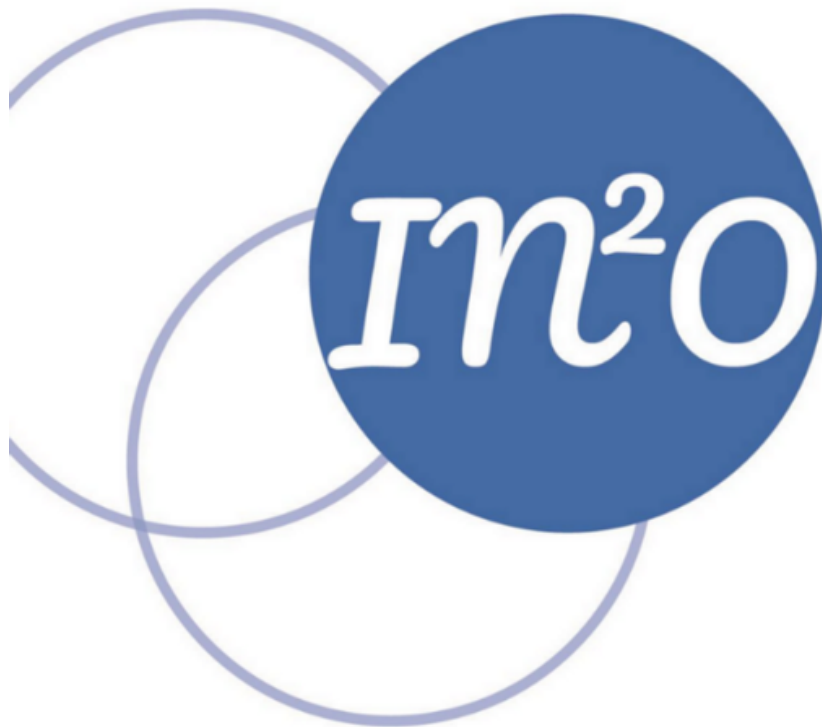




INNOMATH Meeting 2021 december 13th - Élément de solution

Code: 036835

INNOMATH 2021, Patrick Berger



09.12.21



Informations à propos de la route

Nombre d'épreuves::	15
Durée attendue:	~ 03 h 30 min
Longueur:	~ 1.7 km
Niveau scolaire minimal conseillé:	12
Outillage recommandé:	<ul style="list-style-type: none">• Calculatrice• Niveau• Règle pliable
Mots clés::	Thalès, enlargement, proportionality, poster, DINA0, number, slope, percentage, percentage, volume, sphere, units conversion, radius, month, angle, numbers, ordinals, Fibonacci, Combinatorics, sequences, frieze, Combinatorics, number, counting strategy, multiplication, orientation, combinatorics, combination, probabilities

This trail has been designed by gifted students (age 13 to 18) during the Innomath Mid-Day on last Saturday December 4th.



1. Épreuve: Louis the (too much) great



Epreuve

Louis XIV is called "Louis Le Grand" meaning "Louis the great" (the last king Louis of France has been Louis XVIII and yet, XIV > XVIII). This bronze statue made by sculptor François-Frédéric Lemot, dating from 1825, is still larger than life. Estimate its enlargement factor.

Réponse:



Élément de solution:

I have measured 26m from me to the statue and 16m to Ewan who measures 1m80. I compared Louis on his horse to Ewan and I found an enlargement factor of 1.33 (see picture), therefore to be multiplied by 26/16 for the distance, which makes a total of 2.2 of enlargement.



Indice 1

Place a friend whose size you know between you and the statue until their size is the same as the statue and deduce the enlargement factor.

Indice 2

It is the reciprocal of Thales' theorem: by making two objects similar when viewed from a central point, the enlargement factor is the ratio of their distance to the observer.



