Constructing networks of texts

a heuristical method for discovering some collective dimensions of mathematics

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&
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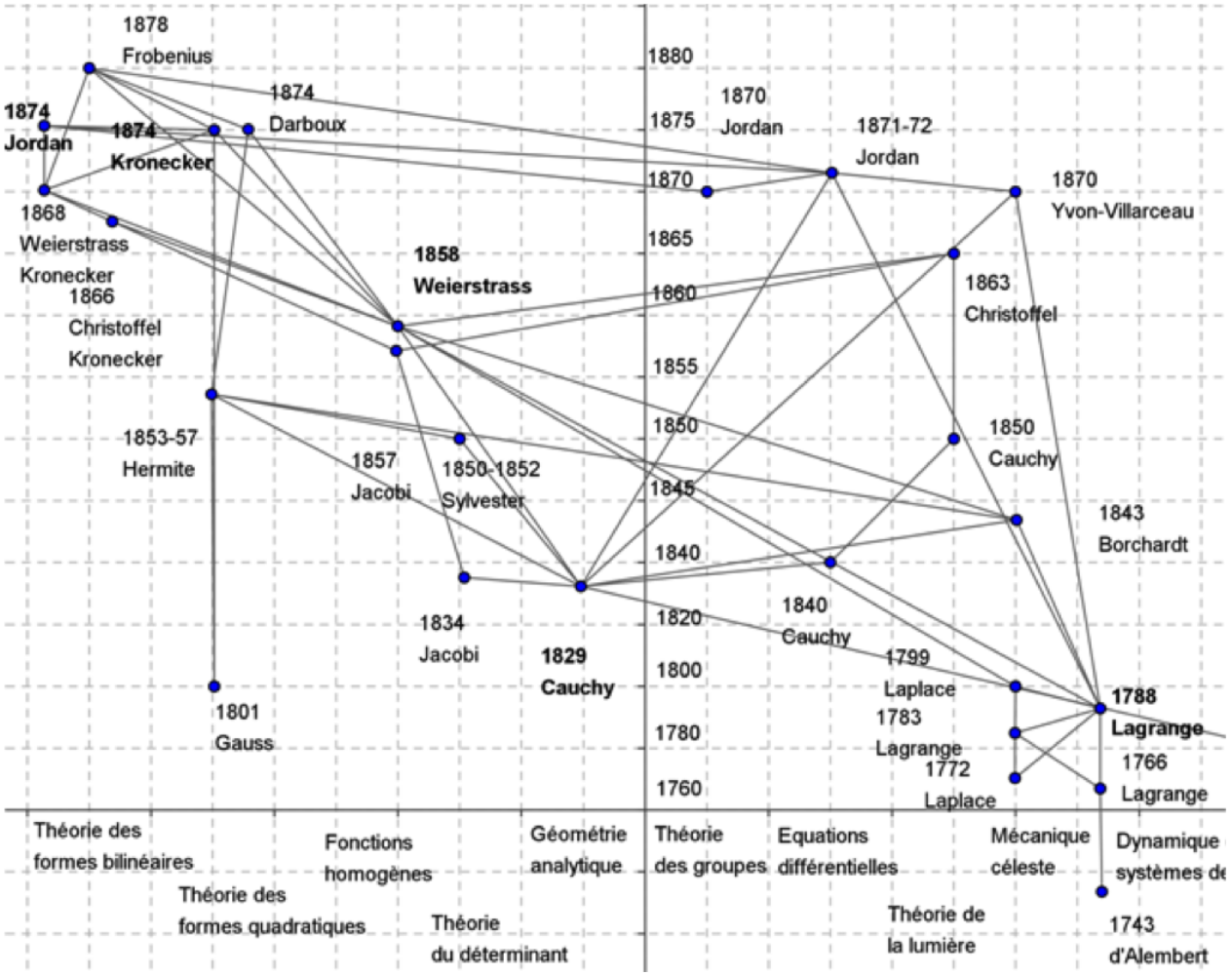
Département humanités et sciences sociales

5 novembre 2013

Networks of texts



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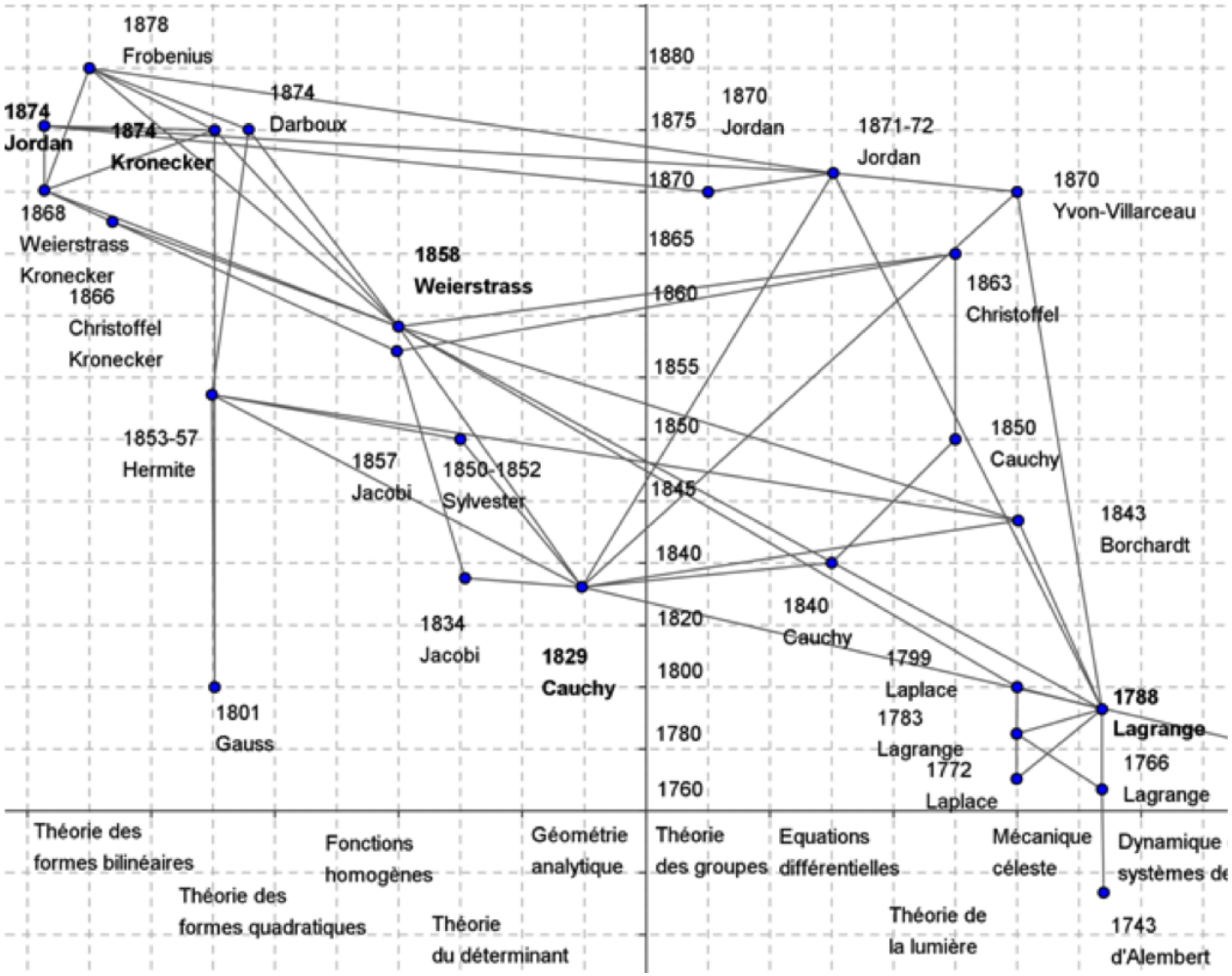

what is it ?

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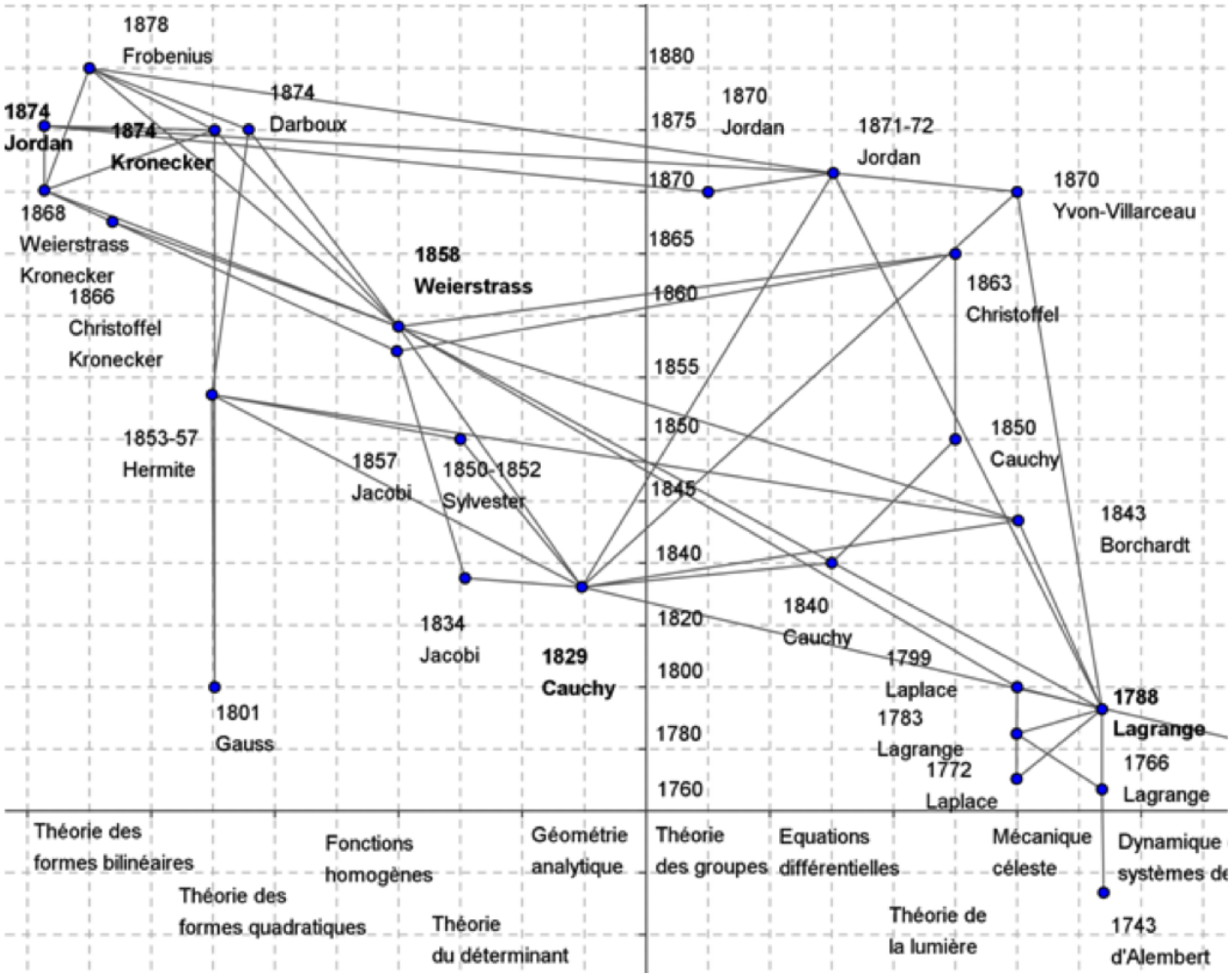

