S_n = matrices symétriques : ${}^tA = A$

A_n = —— anti-symétriques : ${}^tA = -A$

• S_n, A_n sev de M_n

•• $\underset{M_n}{0} \in S_n$ •• $A, B \in S_n$ $\quad {}^t(A+B) = {}^tA + {}^tB$

$= A + B$

donc $A+B \in S_n$

•• $A \in S_n$, $\lambda \in \mathbb{K}$ $\quad {}^t(\lambda A) = \lambda\, {}^tA = \lambda A$ donc $\lambda A \in S_n$

Donc S_n sev de M_n

De même, on montre que A_n sev de M_n

• $\dim S_n = \dfrac{n^2+n}{2}$, $\dim A_n = \dfrac{n^2-n}{2}$

$M_{n \times m}$

$$\begin{array}{cc} & m \\ n & \begin{pmatrix} 1 & 2 & -1 & \dots \\ & & & \\ & & & \end{pmatrix} \end{array}$$

$n \times m$ choix

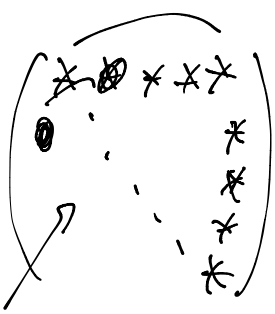
$\leadsto \dim n.m$

•• $\dim S_n$

$n + (n-1) + (n-2) + \dots + 1$

$\shortparallel$

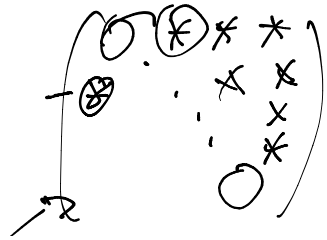
$\dfrac{n^2+n}{2}$



plus de choix possible !

${}^tM = -M$

•• $\dim A_n$



$(n-1) + (n-2) + \dots + 1$

$= n^2 - n$

plus de choix

• $M_n = S_n \oplus A_n$

•• $S_n \cap A_n = \{0\}$ et $\dim S_n + \dim A_n = \dim M_n$

$$M \in S_n \cap A_n : \quad \left.\begin{array}{l} M \in S_n : {}^tM = M \\ M \in A_n : {}^tM = -M \end{array}\right\} M = -M \Rightarrow M = 0$$

Donc $S_n \cap A_n = \{0\}$

et $\dim S_n + \dim A_n = \frac{n^2+n}{2} + \frac{n^2-n}{2} = n^2 = \dim M_n$

•• $\underbrace{S_n \cap A_n = \{0\}}_{\text{déjà fait}}$ et $M_n = S_n + A_n$

$M \in M_n$, on écrit

$$M = \frac{1}{2}\underbrace{(M + {}^tM)}_{\text{symétrique}} + \frac{1}{2}\underbrace{(M - {}^tM)}_{\text{anti-symétrique}}$$

faire un exemple

$$\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 4 \\ 0 & 2 & 3 \end{pmatrix}$$