Indice 3



2. Épreuve: A score of light



Epreuve

Louis XIV's horse looks at a building in Place Bellecour (see picture) where two of my friends live, that my friend Patrick and I want to surprise by inviting them to the restaurant. At nightfall, Patrick is on the spot, but does not know what their windows are. He tells me that 10 windows are lit on the facade. I wonder whether both our friends are at home: What is the probability (in %) that both of our friends' windows are on? Ah, I remember they live under the roof, on the top floor.

Réponse:



Élément de solution:

We will model the situation by the independence of the lighting of each window.

Since there are 7 windows per floor and 4 floors in all, there are $7 \times 4 = 28$ windows.

The number of ways to choose 10 lit windows out of 28 is:

$$\frac{28!}{10! (28-10)!} = 13,123,110.$$

The number of ways to choose 2 windows among the 7 on the top floor, AND to choose 8 windows among the 21 others is:

$$\frac{7!}{2! (7-2)!} \times \frac{21!}{8! (21-8)!} = 4,273,290.$$

The probability of the event sought is then the ratio of these two numbers:

$$\frac{4,273,290}{13,123,110} \approx 0.3256 \dots \text{ or about } 32\%.$$

Indice 1

The probability is the ratio between the number of favorable cases and the number of possible cases. We will assume all the windows as being independent and equiprobable.

Indice 2

Auteur: Patrick Berger

PDF créé par mathcitymap.eu, Groupe de travail MATIS I, Université Goethe (Francfort)



Break down the problem:

- How many possible distributions of the 10 windows?
- How many of these distributions are "favorable" (our 2 windows on the top floor AND 8 in the others)?

Indice 3

The number of combinations of p elements among n is given by $\binom{n}{p} = \frac{n!}{p!(n-p)!}$ Where $n! = 1 \times 2 \times \dots \times n = (n-1)! \times n$.



2. Épreuve: A score of light

1. Sous-tâche: The number of windows



Epreuve

How many windows can you count on this façade?

Réponse:

28

Élément de solution:

There are 4 floors and 7 windows per floor so $7 \times 4 = 28$ windows.

Indice 1

Count the number of windows per floor and the number of floors.

Indice 2

We do not take into account the windows of the shops on the ground floor nor the vents.

Indice 3



2. Épreuve: A score of light

2. Sous-tâche: Choose 10 windows



Epreuve

Determine how many ways you can distribute 10 lit windows on the facade.

Réponse:

13123110

Élément de solution:

The number of ways to choose 10 lit windows out of 28 is: $\frac{28!}{10! (28-10)!} = 13\ 123\ 110$.

Indice 1

It is a problem of choice without order or repetition.

Indice 2

You have to choose k from n : $\binom{n}{k} = \frac{n!}{k! (n-k)!}$ where $n! = 1 \times 2 \times \dots \times n = (n-1)! \times n$.

Indice 3

5. Épreuve: Morris Column



Epreuve

How many A0 posters can we put up on this pillar without overlapping? DIN A0 format: length = 84cm; height = 119cm.

Réponse:

8

Élément de solution:

Data measured using a 1.75m man (see picture)

Circumference: 4.125m

Height: 3.50m

Full-circumference posters:

$$4.125\text{m} / 0.84\text{m} = 4 \text{ posters}$$

Full height posters:

$$3.50\text{m} / 1.19\text{m} = 2 \text{ posters}$$

Total number of posters:

$$4 * 2 = 8$$



Indice 1

You cannot cut posters or put them in "landscape" position.

Indice 2

How many posters in height?

Indice 3

How many posters in the circumference?

6. Épreuve: Bonaparte's hat



Epreuve

Going up the Bonaparte Bridge, you can see that the stone railing forms an angle at its top. Visually, the bridge can recall Bonaparte's cocked hat. So what is the slope (in%) of the guardrail that we see in the photo and which descends from the top of the bridge towards Vieux-Lyon (the old city)?

Réponse:



Élément de solution:

One stone in the railing is 125 cm long. From one end to the other we descend between 5 and 7 cm.

$$5/125 = 0.04 \text{ or } 4\%$$

$$7/125 = 0.056 \text{ or } 5.6\%$$

The slope is therefore $4.8 \pm 0.8\%$

Indice 1

You can measure lengths with your hands and fingers.