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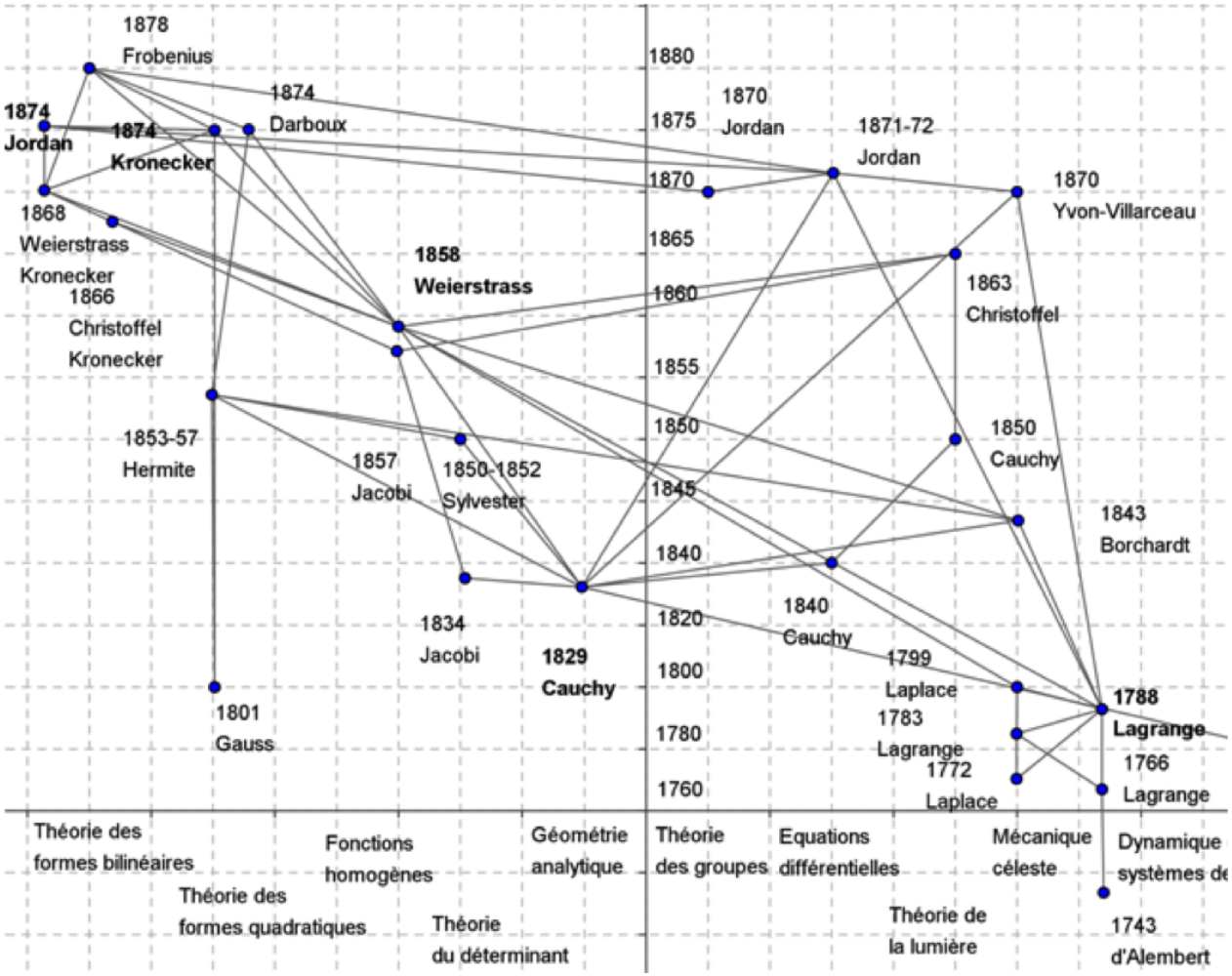

what is it ? what is it made of ? what is made for ?

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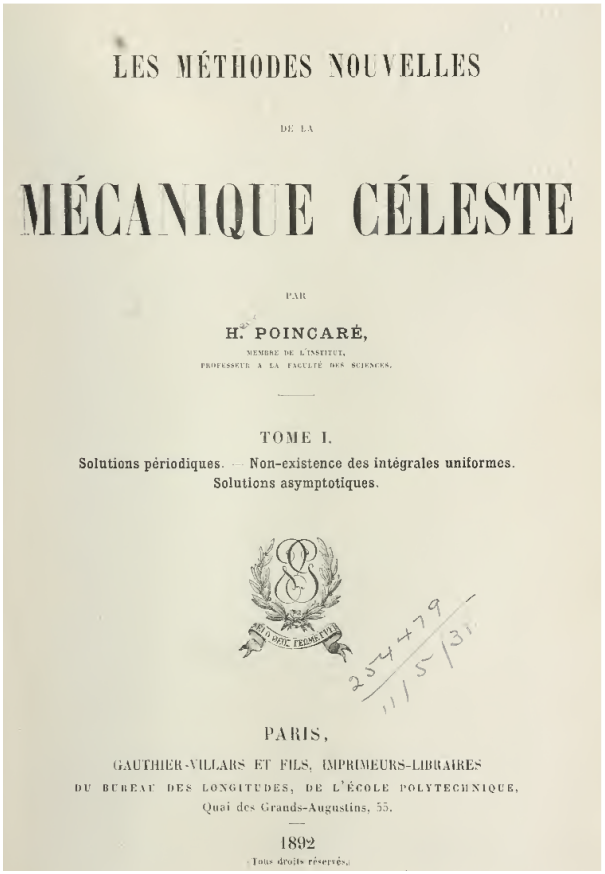
- scale games ... that will lead us to blow up the corpuses that are usually considered in the history of'analysis'.

What's new in Poincaré's *Méthodes nouvelles de la mécanique céleste ?*



LES MÉTHODES NOUVELLES

DE LA

MÉCANIQUE CÉLESTE

PAR

H. POINCARÉ,

MEMBRE DE L'INSTITUT,
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TOME I.

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Solutions asymptotiques.

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Tous droits réservés.

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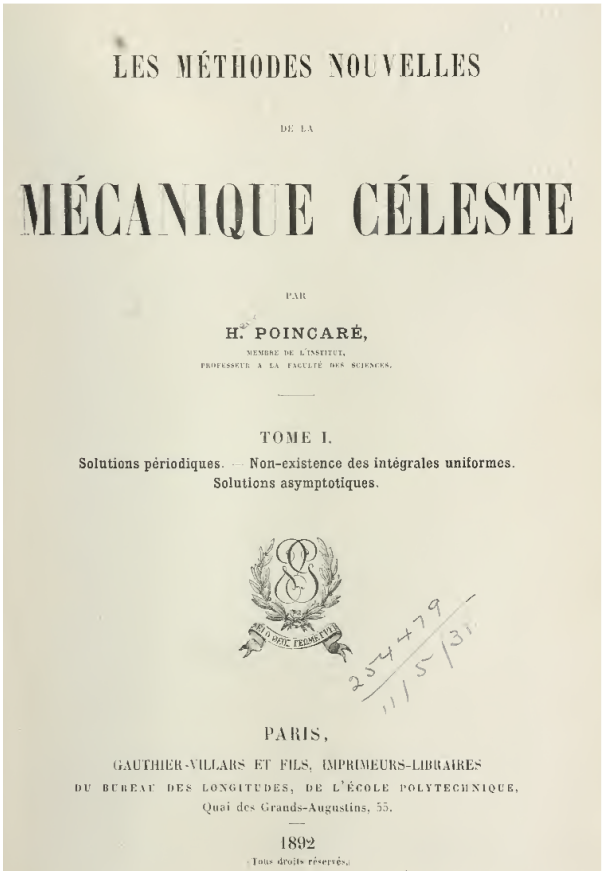

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To identify the collective dynamics that are relevant for grasping individual creativity. 'Mecanics' : another kind of analysis thant the ones Hélène dealt with yesterday (recall the issues about Boussinesq etc.)

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  - the introduction of probabilities into celestial mechanics
- ... a problematic posterity / a problematic position in the history of analysis in the 19th and 20th centuries

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This case study aims at shedding a new light on **some aspects** of Poincaré's celestial mechanics by highlighting some issues that were hardly new and which hardly belonged to celestial mechanics, differential systems, topology or even analysis.

Hardly new

Poincaré, Introduction of the *Méthodes nouvelles*, 1892 :

> *L'étude des inégalités séculaires par le moyen d'un système d'équations différentielles linéaires à coefficients constants peut donc être regardée comme se rattachant plutôt aux méthodes nouvelles qu'aux méthodes anciennes.*

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- ... as the use of linear systems in celestial mechanics dates back to the 18th century : Jean le Rond d'Alembert's, Joseph-Louis Lagrange's, and Pierre-Simon Laplace's great treaties.

Hardly belonging to celestial mechanics

Poincaré's 1881 description of his qualitative approach to differential equations

Ainsi, par exemple, pour étudier une équation algébrique, on commence par rechercher, à l'aide du théorème de Sturm, quel est le nombre des racines réelles : c'est la partie qualitative ; puis on calcule la valeur numérique de ces racines, ce qui constitue l'étude quantitative de l'équation. [...] C'est naturellement par la partie qualitative qu'on doit aborder la théorie de toute fonction et c'est pourquoi le problème qui se présente en premier lieu est le suivant : Construire les courbes définies par des équations différentielles.

