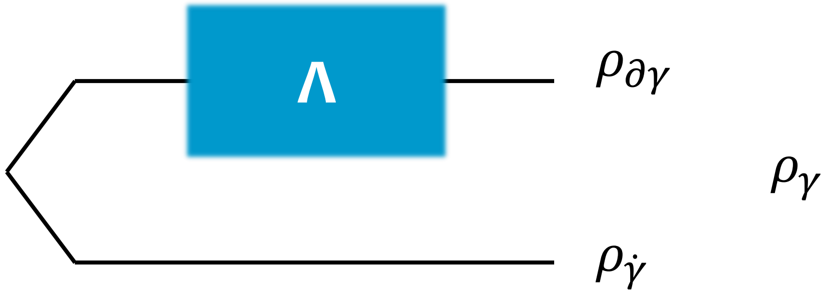
maximally entangled state
of two copies of the bulk

isometry condition $M^\dagger M = \mathbb{I} \quad \Longleftrightarrow \quad \rho_{\dot{\gamma}} = \dfrac{\mathbb{I}}{D_{\dot{\gamma}}}$ reduced bulk state **maximally mixed**

- ✓ Analysis of **isometric character** of **bulk-to-boundary maps** from spin networks with random weights
  - ✓ Map of **homogeneous** graph made of 4-valent vertices **cannot be isometric**.
  - ✓ For generic graph made of 4-valent vertices, increasing the **inhomogeneity** of the spin assignment **increases the holographic character** of the map.

OPEN SPIN NETWORK STATES AS BULK-TO-BOUNDARY MAPS

Study of the holographic properties of the bulk-to-boundary flow of information via the channel/state duality of quantum information theory:



maximally entangled state
of two copies of the bulk

isometry condition $M^{\dagger}M = \mathbb{I} \iff \rho_{\dot{\gamma}} = \frac{\mathbb{I}}{D_{\dot{\gamma}}}$ reduced bulk state **maximally mixed**

- ✓ Generalization to **superposition of spins** and **boundary-to-boundary maps**
  - Random Ising model with **distribution of couplings** determined by the relative sizes of the involved geometries
  - ➢ The **superposition of isometric-map geometries** realizes an **isometric boundary-to-boundary map** if and only if the relative weight of each geometry is inversely proportional to its size

S. LANGENSCHEIDT, D. ORITI, EC, arXiv:2207.07625

SUMMARY

- ✓ Spin networks as **entanglement graphs** in GFT, correspondence with (generalized) **tensor networks**
- ✓ For spin network states with **random vertex weights**, **entropy** calculation mapped into **Ising partition functions**
- ✓ **Area law** for boundary entropy with **corrections** due to the **bulk entanglement** and emergence of a **black hole-like region**
- ✓ Analysis of **isometric character** of **bulk-to-boundary maps** via **channel/state duality**

OUTLOOK

- Generalization of **condensed states** modelling **spherically symmetric geometries** and **quantum black holes**
- Derivation of a "threshold condition" for the **emergence of horizon-like surfaces** in the bulk
- Promotion to the dynamic level
    - Preliminary step: generalization to **superposition of different graphs**
    - Inspiration from previous results:
        - ➢ derivation, from general arguments (energy conservation, gravitational energy as boundary term), of guidelines on the holographic encoding of information in gravitational physics
        - ➢ quasi-local holographic dualities in 3d quantum gravity: bulk quantum geometrodynamics (given by Ponzano-Regge state-sum model) dual to 2d statistical models
        - ➢ LQG boundary dynamics as a 2+1-dimensional SL(2,C) gauge theory

THANKS!