Indice 2

Use the "spirit level" application on your phone to determine the horizontal.

Indice 3

7. Épreuve: Fallen angels



Epreuve

The angels of the arch of the main portal did not particularly appreciate the French revolution where many heads fell. What percentage of angels came out unscathed?

Réponse:



Élément de solution:

We counted $7 + 8 + 9 + 10 = 17 * 2 = 34$ angels on one side, or 68 angels on either side of the 4 additional central angels. Among these 72, 44 have their heads damaged, i.e. a remaining proportion of about 38.9%

Indice 1

Don't forget the 4 central angels.

Indice 2

Some heads are damaged, they are not intact.

Indice 3

8. Épreuve: Water for the pilgrims



Epreuve

On the road to Compostela, pilgrims could come and drink from this fountain. Assuming that the 4 basins of this one were full, how many liters of fresh water were then at their disposal?

Réponse:



Élément de solution:

By modeling each basin as a quarter of a sphere, we must then calculate the volume V of a sphere having for radius R , the depth and / or the width of the basin.

$$V = \frac{4}{3} \pi R^3$$

As the depth of the latter is 40 cm and its width is at most 50 cm, two estimates can be made of its volume in $\text{cm}^3 = \text{mL}$.

For $R = 40$ cm, we find $V \approx 268$ L

For $R = 50$ cm, we find $V \approx 524$ L

So $V = 396 \pm 128$ L



Indice 1

We can model each basin as a quarter of a sphere.

Indice 2

The volume V of a sphere with radius R is $V = \frac{4}{3} \pi R^3$

Indice 3

9. Épreuve: Month of the unicorn



Epreuve

There were 12 statues in niches on the facade, which were destroyed in the revolution. Under the consoles that carried them there are very pretty sculptures that you can only see while sitting with your head in the air. Going from January in the north to December in the south, find the month where a unicorn is hidden.

The unicorn hides in the month of _____ .

Réponse:

The unicorn hides in the month of ****june|juin****.

Type de vérification: normal

Élément de solution:

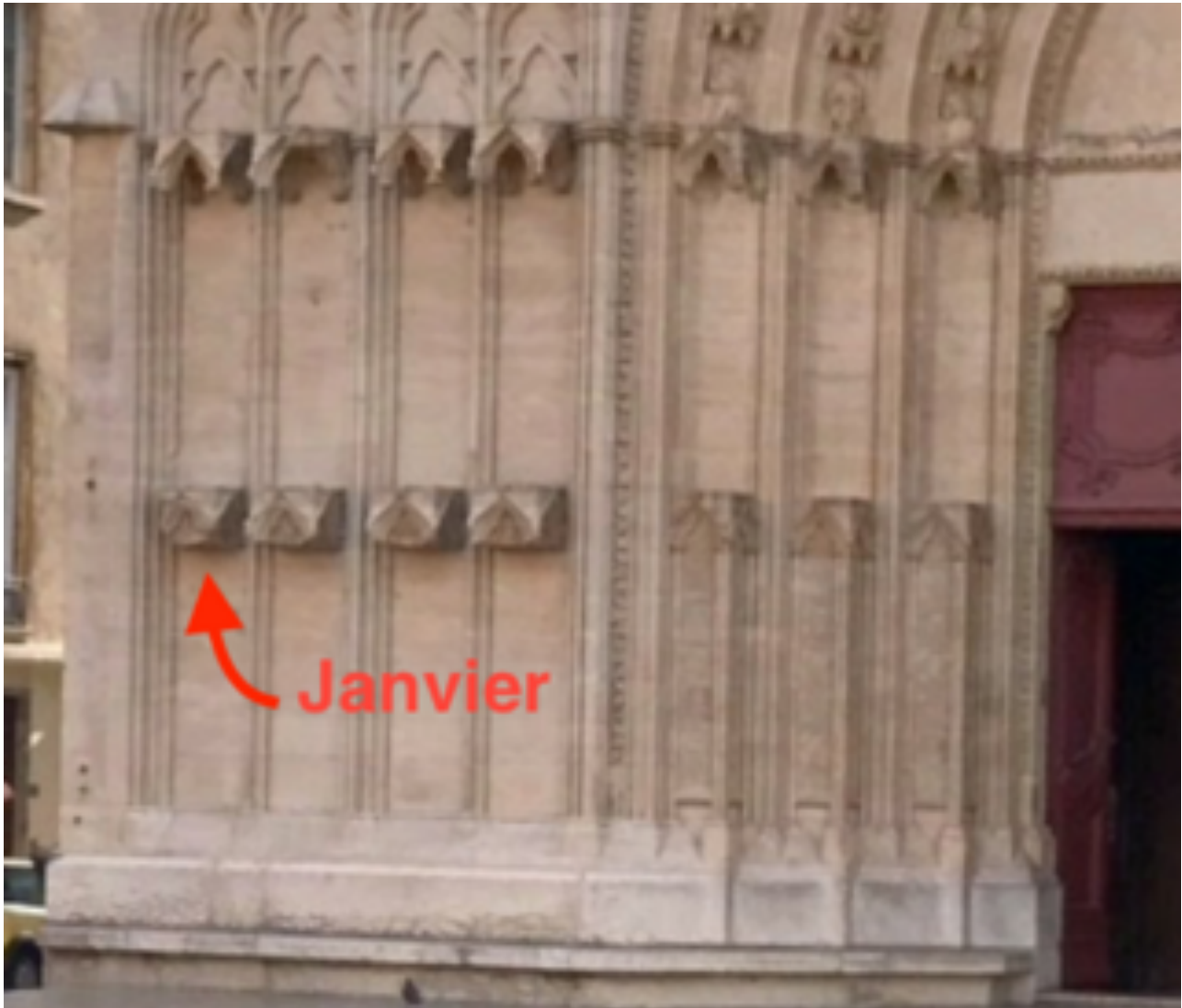
The unicorn is under the console of the 6th missing statue.



Indice 1

Take a break by sitting on the benches of stone.

Indice 2



Indice 3

10. Épreuve: The hidden stairs



Epreuve

You can see a spiral staircase, through the glassless windows of this Renaissance building. Estimate the total number of steps of this staircase.

Réponse:



Élément de solution:

From one floor to another, the staircase makes three quarters of a turn (the landing has a right angle). There are 7 steps to make a quarter of a turn (see image). If there are 21 steps between each landing and 4 floors, we can then estimate that there are 84 steps.



Indice 1

By looking more closely one can see that, from floor to floor, the landings are vertical to each other and form a large flight of steps of a quarter of a turn.

Indice 2

Estimate the fraction of a turn made by a few steps.

Indice 3

11. Épreuve: The number of the three Maries



Epreuve

What is the number, in the street, of the house which gives its name to this street?

Réponse:

7

Élément de solution:

This house is between the 5 and the 9 so it is the 7.

Indice 1

Find the statue above a door.

Indice 2

If you can't find the number, look at the house number before and the house after.

Indice 3

In France, even and odd numbers are located on opposite sides of the street. They increase when the odds are on your left, evens on your right.

12. Épreuve: The stairs of the "Traboule"



Epreuve

Place du Gouvernement is the entrance to a "traboule" (a secret crossway) passing through an elevated interior courtyard. To get there you have to climb a staircase. How many possibilities are there to climb this staircase by going up one or two steps, independently at each step?

Réponse:

4181

Élément de solution:

The staircase has 18 steps. There are several ways to solve this problem. One possibility is to systematically note all the possibilities.

This defines for us the Fibonacci sequence of which each element is the sum of the two preceding ones, which gives:

(1) 1 2 3 5 8 13 21 34 55 89 etc.

