# Surgery and regularity results for spectral problems

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**Abstract**

In this talk we will consider the following shape optimization problem:

$$\min\Big\{F(\lambda_1(A),\dots,\lambda_k(A))\ :\ A\subset\mathbb{R}^N,\ |A|=1\Big\},$$

where we denote $\lambda_i$ the $i$th eigenvalue of the Dirichlet Laplacian and $|\cdot|$ the Lebesgue measure.

We will first show how it is possible to prove, with "spectral surgery" techniques, the existence of an optimal domain for the problem above in the class of quasi-open sets (if $F$ is increasing in each variable and lower semicontinuous), thus generalizing a well-known result by Buttazzo and Dal Maso. Moreover we will discuss how surgery methods can be used when a perimeter contraint is added. In particular we want to geometrically modify an open set so that the first $k$ eigenvalues of the Dirichlet Laplacian and its perimeter are not increasing, its measure remains constant, and both perimeter and diameter decrease below a certain threshold.

At the end, we will discuss the regularity of the eigenfunctions on optimal domains and see that for some very special class of functional, including $\lambda_1+\dots+\lambda_k$, optimal sets are actually open.

(The results discussed in this talk come from joint works with Dorin Bucur, Aldo Pratelli and Bozhidar Velichkov)