SEQUENCES

Niveau(x) concerné(s):	TS- TES
Notions abordées :	Arithmetic and geometric sequences
Nombre de séances :	séances :
Séquence préparée par : ***Sources***	*Jean-Philippe Perret* et Marina Digeon - http://illuminations.nctm.org

Descriptif des séances :

Séance 1 : Travail par groupe

- document : "Table problem"
- Donner le vocabulaire spécifique aux suites lorsque les éléves le demande
- Production attendue : une page A3 par groupe avec les différentes recherches pour une présentation de 3-4 minutes par groupe

Séance 2 :

- document : "**COURS sequences 1.ppt**" en vidéo-projection pour le prof + le document élève "**COURS - sequence 1 intro et arithm- eleve .odt**".
- Exercices classiques : document "exos - sequence arithm.odt"

Séance 3:

- document : "**COURS sequences 2 geom.ppt**" en vidéo-projection pour le prof + le document élève "**COURS - sequence 2 geom- eleve .odt**".
- Exercices classiques : document "exos - sequence geom .odt"

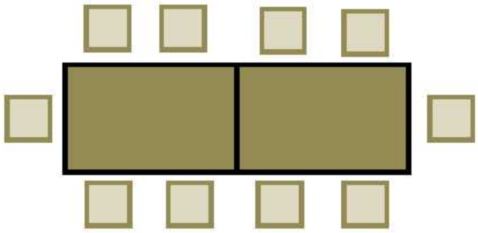
Séance 4 : EXPOSE

- choisir un thème poour les suites et produire à 2 un document élève

Table Problem

We need to work out how many people they can fit on a 6 seater table when put together. Draw the first 3.

Find the n^{th} term of the sequence?



Groupe DNL Maths anglais ***IREM de Lyon***
Pascale Bonnaud, Julien Buil, Marina Digeon, Cécile Durand, Mélanie Estevez, Jennifer Malaval, Julien Mougeot, Jean-Philippe Perret, Julien BUIL,

Arithmetic sequences

In maths, we call a list of numbers in order a …..........................

Each number in a sequence is called a ….................................

$$4, 8, 12, 16, 20, 24, 28, 32, \dots$$

If terms are next to each other they are referred to as …...

When we write out **sequences**, **consecutive terms** are usually separated by commas.

n^{th} term:

The idea behind the n^{th} term is that you can use a formula to generate any term in a sequence from its position, n, in the sequence.
Examples:
a) A sequence has the n^{th} term $4n + 1$. The 10^{th} term is ....
b) A sequence has the n^{th} term $n^2 - 2$. 35 is not in the sequence

Recurrence relations:

Notation : a_k just means the k^{th} term of the sequence (so a_4 is the 4^{th} term, and a_{k+1} is the term after a_k).
Recurrence relations tell you how to work out a term in a sequence from the previous term.
A recurrence relation describes how to work out a_{k+1} from a_k.
Examples:
a) If each term in a sequence is 2 more than the previous term: $a_{k+1} =$ …..........................
b) The sequence 5, 8, 11, 14, 17, … can be defined as: …..
c) A sequence is given by the recurrence relation $a_{k+1} = a_k - 6$ and $a_1 = 20$ with $k \geq 1$
…..

Arithmetic sequences :

When the terms of a sequence progress by …................. …....................... each time, this is called an …......................... sequence.

Definition: A sequence of numbers (a_n) is arithmetic if, for any positive integers n, $\boldsymbol{a_{n+1} - a_n = d}$ where d is a fixed real number, called the ….. of the sequence.
We can also write that: $a_{n+1} = a_n + d$. This equality is …... of the sequence.

Relation between terms:
For any positive integer n, **the formula for the n^{th} term** of an arithmetic sequence is:
For any two positive integers n and p:

Note: The graph of an arithmetic sequence is a set of points lying on a straight line.

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Exercises : arithmetic sequences

Exercise 1 (at home to learn the vocabulary) :

a) The first term of an arithmetic sequence is equal to 6 and the common difference is equal to 3. Find a formula for the n th term and the value of the 50 th term.
b) An arithmetic sequence has a common difference equal to 10 and its 6 th term is equal to 52. Find its 15 th term.
c) An arithmetic sequence has its 5 th term equal to 22 and its 15 th term equal to 62. Find its 100 th term.
d) Find the sum of the first 50 even positive integers.

Exercise 2:

A single square is made from 4 matchsticks. Two squares in a row need 7 matchsticks and 3 squares in a row need 10 matchsticks. This process defines a sequence.
Determine for this sequence:
a) The three first term.

b) The common difference.
c) The formula for the n^{th} term.
d) How many matchsticks are in a row of 25 squares.
e) If there are 109 matchsticks, calculate the number of squares in the row.

Exercise 3:

The third term of an arithmetic sequence is –7 and the 7th term is 9. Determine:
a) the first term a_1 and the common difference d.
b) the 51th term.