To find the value more quickly, it is possible to use the approximate formula $F_n \approx \frac{\phi^{n+1}}{\sqrt{5}}$ where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

Indice 1

Systematically note all the possibilities for 1, 2 or 3 steps then look for how to deduce the possibilities for 4 steps.

Indice 2

To go to the last step N, one of two things, you start by going up one step (then you go up N-1) or by going up two steps at once (then you go up N-2) .

Indice 3

For the more mathematical among you, you can save time with the approximation formula of the n-th term



of the Fibonacci sequence involving the $(n + 1)$ -th power of the golden ratio $\phi = \frac{1 + \sqrt{5}}{2}$.

13. Épreuve: The frieze factor



Epreuve

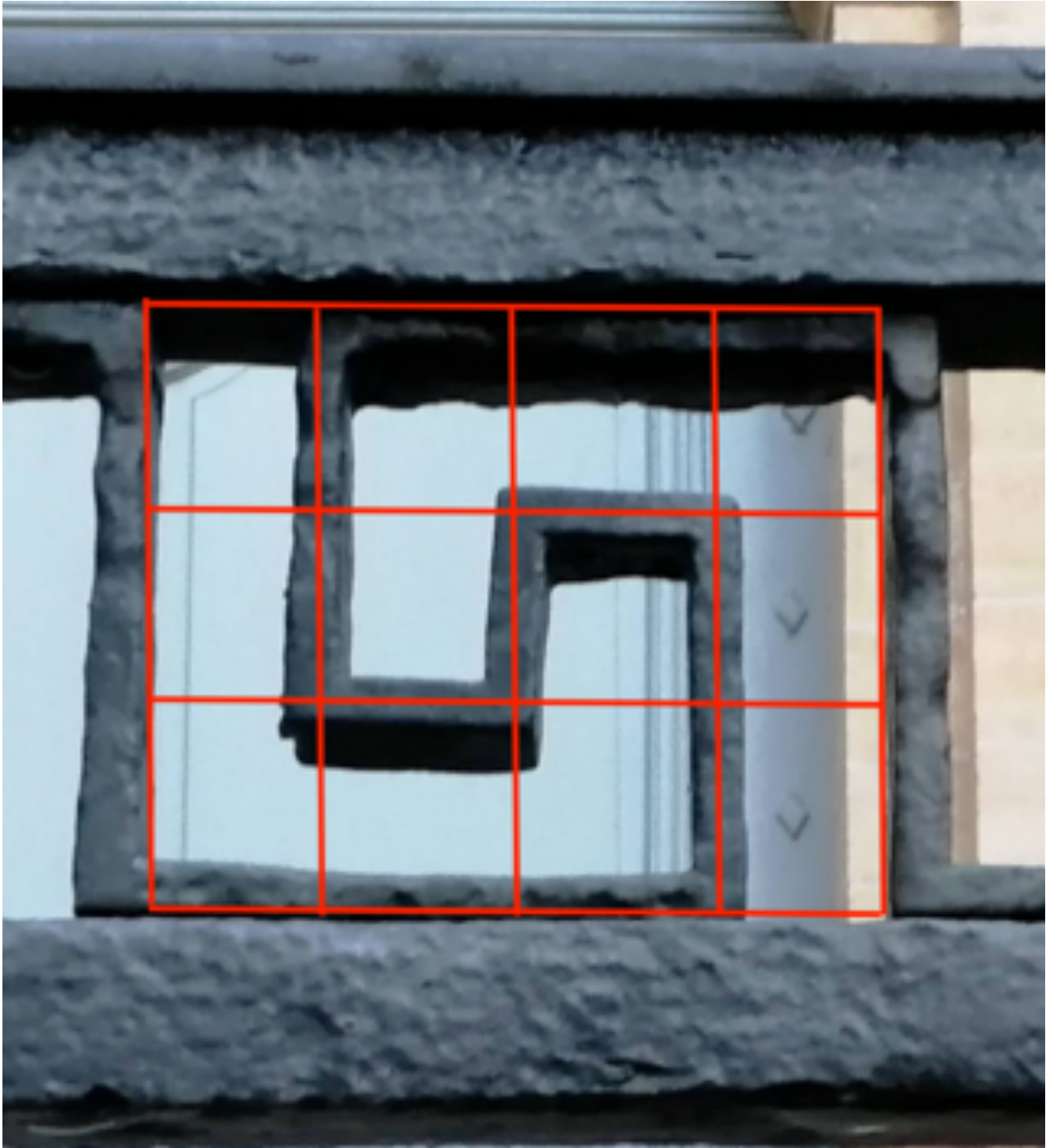
A Greek frieze adorns the top of this metal railing. By which factor is this frieze longer than the railing itself?

