

BACCALAURÉAT-Session 2017

Epreuve de Discipline Non Linguistique

Mathématiques/Anglais

Topic: probabilities

The logic of drug testing

London 2012 vowed to be the cleanest Olympics ever, with more than 6,000 tests on athletes for performance enhancing drugs. But when an athlete does fail a drug test can we really conclude that they are cheating? John Haigh does the maths.*



In order to see whether an athlete has sought an unfair advantage using performance-enhancing drugs, various tests can be made. Their detail depends on what banned substance is being investigated, but the logic behind all such tests is identical: some “measurement ” is made, and if its value exceeds some threshold, this is seen as sufficient evidence of cheating.

How likely is it that an athlete who fails the test really is a cheat, that is, how reliable is this accusation? In the language of probability, we seek** the conditional probability that the athlete is guilty given they have failed the test. Three numbers are required to work this out:

- ✓ The first is the so-called sensitivity of the test: in probability terms, this is the conditional probability that the athlete fails the test given he is guilty.
- ✓ A second number is the test’s specificity: the conditional probability that the athlete passes the test given he is not guilty.
- ✓ The final quantity is the actual proportion of drug users in the relevant population, that is the group of athletes who might be tested. This is hard to know with precision, but we can make reasonable estimates.

*vowed: promised

**seek: look for

Adapted from Plus magazine, John Haigh.

Questions

1. Make a short presentation of the text.
2. Suppose the proportion of cheats is 1%, the sensitivity is 95%, and the specificity is also 95%.
Let G and F be the events “The athlete is guilty” and “The athlete fails the test”.
 - a. Draw a tree diagram to show this information.
 - b. An athlete is chosen at random. Calculate $P(F)$ (probability of the event F).
 - c. Show that the probability that an athlete who fails the test really is a cheat is about 16%.
3. What do you think of the reliability of this test? How could the Olympic Games be made cleaner?