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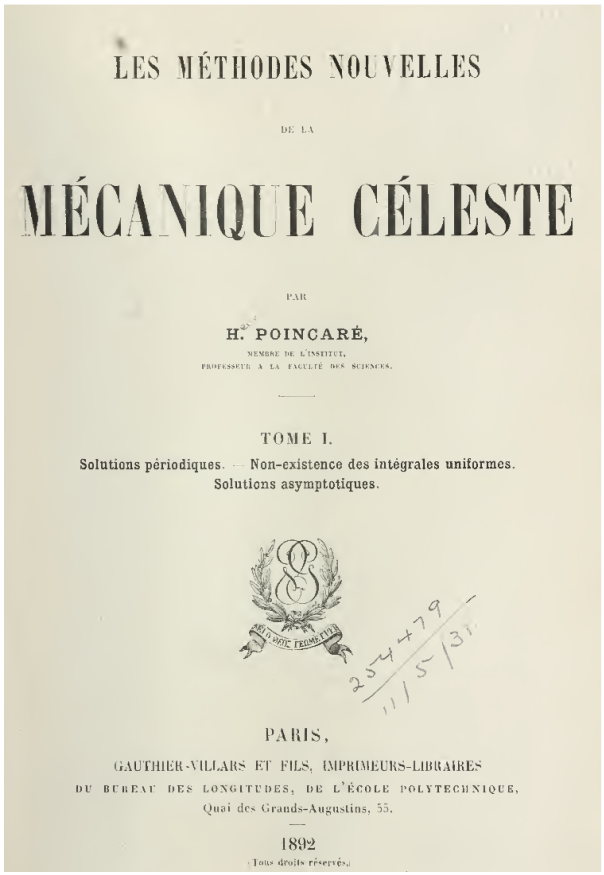
1. Hardly new : linear systems, a direct relationship to Lagrange's *Mécanique analytique*...
2. Hardly belonging to celestial mechanics : Sturm theorem, the long run, the collective dimensions of Poincaré's *Méthodes nouvelles*

Reminder : Poincaré's approach to the three-body problem

- King Oscar II's price (Barrow-Greene 1996)
- The error in Poincaré's memoir : the introduction of homoclinic solutions (Anderson 1994)
- The writing of the *Méthodes nouvelles* from 1892 to 1899.
- In this case study : a focus on the role of periodic trajectories in the strategy Poincaré developped in celestial mechanics

The issue of the stability of the solar system



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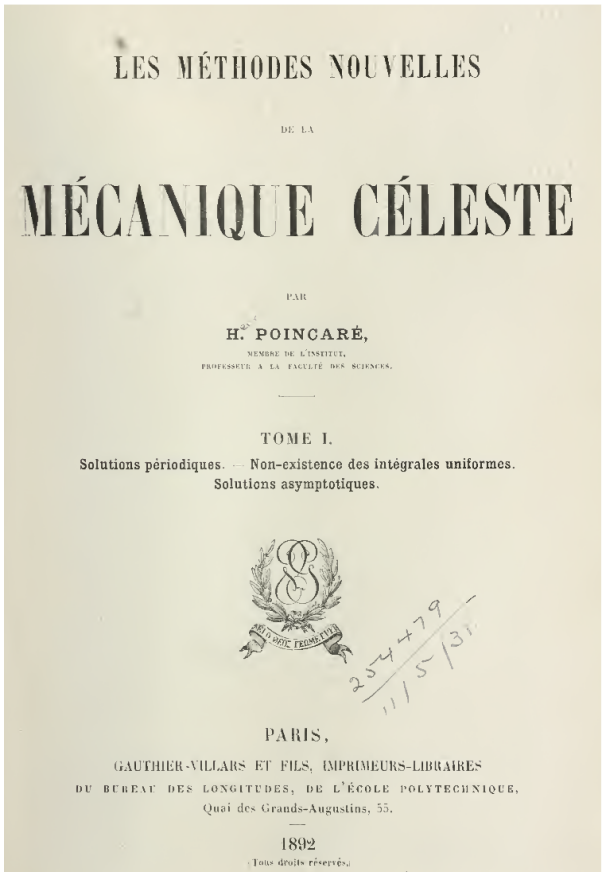

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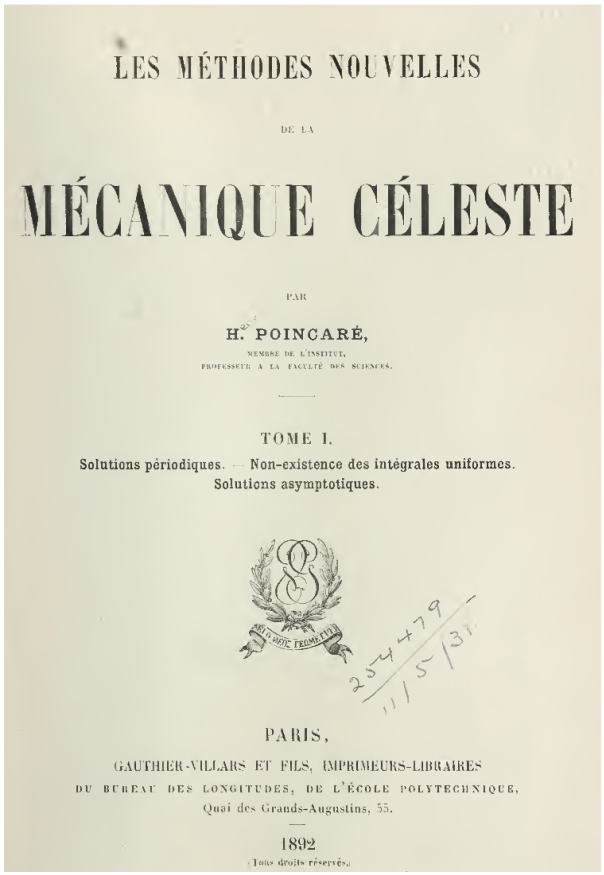

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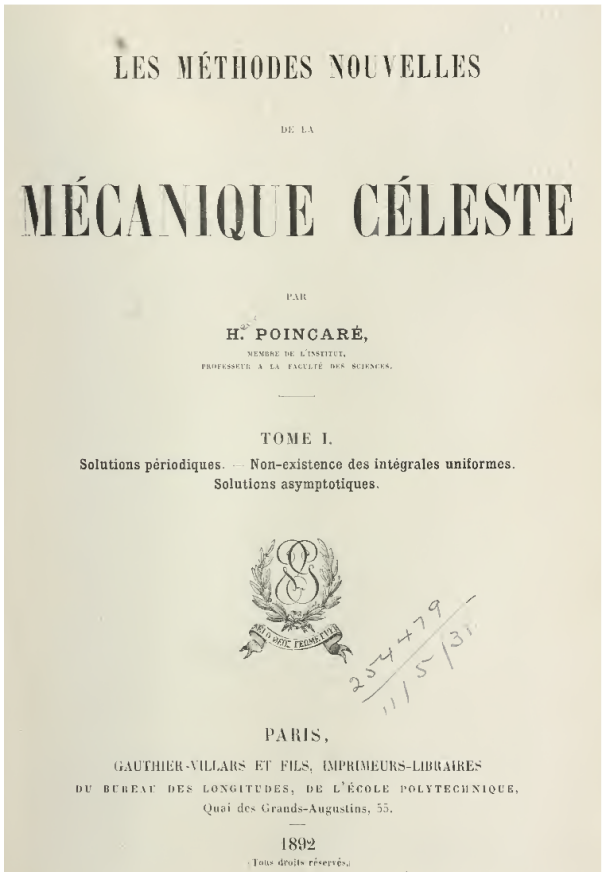
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- as well as the mathematical problem of the stability of the solar system

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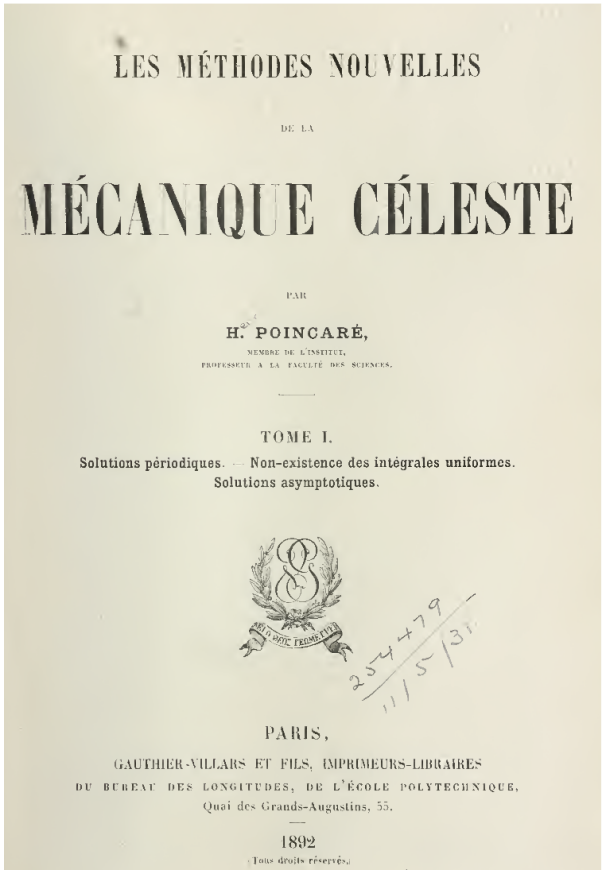