Réponse:

4

Élément de solution:

If we cut each rectangle by 3×4 squares. Horizontally it makes 4 units and in length $3 + 3 + 2 + 1 + 1 + 1 + 2 + 3 = 16$, i.e. 4 times more.



Indice 1

Analyse one pattern.

Indice 2

A rectangle can be broken down into small squares.

Indice 3

14. Épreuve: Mixed coats of arms



Epreuve

The building at 2 Place du Change has a door from another era that is worth a detour. And if you look up, you will see coats of arms hanging from the facade. If these coats of arms were to come off (during a revolution for instance!) and we had to hang them back up, having forgotten their original order, in how many ways could we do it?

Réponse:

24

Élément de solution:

There are 4 coats of arms above the windows on the 2nd floor.

There are 4 options for the first coat of arms, then 3, then 2, then 1 for the last. In all this makes $4 \times 3 \times 2 \times 1 = 24$ possibilities.



Indice 1

The coats of arms are aligned horizontally above the windows on the second floor.

Indice 2

If we have to hang up the first coat of arms on the left, how many possibilities do we have? And, once this one is hooked, how many possibilities do we have for the second one? Etc.

Indice 3

15. Épreuve: College chairs



Epreuve

As in the engraving "Relativity" by Maurits Escher, the directions are surprising on this facade. Estimate the number of oddly oriented chairs.

Réponse:



Élément de solution:

You have to look from both sides to be convinced that there are indeed 29 chairs.



Auteur: INNOMATH 2021



Indice 1

Organize your counting.

Indice 2

Take a view from both sides

Indice 3

16. Épreuve: Buren's black bands



Epreuve

Estimate the number of small strips of black marble it took to pave the Place des Terreaux.

Réponse:



Élément de solution:

There are 30 slats on one side of a square, 24 in the other direction. There are 17 by 7 large squares in the square (not all of them whole). So that's about 6,500 slats of black marble.

Indice 1

How many are there on one side of a large square? On the other side? How many squares?

Indice 2

Be careful, the slats are oriented east-west, there is less in the other direction!

Indice 3

Not all the squares are complete, but that's not necessarily a big deal for an estimation!

17. Épreuve: Horse from the future



Epreuve

On the facade of the Town Hall, there is a statue of Henri IV. His white horse is looking towards another horse, present in another emblematic place of Lyon, which one?

- A) Part-Dieu
- B) Saint John's Cathedral
- C) Croix-Rousse
- D) Bellecour square
- E) Fourvière's Basilica
- F) Square of the Terreaux

Réponse:

- Part-Dieu
- Saint John's Cathedral
- Croix-Rousse
- Bellecour square
- Fourvière's Basilica
- Square of the Terreaux

Élément de solution:

He looks at the horse of his grandson Louis XIV who put an end to the Edict of Nantes. This statue is in Place Bellecour. It is a work by Lemot, dating from 1825 but using a similar statue from 1713, destroyed during the revolution.

Indice 1

These are two statues. You may have crossed the other

Indice 2

One horseman promulgated an edict which the other horseman revoked.

Indice 3

These two horsemen are kings of France, grandfather and grandson, one says "the good", the other, "the great".