Exercise 4 (at home):

In an arithmetic sequence, the first and seventh terms are x^2 and $6 + x - 5x^2$ respectively. If the common difference is x, determine the possible values of x.

Exercise 5 (at home):

The twelfth term of an arithmetic sequence is 5, and the common difference between successive terms is 3. Determine which term has a value of 47.

Exercise 6:

A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic sequence, until the top row has 4 cans. How many cans, in total, are in the display?

Geometric Sequences

- **With geometric sequences, rather than adding, you get from one term to the next by ….................. by a constant called the ….....................................**
- **<u>Definition:</u>** A sequence of numbers (a_n) is geometric if, for any positive integers *n*, $\frac{a_{n+1}}{a_n}=r$ where *r* is a fixed real number, called the …................................. of the sequence.
  We can also write that: $a_{n+1}=r\times a_n$. This equality is ….. of the sequence.

  <u>Examples:</u>
  a) 1, 2, 4, 8, 16, 32, 64, … is a geometric sequence …...

  b) If the **common ratio is negative**, the signs of the sequence will **alternate**: 2, –6, 18, –54, 162, –486 … is a geometric sequence where the common ratio is –3.
  c) The common ratio might **not** be a **whole number**: 16, 12, 9, $\frac{27}{4}$, $\frac{81}{16}$, $\frac{243}{64}$, … is a geometric sequence with common ratio …........

- **<u>Relation between terms:</u>**
  You get each term by multiplying the first term by the common ratio some numbers of times. In other words, each term is the **first term** multiplied by **some power** of the **common ratio**.

  For any integer $n > 0$, **the formula for the n^{th} term** of a geometric sequence is ….......................

  For any two positive integers *n* and *p* ....

Exercises : Geometric Sequences

Exercise 1(at home):
a) A geometric sequence is 24, 12, 6, … What is the 9^{th} term?
b) The sixth and seventh terms of a geometric sequence are -2673 and 8019 respectively. What is the first term?
c) The 14^{th} term of a geometric sequence is 9216. The first term is 1.125. Calculate the common ratio.

Exercise 2:
One year, Rob invested in a tangerine farm, and at the end of the year earned £2000 from his investment. He earned money from the investment every year and his annual earnings from the investment increased by 4% each year. Find, to the nearest pound, the total amount he received in the first 8 years of his investment.

Exercise 3:
$k + 10$, k , $2k – 21$, … is a geometric progression, k is a positive constant.
a) Show that $k^2 – k – 210 = 0$.
b) Find the common ratio of this sequence and find the sum of the first 10 terms.

Exercise 4 (at home):
To raise money for charity, Alex, Chris and Heather were sponsored £1 for each kilometer they ran over a 10-day period. They received sponsorship proportionally for partial kilometers completed.

- Chris ran 2 km on day 1 and on each subsequent day ran 20% further than the day before.
- Heather ran 1 km on day 1, on each subsequent day she ran 50% further than the previous day.
- Alex ran 3 km every day.

By the end of the 10 days, who did raise the maximum money?

Exercise 5 :
A company wants to recruit people to work for a ten month period. They offer two different types of pay scheme.
a) Starting salary of £ 1,000 per month, increasing by £ 100 per month.
b) Starting salary of £ 900 per month, increasing by 10% per month.
Which pay scheme is more interesting?

<u>Learn maths by the others</u>

You will have to prepare a lesson on the topic assigned to you and your partner. It has to be well-structured, easy to understand and well-documented. The aim is to inform and entertain the reader and the listener!

If you think that your teachers' lessons are boring, try to make yours more appealing!!
You can for instance make a glossary of the keywords and expressions of the topic, you can illustrate it with drawings, examples…

Quote your sources : to copy a website is forbidden and will be punished!!

Your lesson must be handed as a .doc file or .odt **<u>and</u>** as a pdf file.
Deadline : ……….

Then, you will expose it with the help of a poster (about 5-10 minutes) : …...

Here are the different subjects :

- Who is Fibonacci ?
- Fibonaci's numbers
- Wheat and chessboard problem :
- Koch Snowflake :
- Sierpinski triangle :
- Tower of Hanoi :
- The Fibonacci rabbit sequence :

Élève : Date d'oral :

Sujet choisi :

Critères et compétences	Appréciations	Note
Document écrit (résumé de l'exposé)		Total : /10
Présentation du document		Titre(s) /1 Sources /1 Informations importantes mises en valeur : /1
Langue		Orthographe : /1 Grammaire : /1
Contenu mathématique		Sujet respecté : /1 pertinence des informations /1 contenu « complet » /1 « valeur ajoutée » /2
Oral		Total : /15
Contenu, cohérence		Sujet respecté : /1 Discours construit Apport d'information Maîtrise du contenu, explications /3
Savoir être		Consultation des notes Tenir compte de l'auditoire volume sonore se montrer convaincant /4
Vocabulaire		Vocabulaire approprié/adapté : /2
Prononciation, intelligibilité		Aisance, prononciation...: /2
Production (diaporama, affiche)		Production : /3
		Note finale : /25