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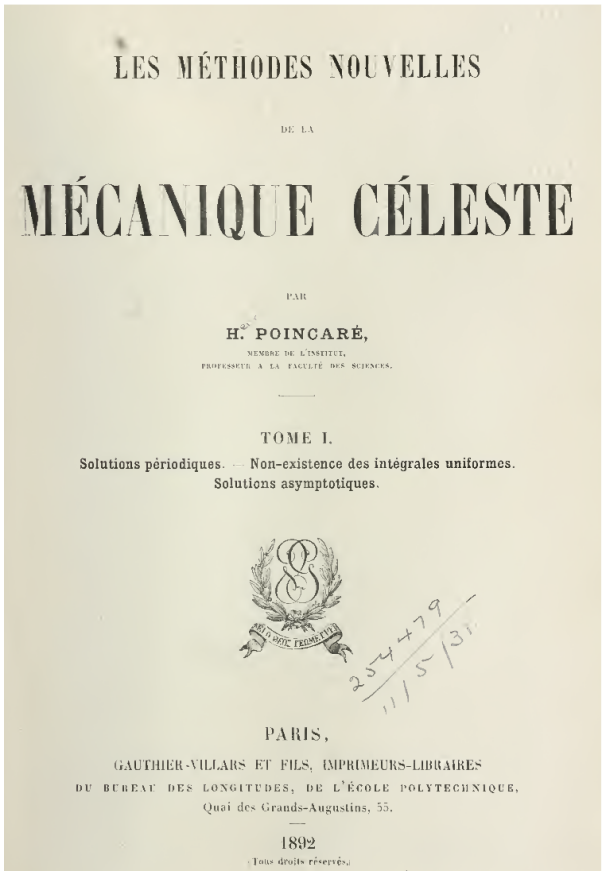
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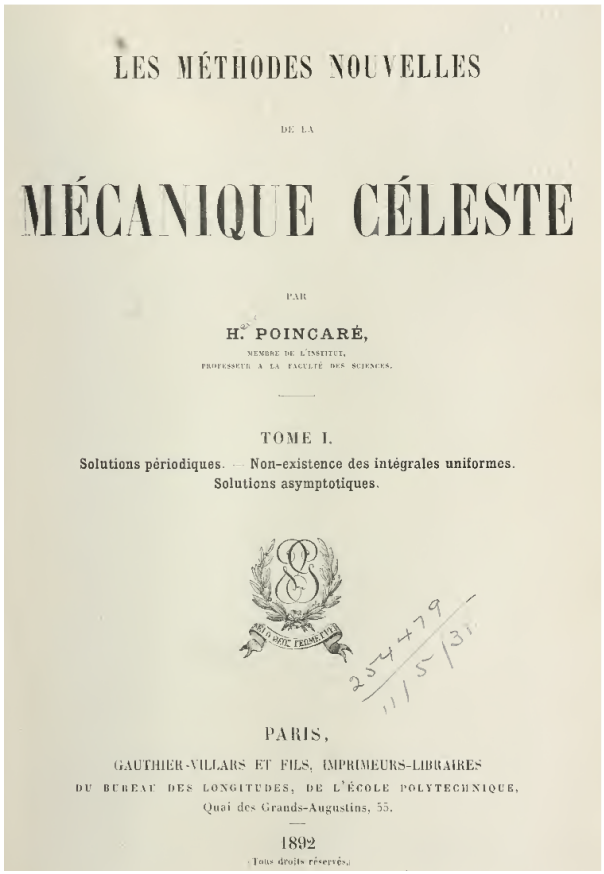
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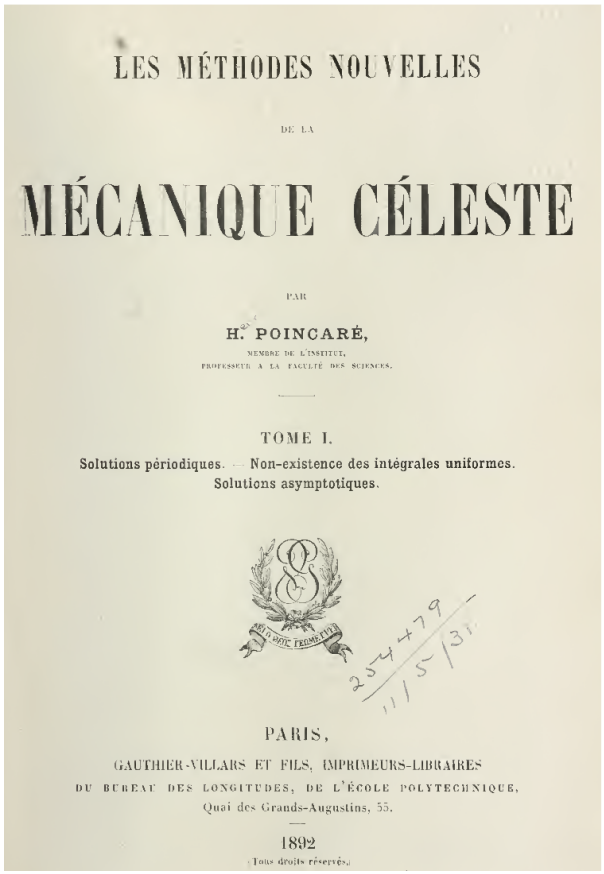
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- Expressing the trajectory of a celestial body assimilated to a point of coordinates $(x_1, ..., x_n)$ in function of the time t by some analytic functions X_i of the coordinates
- a differential system which cannot be exactly solved.
- Approaching general solutions by some particular ones ...

Approximations by periodic solutions

Poincaré 1891 :

> *Si l'on se donne ces conditions initiales du mouvement, on aura défini une solution particulière du mouvement. [...] La position et la vitesse initiales de notre satellite auraient pu être telles que la Lune fût constamment pleine ; elles auraient pu être telles que la Lune fût constamment nouvelle [...] dans une des solutions possibles, la Lune, d'abord nouvelle, commence par croître ; mais, avant d'atteindre le premier quartier, elle se met à décroître pour redevenir nouvelle et ainsi de suite ; elle aura donc constamment la forme d'un croissant.*

While some of these particular solutions "are only interesting because of their weirdness", others have "astronomical applications" : periodic trajectories.

Approximations by periodic solutions

Poincaré 1891 :

> *Ce sont celles où les distances des trois corps sont des fonctions périodiques du temps ; à des intervalles périodiques, les trois corps se retrouvent donc dans les mêmes positions relatives.*

Solutions périodiques et approximations

The core of Poincaré's strategy : small variations of periodic solutions / the description of more complex trajectories (e.g. asymptotic and homoclinic solutions)

Poincaré 1891 :

> *En effet, il y a une probabilité nulle pour que les conditions initiales du mouvement soient précisément celles qui correspondent à une solution périodique. Mais il peut arriver qu'elles en diffèrent très peu, et cela a lieu justement dans les cas où les méthodes anciennes ne sont plus applicables. On peut alors avec avantage prendre la solution périodique comme première approximation. [...] D'ailleurs, ce qui nous rend ces solutions périodiques si précieuses, c'est qu'elles sont, pour ainsi dire, la seule brêche par où nous puissions essayer de pénétrer dans une place jusqu'ici réputée inabordable.*

Poincaré's equations of variations

Let ϕ be a given periodic solution.

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$$\frac{dx_i}{dt} = X_i \ (i = 1, ..., n)$$

one gets the "équations aux variations" which express the difference $\xi_i(t)$ between the coordinates, $x_i(t)$ and $\phi_i(t)$, of the two trajectories,.

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Suppose ξ_i is very small and neglect all the terms of a degree higher than the first :

$$\frac{d\xi_i}{dt} = \sum_{j=1,n} \frac{\delta x_i}{\delta x_j} \xi_j \ (i, j = 1, ..., n)$$

a system of linear equations with periodic functions of t as coefficients (say of a period 2π)

Poincaré's equations of variations

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- Linearity : a solution is a linear combination of n independent solutions $\psi_i(t)$

Poincaré's equations of variations

$$\frac{d\xi_i}{dt} = \sum_{j=1,n} \frac{\delta x_i}{\delta x_j} \xi_j \ (i,j = 1,...,n)$$

- Linearity : a solution is a linear combination of n independent solutions $\psi_i(t)$
- Periodicity : $\psi_j(t + 2\pi)$ are solutions

Poincaré's equations of variations

$$\frac{d\xi_i}{dt} = \sum_{j=1,n} \frac{\delta x_i}{\delta x_j} \xi_j \ (i,j = 1,...,n)$$

- Linearity : a solution is a linear combination of n independent solutions $\psi_i(t)$
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- one thus eventually get a **linear system with constant coefficients**

Periodic solutions and linear systems

The whole strategy of "approximations" by periodic solutions : forcing linear systems with constant coefficients ...

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A strategy molded on a specific approach to the secular inequalities in planetary theory developped in the 18th century.

Changing the scale of analysis

From the small oscillations of swinging strings to the ones of periodic trajectories

Lagrange, 1766 : the small oscillations $\xi_i(t)$ of a string loaded with n bodies

Lagrange 1774 : the small oscillations of planets on their orbits : the "secular inequalities"

Lagrange 1788 : the small oscillations of a general mechanical system of n bodies

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A (symetric) system of n linear equations with constant coefficients :

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The secular equation

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Back to Poincaré's variations of periodic trajectories

The transfer of the same mathematization to the oscillations of a periodic trajectory $x_i(t) = \phi_i(t) + \xi_i(t)$

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If the equation in S has n unequal roots, the small oscillations of a periodic solution are given by :

$$\xi_i(t) = C_1 e^{\alpha_1 t}\lambda_{1,i}(t) + C_2 e^{\alpha_2 t}\lambda_{2,i}(t) + \ldots + C_n e^{\alpha_n t}\lambda_{n,1}(t)$$

where $\lambda_{i,j}(t)$ are convergent trigonometic sums with the same periodicity as $\phi_i(t)$.

The algebraic nature of the roots / mechanical nature of the motion

The behaviour of the set of trajectories generated by the oscillations of a periodic solution is controlled by the algebraic nature of the roots of the "equation in S" (which Poincaré designated as the "exposants caractéristiques")

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especially by their order of multiplicity.

Lagrange's criterion of stability

Lagrange, *Mécanique analytique*, 1788

1. A mechanical system is stable iff the α_i are *real, negatives and distinct*. In this situation a particular solution has the form $sin(\alpha_i t)$
2. In the case of imaginary roots : some exponential oscillations
3. In the case of a multiple root : $tsin(\alpha_i t)$: "le temps sort du sinus" and generates some non periodic unbounded oscillations (false :Weierstrass 1858, Jordan 1871)

Mechanical stability and algebraic multiplicity

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- If the periodic solution remains "stable", the approached trajectories will remain close to the periodic solution
- unstable periodic solutions support the introduction of more complex trajectories such as asymptotic solutions... (Roque 2011)

Hardly new ...

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> *L'étude des inégalités séculaires par le moyen d'un système d'équations différentielles linéaires à coefficients constants peut donc être regardée comme se rattachant plutôt aux méthodes nouvelles qu'aux méthodes anciennes.*

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- Poincaré's revendication of a direct relationship to the great treaties of Lagrange and Laplace
- Jumping over most of the non linear developments in celestial mechanics since Poisson in 1808 ?

Collective dimensions ?

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(a few of) the transfers of Lagrange's criterion to other problems of mechanical stability in the 19th century :

- equilibrium figures - rotating fluid and solid bodies (Lagrange, Jacobi, Riemann, Dirichlet etc.),
- elasticity (on the model of Fresnel's theory of light)
- heat theory Fourier,
- the molecular theory of light (Cauchy, Christophel, etc.),
- ... at the heart of the second edition of Thomson and Tait's *Treatise on Natural Philosophy* (1879-1883) (Darigol 2002, Wise 2005).

