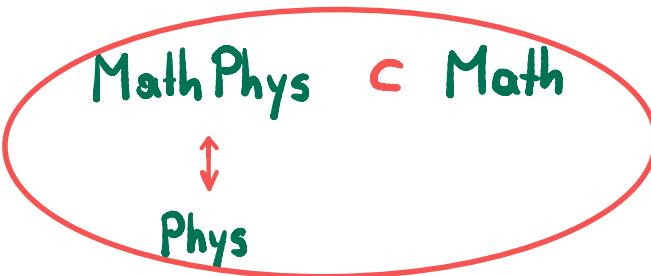


M2R Mathematical Physics



Goals:

- understand fundamental physics as mathematician
- see examples of physics inspiring new mathematics

classical

differential geometry

manifolds

- symplectic
- Riemannian
- fiber bundles & connections

symmetries

Lie group(oid)s
Lie algebras/oids

groups

- rotations
- $\mathbb{Z}/2\mathbb{Z}$
- $\pi_1(M)$

quantum

Linear algebra
functional analysis

Hilbert spaces

- Spectral theory
- C^* algebras
- Quantum Information

Structure winter semester:

2 weeks intensive: **manifolds** & **Hilbert spaces**

3 introductory courses:

1. Symplectic Geometry & Lie groupoids

 ↳ (classical) Mechanics

2. Fiber Bundles & Gauge Theories

 ↳ Gen.Rel., Electromag., Yang-Mills Theories

3. Quantum Mechanics & Quan. Info. Th.

Example: The 2-body problem

1. Classical → Kepler problem

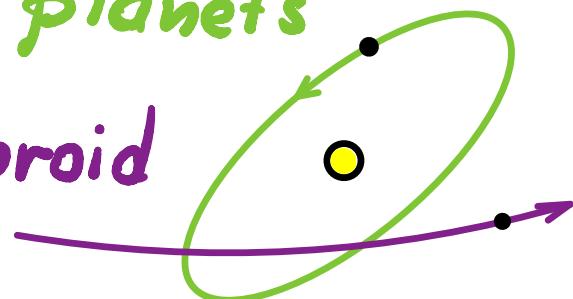
$$H = \frac{1}{2} \vec{P}^2 + \frac{1}{\|\vec{x}\|} \in C^\infty(T^*(\mathbb{R}^3 \setminus \{0\}))$$

evident symmetry group $SO(3)$, but:

$H = E < 0$ group $SO(4)$ → planets

$H = E > 0$ $SO(3, 1)$ → meteoroid

→ example for groupoid symm.



2. Quantum \rightarrow Hydrogen atom

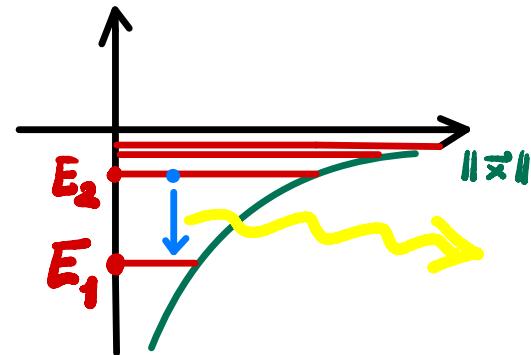
$$\hat{H} \psi_t = -i\hbar \frac{\partial}{\partial t} (\psi_t) \quad \text{Schrödinger eq.}$$

$$\psi_t \in L^2(\mathbb{R}^3), \quad \hat{H} = -\frac{\hbar^2}{2} \Delta + \frac{1}{\|\vec{x}\|}$$

possible energy levels $E \in \text{Sp}(\hat{H})$

for $E < 0$ "quantized" $\sim \frac{1}{n^2}$

(means discrete spectrum)



Structure summer semester:

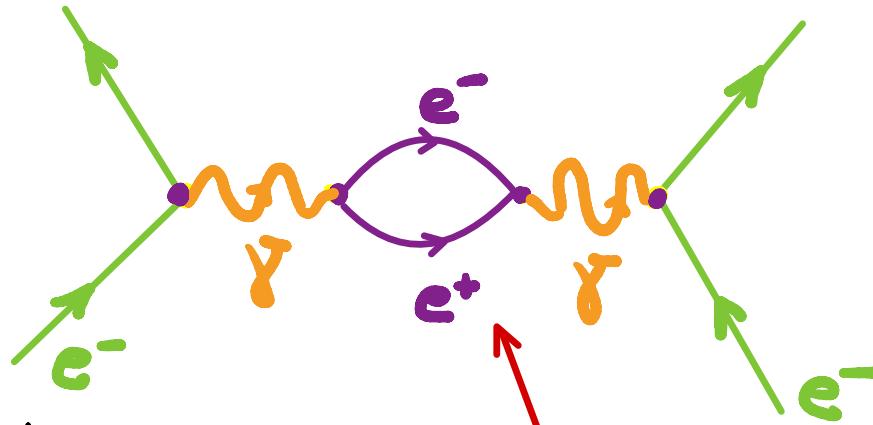
4 advanced courses:

- ① Quantum Field Theory & Renormalization
- ② Topological Phases of Matter
- ③ Seiberg-Witten Invariants
- ④ Poisson Sigma Models

1.

