Lectures on Azumaya algebras at IMAR
Winter 2025

The goal of these lectures is to continue our study of the theory of Azumaya algebras over rings and schemes, which is the globalization of the theory of central simple algebras over a field. This theory is mostly due to Alexander Grothendieck and is related to descent techniques and non-abelian étale cohomology, which are fundamental tools in Algebraic Geometry and Arithmetic Geometry. Various references are listed below.

The lectures will be on Mondays in the Foiaș lecture room (8th floor) at 2 PM.

January 20, (Cameron Ruether): *The étale and flat sites over a scheme, Azumaya algebras over schemes, differences from Azumaya algebras over rings.*

January 27, (Philippe Gille): *The Severi-Brauer Scheme associated to an Azumaya algebra.*

February 3, (TBD): *Quasi-coherent modules on a scheme and its site, triviality of cohomology for quasi-coherent modules over affines..*

February 10, (TBD): *Infinitesimal thickenings of schemes, introduction to deformation problems, the Lie algebra.*

February 17, (TBD): *Classification and obstruction theory for deformations of vector bundles and Azumaya algebras.*

February 24, (TBD): *Functoriality of deformation obstructions, relative deformation problems.*

Further topics will be decided as the lectures proceed.

References

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[4] P. Gille and T. Szamuely, *Central simple algebras and Galois cohomology*, Second edition, Cambridge Studies in Advanced Mathematics **165**, Cambridge University Press, Cambridge, 2017.

[5] J. Giraud, *Cohomologie non-abélienne*, Springer (1970).

[6] A. Grothendieck, *Le groupe de Brauer : I. Algèbres d'Azumaya et interprétations diverses*, Séminaire Bourbaki no. 9 (1966), Talk no. 290, 21 p.

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[8] R. Hartshorne, *Deformation theory*, Berlin: Springer (2010; Zbl 1186.14004).

[9] M.-A. Knus, *Quadratic and Hermitian Forms over Rings*, Grundlehren der mathematischen Wissenschaften **294** (1991), Springer.

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[11] M.-A. Knus, M. Ojanguren, *Théorie de la Descente et Algèbres d'Azumaya*, Lecture Notes in Mathematics **389** (1974), Springer.

[12] J.S. Milne, *Étale Cohomology*, Princeton University Press (1980).

[Se1] J-P. Serre, *Cohomologie galoisienne*, cinquième édition, Springer-Verlag, New York, 1997.

[13] Stacks project, https://stacks.math.columbia.edu Cambridge University Press,

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