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Changing the scale of analysis : the long run / the collective dimensions of Poincaré's strategy / beyond celestial mechanics

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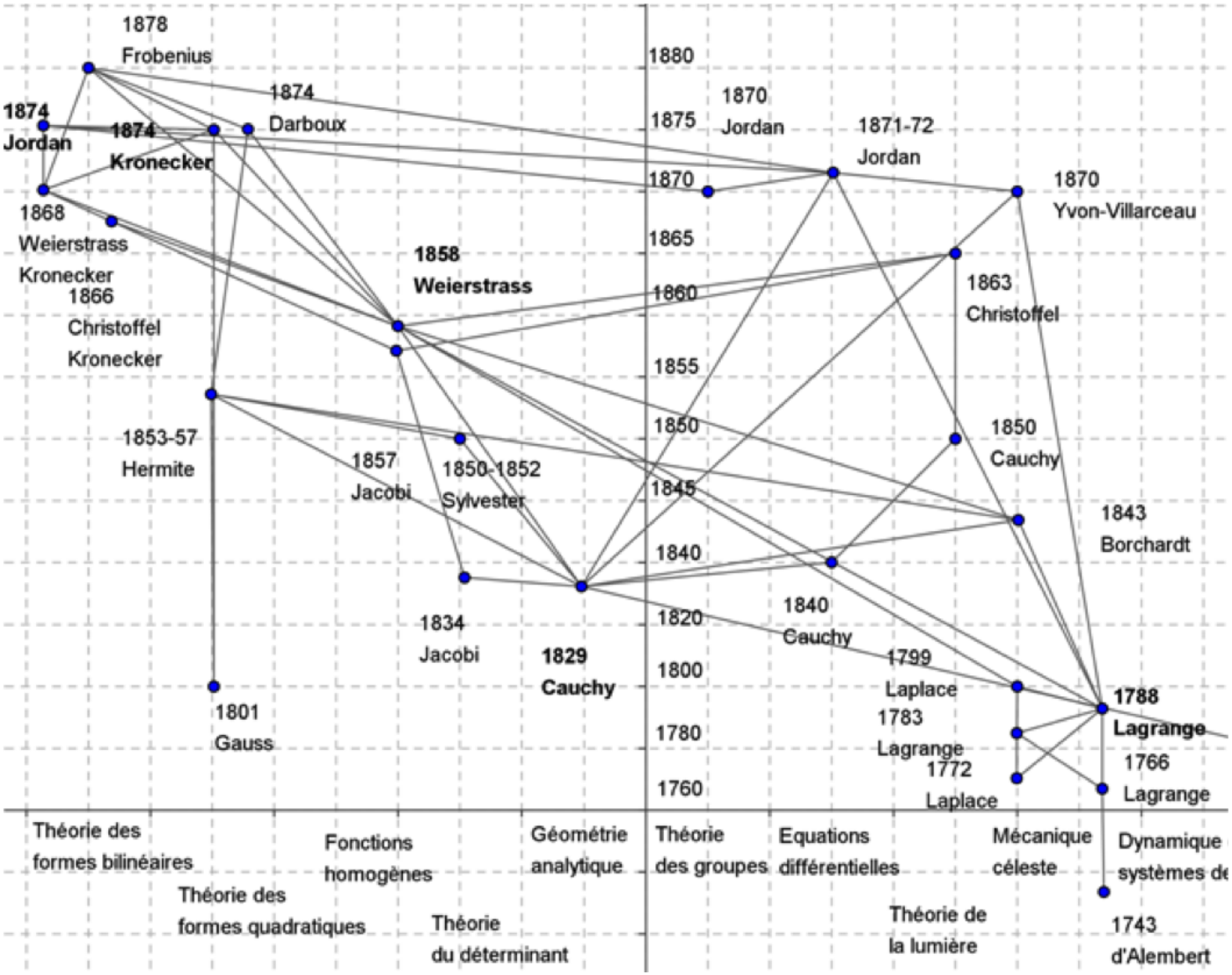
- Great many references to the secular equation (or "equation in S") in the 19th century...
- ... most of the time with little interest for celestial mechanics...

The equation to the secular inequalities in planetary theory



The secular equation : a shared algebraic culture in the 19th century

A mathematical culture (Brechenmacher 2007), in the particularist and interactionist sense developped in social sciences (Sapir1949 ; Bastide 1971).

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- Forms of representations
- Operatory procedures
- Ideals
- Interconnections between scientific domains
- Values

The analytic representation

One widely shared meaning of 'analytic'

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One widely shared meaning of 'analytic' . Explicit expressions for the equations and their solutions

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$$\xi_i(t) = C_1 e^{\pm\sqrt{\alpha_1}t} + C_2 e^{\pm\sqrt{\alpha_2}t} + \ldots + C_n e^{\pm\sqrt{\alpha_n}t}$$

Polynomial procedures for manipulating linear systems

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A rational expression of the coordinates $(x_i^{\alpha_j})$ of the solutions of symetric linear systems of n equations with constant coefficients.

$$x_i^{\alpha_j} = \frac{\Delta_{1i}}{\frac{\Delta}{S-\alpha_j}}(\alpha_j)$$

Involving

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- $\Delta(S) = \begin{vmatrix} A_{1,1}-S & A_{1,2} & \dots & A_{1,n} \\ A_{2,1} & A_{2,2}-S & \dots & A_{2,n} \\ \dots & \dots & \dots & \dots \\ A_{n,1} & A_{n,2} & \dots & A_{n,n}-S \end{vmatrix}$, the (polynomial) characteristic determinant of the system A, i.e. the one that generates the "equation in S" : $det(A - SI)$
- its (polynomial) successive minors $\Delta_{1i}(S)$ (developments / first line and ith column)

Specific ideals

Specific ideals of generality : dealing with n variables

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The special nature of the secular equation : the equation is of degree n and thus cannot be solved by radicals in general but the nature of its roots can be deduced from the symetry of the linear system which generates the equation : the roots of the secular equation are always real.

Interconnections between scientific domains

In the long run of the 19th century, the special nature of the secular equation has supported various analogies between different branches of the mathematical sciences

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Specific interconnections between various branches of the mathematical sciences :

- swinging strings,
- celestial mecanics,
- heat theory,
- elasticity,
- the theory of light ,
- analytic geometry ,
- Sturm theorem,
- complex analysis
- algebraic theory of quadratic forms,
- molecular theory of light, etc.

Interconnections between scientific domains : the example of Cauchy

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The problem of the determination of the principal axis of a conic (with real coefficients) (Fresnel's theory of light, elasticity theory, the teaching of analytic geometry at polytechnique).

$$f(x_1, x_2, ..., x_n) = A_{11}x_{12}^2 + A_{22}x_{22}^2 + ... + A_{nn}x_{n2}^2 + 2A_{12}x_1x_2 + 2A_{13}x_1x_3 + ...$$

To turn the above function into a sum of squares

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Interconnections between scientific domains : analysis

The analogy supported by the secular equation had actually been pointed out to Cauchy by Sturm, who had been especially interested in this equation in connection with the statement of his theorem on the number of real roots of an algebraic equation. (Hawkins 1975)

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The Sturm theorem had been stated in the late 1820s in the framework of researches on linear differential equations. (Sinaceur 1992) in connection with Fourier's aim of counting the roots of any equations, i.e., both algebraic and transcendental equations.

Sturm's theorem has then been considered as giving rise to a **"general notion" of analysis**, that one could apply to the transcendental functions encountered in problems of celestial mechanics, swinging strings, heat theory, waves propagations, etc.,

As well as a general tool of *a priori* analysis, that is a **"qualitative" analysis of equations analyzed "by themselves" ("en elles-mêmes")**, whether these were **algebraic, transcendental or differential equations**.

Equations

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At the beginning of the 19th century, the "theory of equations," and more generally "algebra," were usually considered as a "specie" of an "analytic gender" altogether with "differential analysis," "infinitesimal analysis," "geometric analysis," and the "analysis of curves." (Sinaceur 1992)

Shared values : problems of multiple roots

$$f(x_1, x_2, ..., x_n) = \Delta_{n-1} X_1^2 + \frac{\Delta_{n-2}}{\Delta_{n-1}} X_2^2 + ... + \frac{\Delta}{\Delta_1} X_n^2$$

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It was for overcoming the problems posed by multiple roots that Cauchy developed his Residue theory in the mid 1820s : a fully homogeneous and general solution to systems of n linear differential equations with constant coefficients, whatever the multiplicity of roots.

$$y_i(t) = \Sigma_{j;k=1}^{n} a_j Res_{s=\alpha_k}[\frac{\Delta_{1i}(s)}{\Delta(s)}]e^{st}, \ i = 1, 2, ..., n$$

The royal road to me ?

From Cauchy to Leopold Kronecker, several mathematicians appealed to the example of the secular equation to blame the generic tendency of algebraic reasonings which pay little attention to singularity : an emblematic example of the development of Analysis as a march toward rigor, esp. Weierstrassian's rigor (Kronecker 1874).

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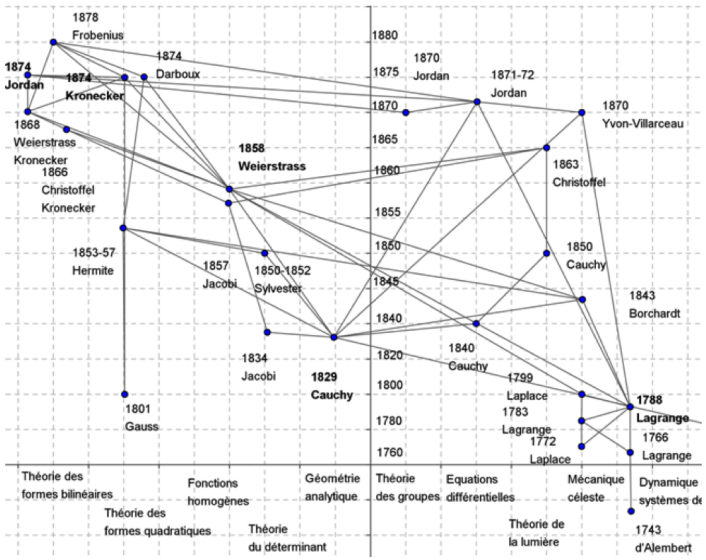

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Values, not only of rigors, but also to provide an homogeneous solution whatever the multiplicity of roots

The slow weathering of the shared culture of the secular equation

- Yet, from the 1850s on, different homogeneous algebraic approaches to the problem of the multiplicity of roots
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- Different local algebraic cultures.... The slow weathering of the shared culture of the secular equation... a strong structuration of the algebraic practices at use at the end of the 19th century (esp. Klein vs. Poincaré).
- looking more closely at one of these local cultures...

Hermite's approach to Sturm theorem

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The core of the algebraic theory of forms : a local mathematical culture in the 1860s-1870s

Hermite's approach to Sturm theorem

To the secular equation one can associate a quadratic form :

$$f(x_1, x_2, ..., x_n) = A_{11}x_1^2 + A_{22}x_2^2 + ... + A_{nn}x_n^2 + 2A_{12}x_1x_2 + 2A_{13}x_1x_3 + ...$$

which can be transformed into a sum of squares by Lagrange's procedure :

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The number of positive and negative signs is an invariant of the quadratic form (Sylvester's inertia law) that provides the number of real distinct roots of the secular equation (and more generally an algebraic proof of Sturm theorem).