QFT

example :



Feynman-diagram for electron
electron scattering

"naively divergent"

↓
Renormalization

→ Connes - Kreimer Hopf algebra

2.

Topology & Matter

Idea:

$[\psi]$ $\xleftrightarrow{1:1}$ Topological
homotopy classes Phases

respecting a symmetry group G

Toy model:



spin
chain

$\psi \in (\mathbb{C}^2)^{\mathbb{Z}^d}$

Ogata'21

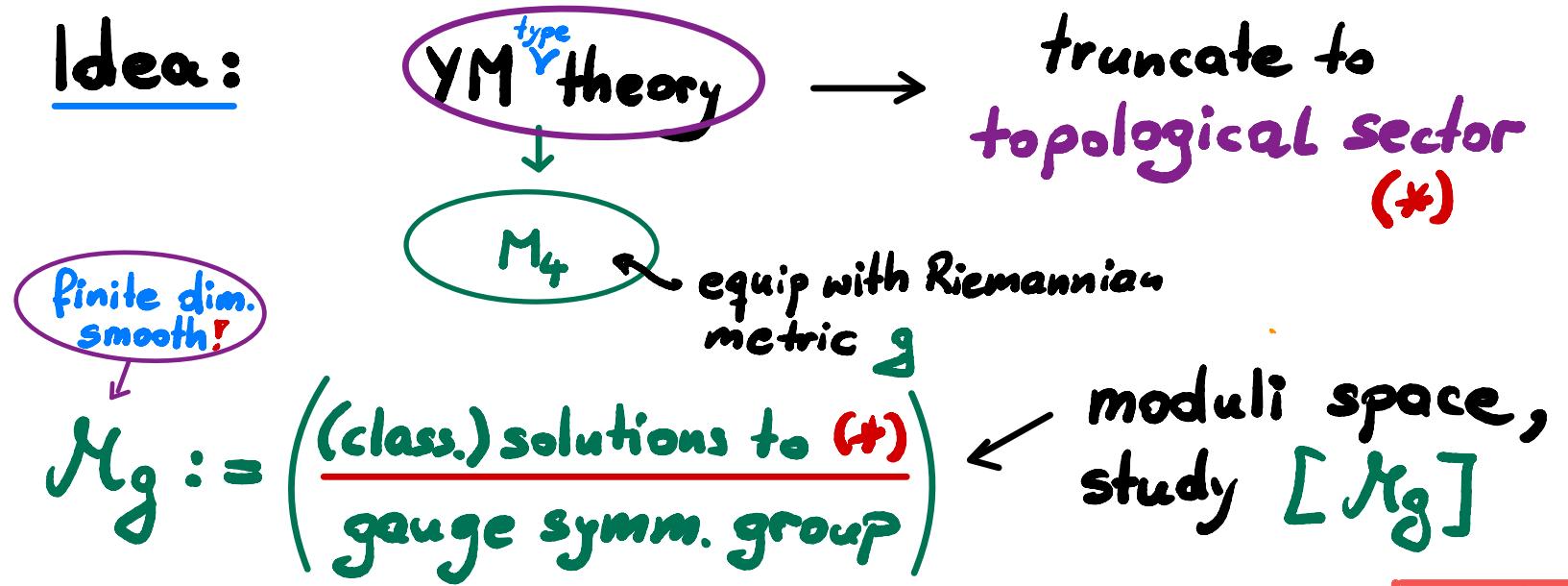
$$\{[\psi]\} \cong H^{d+1}(G, U(1))$$

$d = 1, 2$

group cohomology

3. Seiberg-Witten invariants

Idea:



Apply: Thm. [Donaldson] Many topological 4-manifolds do not admit a smooth structure.

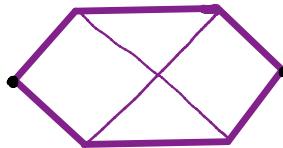
easier
w. SW
invar.

4.

Poisson Sigma Models

- Toy models : 2-dim. gravity

example:



Penrose diagram
of the Schwarzschild
black hole

- Integration of Lie algebroids CF^2
construction via sympl. quotient $\rightarrow [TI \rightarrow LA]$
 \curvearrowleft homotopy classes
- Deformation quantization of Poisson manifolds
Feynman diagrams of PSM \rightarrow Thm. [Kontsevich]