Lectures on Azumaya algebras at IMAR

The goal of these lectures is to study the theory of Azumaya algebras over a ring (or a scheme) which is the globalization of the theory of central simple algebras over a field. This theory is mostly due to Alexander Grothendieck and is related to descent techniques and non-abelian étale cohomology, which are fundamental tools in Algebraic Geometry and Arithmetic Geometry. We will mostly follow the recent book by Colliot-Thélène and Skorobogatov [1]. Additional references are listed below.

The lectures will be on Mondays in the Foiaș lecture room (8th floor) at 2 PM.

October 7, lecture 1 (Cameron Ruether): *Central simple algebras: Wedderburn's theorem and characterizations, Skolem-Noether theorem.*

October 14, lecture 2 (Ilan Zysman): *Central simple algebras: reduced norms and traces, splitting fields and Galois cohomology, Brauer groups.*

October 21, lecture 3 (Philippe Gille): *Splitting fields, example of local fields*

October 28, lecture 4 (Philippe Gille): *Brauer group of $k(t)$. Application to the rationality problem.*

November 4, lecture 5 (Margot Bruneaux): *Azumaya algebras: definition over rings, faithfully flat splitting, the failure/analogue of Skolem-Noether.*

November 11, lecture 5 (Cameron Ruether): *Azumaya algebras: Morita equivalence and Brauer equivalence.*

November 18, lecture 6 (TBD): *Étale cohomology, classification of Azumaya algebras by $H^1_{\acute{e}t}(R, \mathrm{PGL}_n)$.*

Further topics will be decided as the lectures are proceeding.

References

[1] J.-L. Colliot-Thélène, A. Skorobogatov, *The Brauer–Grothendieck Group*, Springer (2021).

[2] T. J. Ford, *Separable Algebras*, Graduate Studies in Mathematics **183**, Amer. Math. Soc, Provident, RI, (2017).

[3] O. Gabber, *Some theorems on azumaya algebras*, LNM 844 (1981), pp. 129-209.

[4] P. Gille and T. Szamuely, *Central simple algebras and Galois cohomology*, Second edition, Cambridge Studies in Advanced Mathematics **165**, Cambridge University Press, Cambridge, 2017.

[5] J. Giraud, *Cohomologie non-abélienne*, Springer (1970).

[6] A. Grothendieck, *Le groupe de Brauer : I. Algèbres d'Azumaya et interprétations diverses*, Séminaire Bourbaki no. 9 (1966), Talk no. 290, 21 p.

[7] A. Grothendieck, *Le groupe de Brauer : Le groupe de Brauer : II. Théories cohomologiques*, Séminaire Bourbaki, no. 9 (1966), Talk no. 297, 21 p.

[8] M.-A. Knus, *Quadratic and Hermitian Forms over Rings*, Grundlehren der mathematischen Wissenschaften **294** (1991), Springer.

[9] M.-A. Knus, A. Merkurjev, M. Rost and J.-P. Tignol, *The Book of Involutions*, Amer. Math. Soc. Colloquium Publ., **44**, Amer. Math. Soc., Providence, RI (1998).

[10] M.-A. Knus, M. Ojanguren, *Théorie de la Descente et Algèbres d'Azumaya*, Lecture Notes in Mathematics **389** (1974), Springer.

[11] J.S. Milne, *Étale Cohomology*, Princeton University Press (1980).

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[12] Stacks project, https://stacks.math.columbia.edu Cambridge University Press,

[13] A. Vistoli, *Notes on Grothendieck topologies, fibered categories and descent theory*, Mathematical Surveys and Monographs **123** (2005), American Mathematical Society, 1-104.