A new definition of the notion of multiple root

Darboux 1874, "Sur la theorie algébrique des formes quadratiques"

> *Ainsi une racine multiple pourra être considéré comme simple si elle n'annule pas tous les mineurs [de Δ] du premier ordre ; comme double si, annulant tous les mineurs du premier ordre, elle n'annule pas tous ceux du second et ainsi de suite.*

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Poincaré transfered this definition from the roots of algebraic equations to the solutions of differential equations : "multiplicity of periodic trajectories"
Poincaré 1892

> *Si le déterminant d'une substitution linéaire est nul, ainsi que tous ses mineurs du premier, du second, etc., du (p-1)^e ordre, l'équation en S aura p racines nulles.*

The legacy of the 'algebraic theory of forms'

Moreover, Hermite's approach supports the analysis of the variation of the number of distinct roots of the "equation in S" in function of S

Darboux 1874, "Sur la theorie algébrique des formes quadratiques"

> *Le nombre de carrés positifs de la forme ne peut changer que si S passe par une racine de l'équation [...], et dans ce cas le nombre de carrés positifs de la forme ne peut varier d'une quantité supérieure à l'ordre de multiplicité de la racine considérée.*

The legacy of the 'algebraic theory of forms'

Transfered by Poincaré to the perturbations of periodic solutions in function of a small parameter : a key role in the statements relative to the existence of periodic solutions and to their behaviours after perturbations :

Poincaré 1892

> *J'observe d'abord qu'une solution périodique ne peut disparaître quand μ passe de la valeur $-\epsilon$ à la valeur $+\epsilon$ que si pour $\mu = 0$, l'équation admet une racine multiple ; en d'autres termes une solution périodique ne peut disparaître qu'après s'être confondue avec une autre solution périodique [...] avec laquelle elle aura échangé sa stabilité.* ***Donc les solutions périodiques disparaissent par couples à la façon des racines réelles des équations algébriques.***

Conclusions of the case study

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  - at a smaller scale : the legacy of Hermite's theory of algebraic forms

Disciplinary categories

- Not only did some specific practices for manipulating linear systems emerge from mechanical works but the secular equation generated a shared mathematical culture in the 19th century by supporting the circulation of some procedures between various domains ... until Poincaré used them again in celestial mechanics...

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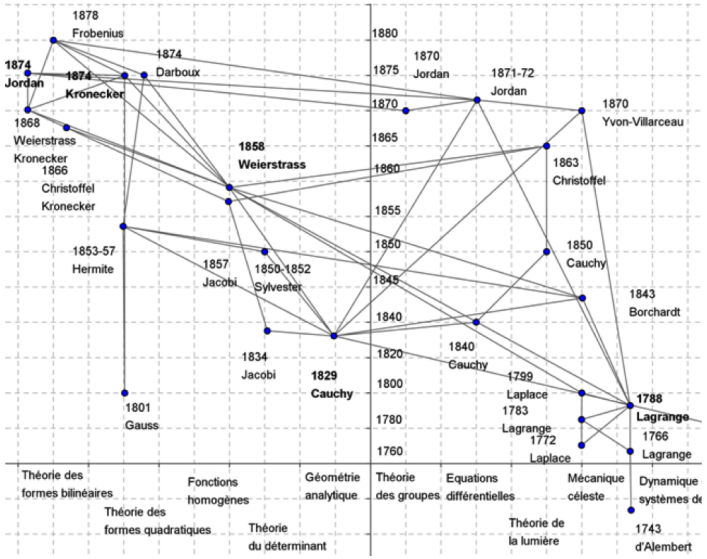
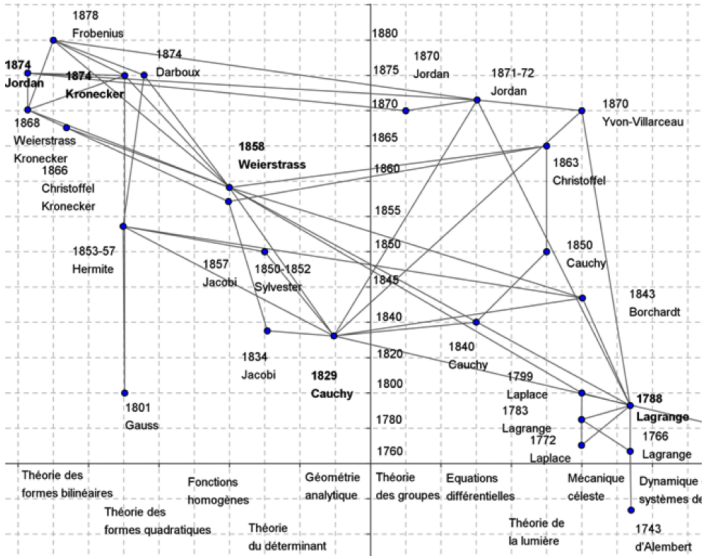
- The significations of categories such as "analysis", "qualitative", "equation," etc. are implicitely defined within such spaces of circulations.

Constructing networks of texts

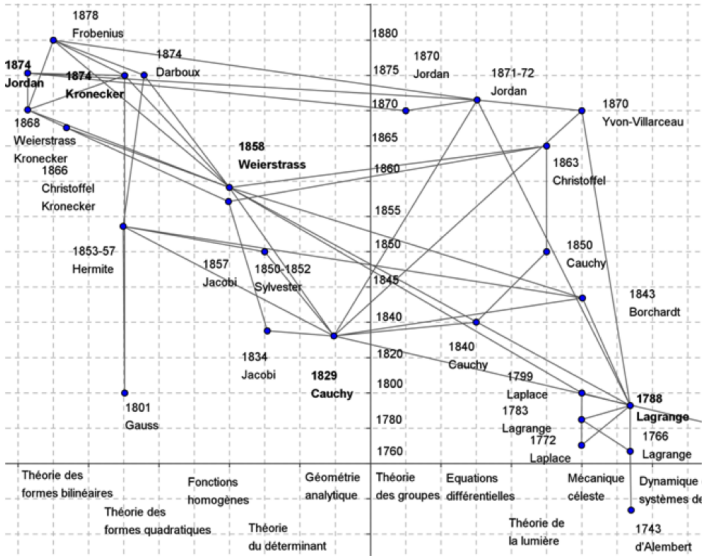


Constructing networks of texts



a problem : the secular equation

Constructing networks of texts



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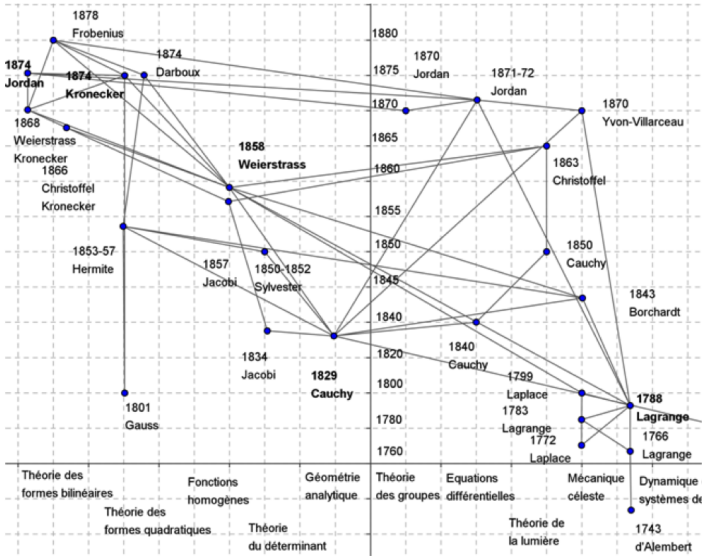
A heuristical method for finding new results / a discipline for reading texts

Constructing networks of texts



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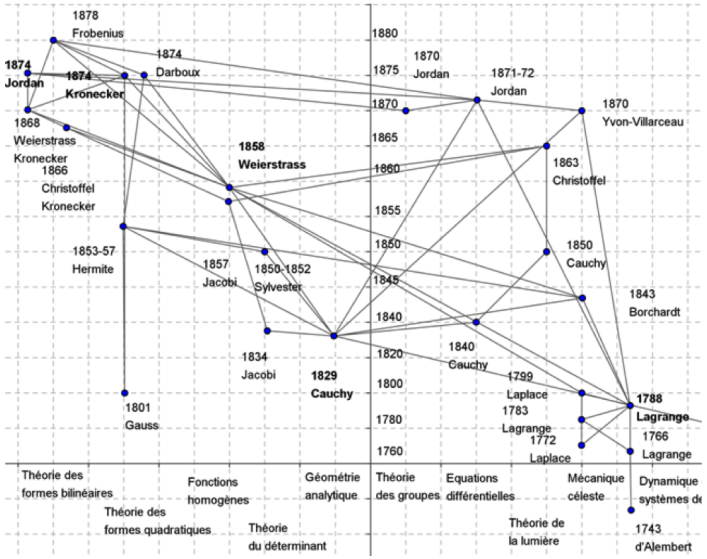
Gives access to the implicit significations taken on by categories such as 'analysis' etc. in some local spaces and times.

Constructing networks of texts



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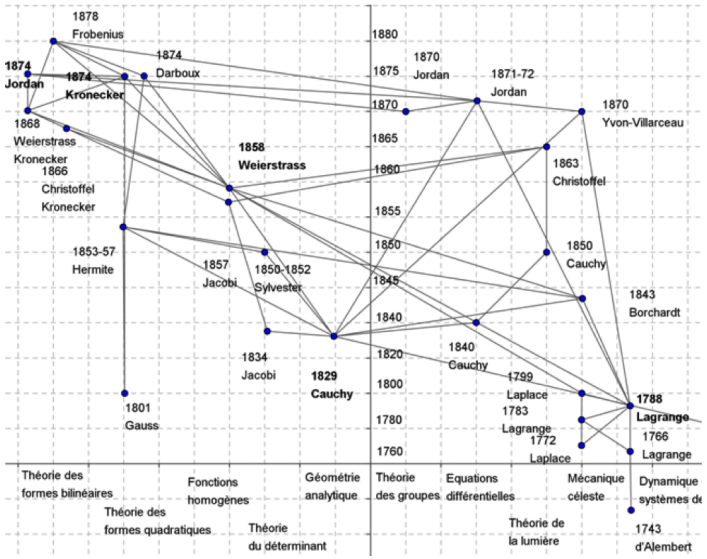
The discovery of some implicit forms of references : "the secular equation", "The analytic representation of substitutions" etc. These allow to complete the initial web of explicit references.

