Φ alternée ssi Φ anti-symétrique

$\Phi(x,x)=0$ $\qquad\qquad$ $\Phi(y,x)=-\Phi(x,y)$

- $\boxed{\Leftarrow}$ Φ anti-symétrique

$$\Phi(x,x) = -\Phi(x,x) \text{ donc } \Phi(x,x)=0$$

- $\boxed{\Rightarrow}$ Φ alternée

$$\underbrace{\Phi(x+y,\, x+y)}_{=0} = \Phi(x, x+y) + \Phi(y, x+y)$$

car Φ alternée

$$= \underbrace{\Phi(x,x)}_{=0} + \boxed{\Phi(x,y) + \Phi(y,x)} + \underbrace{\Phi(y,y)}_{=0}$$

car Φ alternée

donc $\Phi(x,y) + \Phi(y,x) = 0$

$\Phi(x,y) = -\Phi(y,x)$ $\quad$ Φ anti-sym.

$$\Phi(x,y) = \underbrace{x_1 y_1} + x_2 y_1 + \underbrace{x_1 y_2} + x_2 y_2$$

$$= x_1(y_1+y_2) + x_2(y_1+y_2)$$

$$= (x_1, x_2)\begin{pmatrix} y_1+y_2 \\ y_1+y_2 \end{pmatrix}$$

$$= {}^t X \begin{pmatrix} y_1+y_2 \\ y_1+y_2 \end{pmatrix}$$

$\mathbb{R}^2$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$\Phi(x,y) = {}^t\underline{X}AY$

$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$$= {}^t X \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\Phi(x,y) = {}^t X \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} Y$$

écriture matricielle de Φ

↑ matrice de Φ (sur la base canonique)

- Φ est symétrique
- $\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0$ $\quad \Phi$ est dégénérée

$\boxed{\mathbb{R}^3}$

$$\Phi(x,y) = -2x_1y_2 + x_1y_3 + 2x_2y_1 - x_2y_3 - x_3y_1 + x_3y_2$$

$$= x_1(-2y_2+y_3) + x_2(2y_1-y_3) + x_3(-y_1+y_2)$$

$$= (x_1, x_2, x_3)\begin{pmatrix} -2y_2+y_3 \\ 2y_1-y_3 \\ -y_1+y_2 \end{pmatrix}$$

$$= (x_1, x_2, x_3)\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 0 & \boxed{-2} & \boxed{1} \\ \boxed{2} & 0 & \boxed{-1} \\ -1 & 1 & 0 \end{pmatrix} \end{array}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$