Constructing networks of texts



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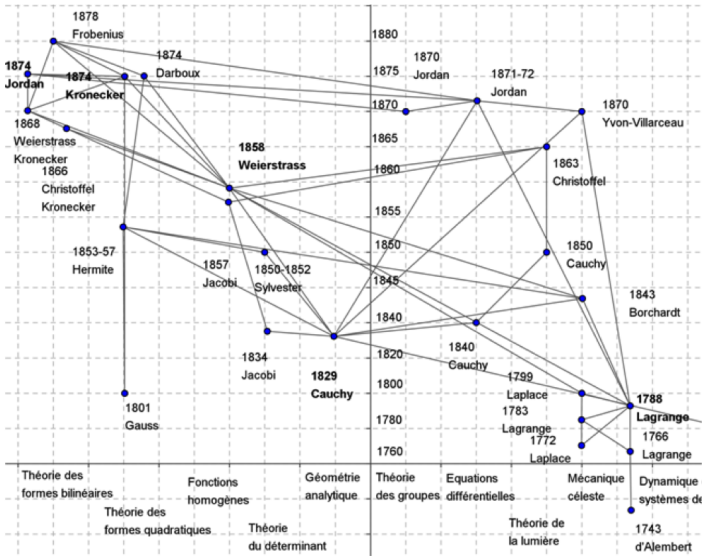
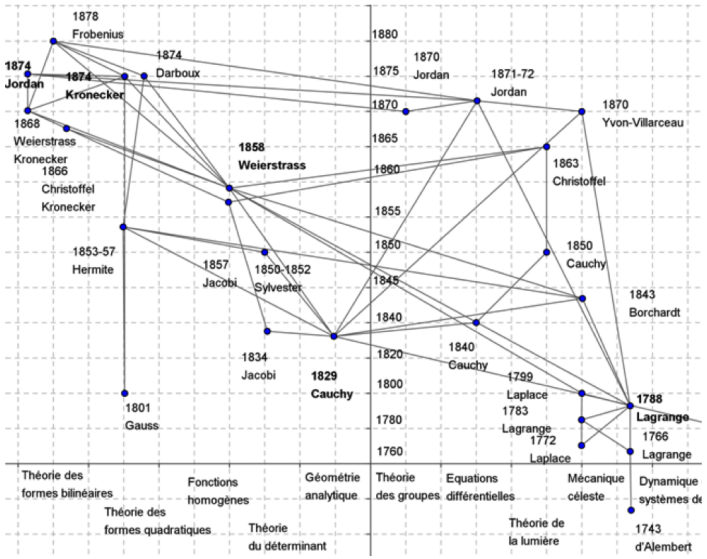


The dangers of reifications : the various practices of intertexual references... the necessity to read, and even to study the texts...

Constructing networks of texts



The dangers of reifications : the various practices of intertexual references... the necessity to read, and even to study the texts...
The network : an artefact, that cannot be dissociated from its point of origin = a text and a problem

Constructing networks of texts

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Other issues give rise to other networks

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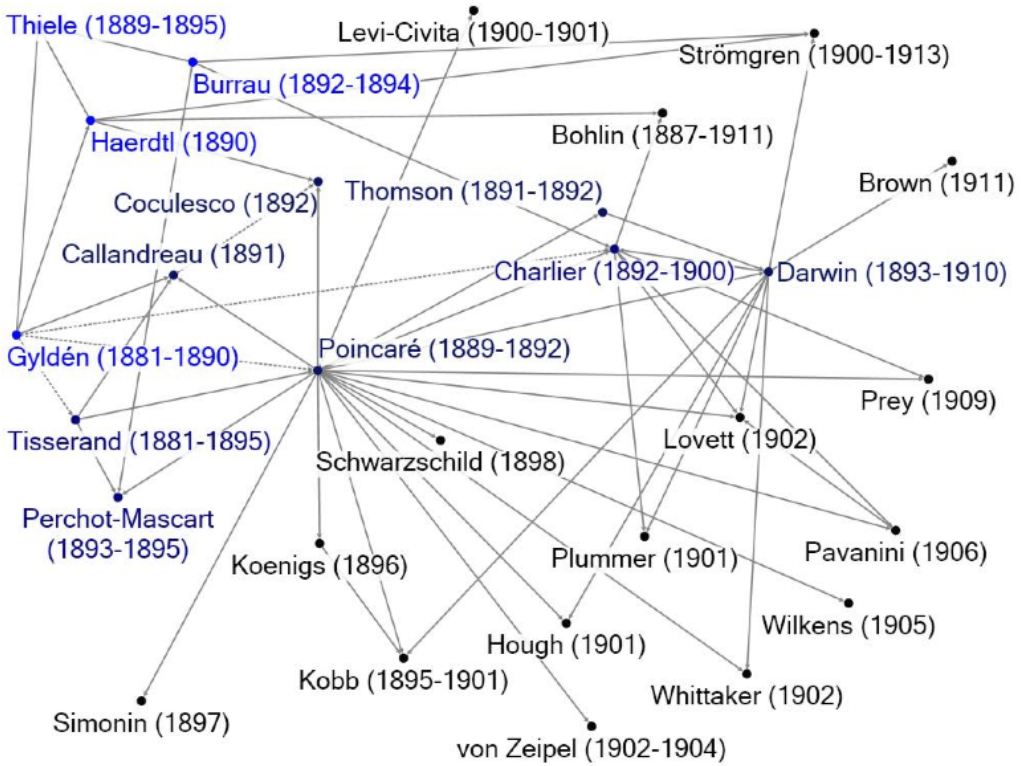
Other issues give rise to other networks such as the network of astronomical texts identified in (Roque 2013). Other collective dimensions : the multiplicity of times and spaces in which a text takes its significations



Constructing networks of texts

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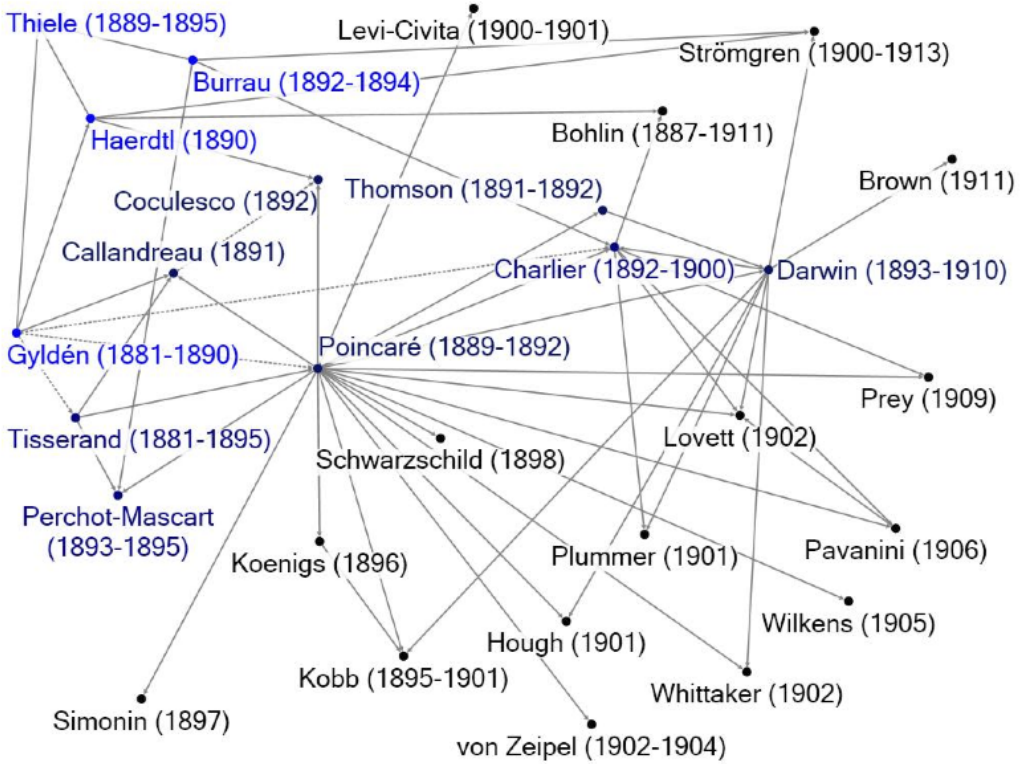

What is it ?

Constructing networks of texts

Roque 2013

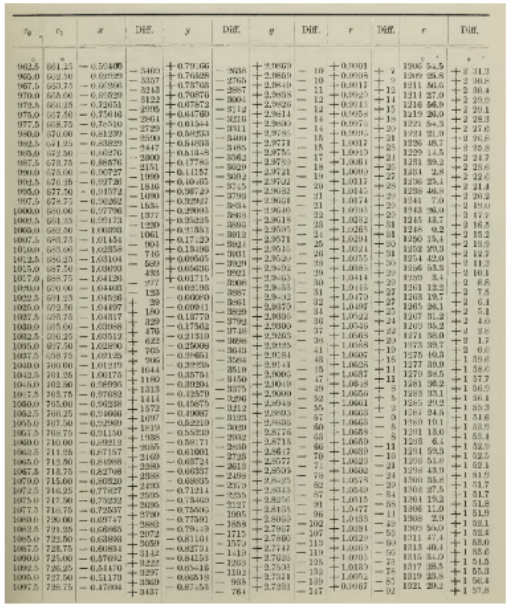

FIGURE 2. Haerdtl fournit 11 pages des tables comme celle-ci, à fin de décrire le mouvement de C

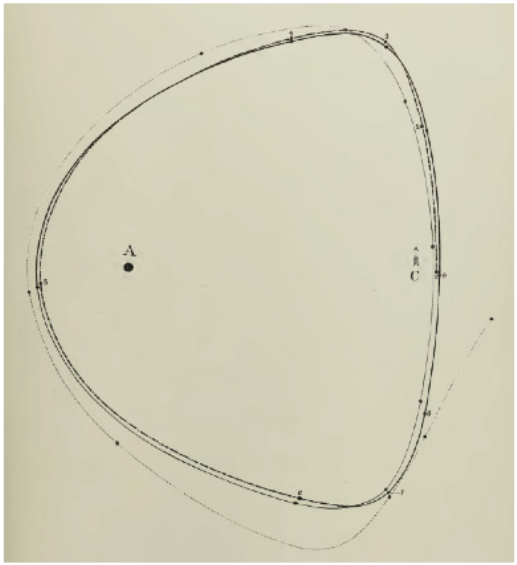

FIGURE 3. Mais il propose aussi des dessins pour montrer ce qui se passe dans le voisinages des orbites périodiques

Constructing networks of texts

Roque 2013

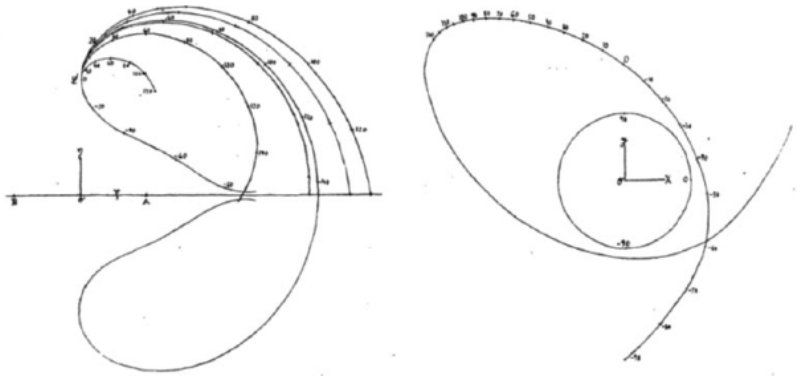

FIGURE 5. Dessins proposés par Burrau montrant les comportements possibles des orbites

What is it ?

François Lê's work on the theorem of the 27 lines on a cubic surfaces

Networks of texts and the collective dynamics of mathematics

Sometimes, networks of texts coincide with networks of actors,

Networks of texts and the collective dynamics of mathematics

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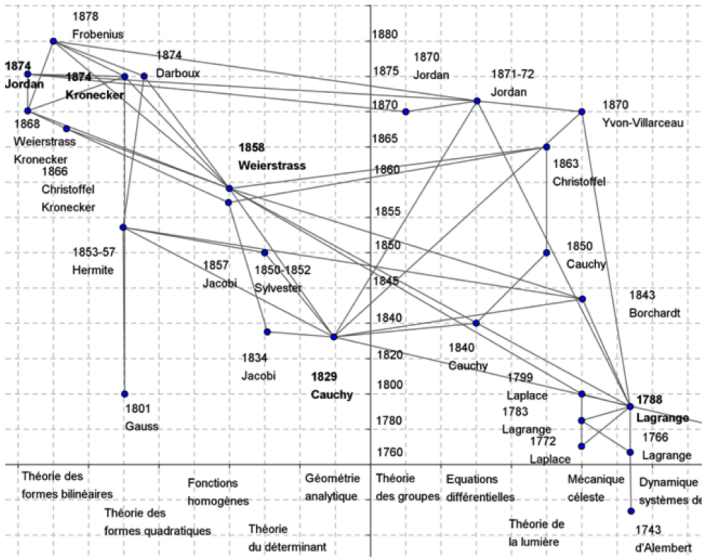
Networks of texts and the collective dynamics of mathematics

Sometimes, networks of texts coincide with networks of actors, the ressources of sociology .... such as in the case of the « research field »(in the sense of Bourdieu) of "Arithmetic algebraic analysis" introduced in Goldstein and Schappacher 2007.

Networks of texts and questions



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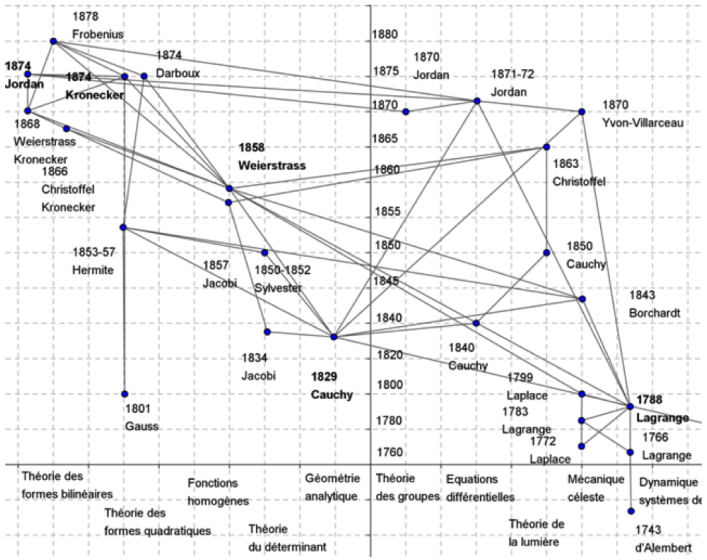
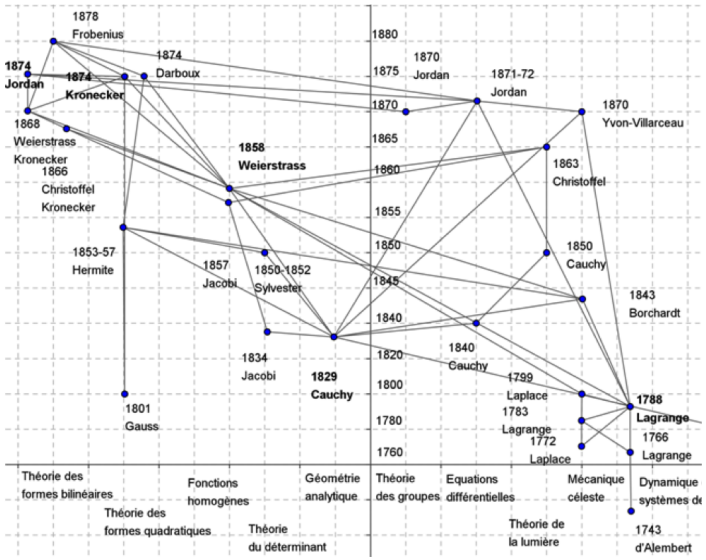


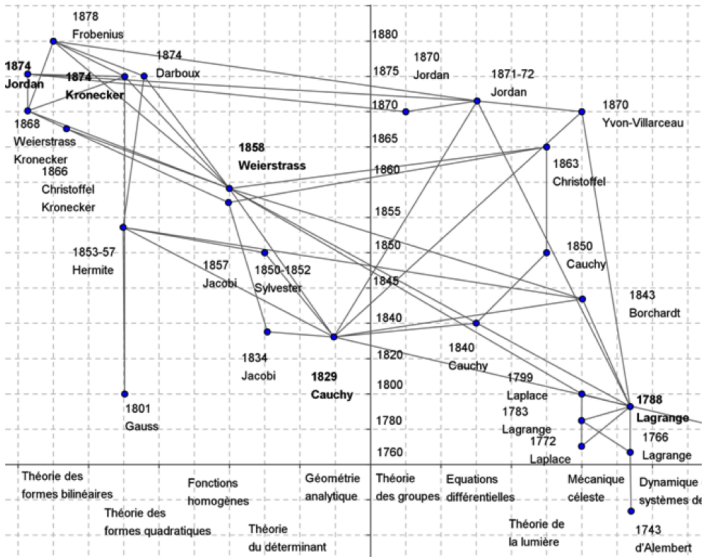
Most of the time it doesn't...

Networks of texts and questions



Most of the time it doesn't... A « **mathematical culture**, in the particularist and interactionist sense developped in social sciences to avoid any reification of the notion of culture (Sapir1949 ; Bastide 1971).

Networks of texts and questions



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A space of interactions between the different concrete actions of individuals and groups of individuals, a space of circulations of **mathematical pratices** : forms of représentations, procedures, ideals, values, interconnections between various domains.

Networks of texts and the collective dynamics of mathematics

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- the dynamics of networks of texts... processes of acculturations

Networks of texts and the collective dynamics of mathematics

Poincaré 1891, *Méthodes nouvelles de la mécanique céleste.*

2° Que si le déterminant Δ est nul ainsi que tous ses mineurs jusqu'aux mineurs de l'ordre p inclusivement, il en sera de même du déterminant

$$\Delta' = \begin{vmatrix} a'_1 & a'_2 & a'_3 \\ b'_1 & b'_2 & b'_3 \\ c'_1 & c'_2 & c'_3 \end{vmatrix}.$$

Les mineurs d'ordre p de Δ' sont, en effet, des combinaisons linéaires des mineurs d'ordre p de Δ;

3° Que l'on peut choisir les λ de façon à ramener la substitution (2) à une forme aussi simple que possible, dite *forme canonique*. Voici en quoi consiste cette forme :

Si l'équation en S a toutes ses racines simples, on peut annuler à la fois $a'_2, a'_3, b'_1, b'_3, c'_1, c'_2$.

Si l'équation en S a une racine double, on peut annuler à la fois $a'_2, a'_3, b'_3, b'_1, c'_1$; on a alors $b'_2 = c'_3$.

Some crucial individual innovations of Poincaré : an acculturation to Jordan's algebraic practices with substitutions groups within a mathematical culture dominated by Hermite's legacy. (Brechenmacher 2013)

Open questions

Dynamic interactions between the diversity of spaces and times within which individuals are interacting : the local, the national, institutions... i.e. the approaches stressed by Hélène in her lecture.

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- While Poincaré's *Méthodes nouvelles* have often been considered as isolated,we are now submerged by collective dimensions...

Attemps to solve these open problems : systematic investigations of some editorial enterprises

Journals... Jenny Boucard, 2011, *Un "rapprochement curieux de l'algèbre et de la théorie des nombres" : études sur l'utilisation des congruences en France de 1801 à 1850*

Catherine Goldstein, 1999, Sur la question des méthodes quantitatives en histoire des mathématiques : le cas de la théorie des nombres en France (1870-1914)

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- The cluster 'Dirichlet' : citations to the analytic works of Dirichlet. Complex analysis in the legacy of Riemann in connection with Dirichlet's series $\sum \frac{a_n}{n^s}$ with s running in the complex field.

CaaFÉ : A collective project on the circulation of algebraic and arithmetic knowledge between France and the U.S.A. (1870-1945)

Attemps to solve these open problems : systematic investigations of some editorial enterprises

@ *Laguerre, Edmond Nicolas* — *Sur la détermination d'une limite supérieure des racines d'une équation et sur la séparation des racines* — *1880* — b\s

buts

Tous

Biblio [3]

Identification : Aucun

← @ *Hermite, Charles*	*Sur la fonction exponentielle*	*1873*	b\s ↔
Cite			
← Petersen, Julius	Theorie der algebraischen Gleichungen	1878	b\s ↔
Cite			
← @ *Laguerre, Edmond Nicolas*	*Sur la règle des signes de Descartes*	*1879*	b\s ↔
Cite			

Personnes [4]

Identification : Aucun

← Descartes, René		1596	1650 b\s ↔
Cite			
← Newton, Isaac		1642	1727 b\s ↔
Cite			
← Budan de Boislaurent, Ferdinand François Désiré		1761	1840 b\s ↔
Cite			

sources

Tous

Biblio [5]

Identification : Aucun

@ *Candèze, Gabriel*	*Sur une règle de M. Laguerre*	*1880*	b\s → ↔
Cite			
@ *Lévy, Lucien*	*Sur le même théorème*	*1880*	b\s → ↔
Cite			
@ *Lucas, Edouard*	*Sur l'extension du théorème de Descartes.*	*1880*	b\s → ↔
Cite			
@ Vénard, Ch.	Sur une règle de M. Laguerre	1880	b\s → ↔
Cite			
@ Laguerre, Edmond Nicolas	Œuvres	1898	b\s → ↔
Etude sur - Cite			

Personnes [0]

Revues [0]

Institutions [0]

Attemps to solve these open problems : systematic investigations of some editorial enterprises

Nom	Titre
Lucas, Félix	*Résolution des équations par l'électricité*
Lucas, Félix	*Résolution immédiate des équations au moyen de l'électricité*
Lucas, Félix	*Détermination électrique des lignes isodynamiques d'un polynôme quelconque*
Lucas, Félix	*Généralisation du théorème de Rolle*
Lucas, Félix	*Résolution électromagnétique des équations*
Demanet, Armand	*Résolution hydrostatique de l'équation du troisième degré*
Meslin, Georges	*Sur une machine à résoudre les équations*
Meslin, Georges	*Sur une machine à résoudre les équations*
Moritz, Robert Edouard	*Some physical solutions of the general equation of the nth degree*
Russell, Alexander & Alty, J.N.	An electromagnetic method of studying the theory of and solving algebraic equations of any degree
Russell, Alexander & Alty, J.N.	An electromagnetic method of studying the theory of and solving algebraic equations of any degree

What's new in Poincaré's Méthodes nouvelles

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Or how, when looking for 'analysis' one eventually collides, not with Mars or Venus ...

What's new in Poincaré's Méthodes nouvelles

Or how, when looking for 'analysis' one eventually collides, not with Mars or Venus ... but with 'number theory'...