```
> restart:
  with(LinearAlgebra):
  d:=2;
```

$$d := 2 \tag{1}$$

We define the Lax matrix using Theorem 4.1

```
> checkLRed12:=-t[2*d+1]*lambda^(2*d):
  for k from 0 to d-1 do checkLRed12:=checkLRed12-t[2*d+1]*Qinfty
  [2*k]*lambda^(2*k): od:
  checkLRed12:=checkLRed12;
```

$$checkLRed12 := -t_5 \lambda^4 - t_5 \, Qinfty_2 \, \lambda^2 - t_5 \, Qinfty_0 \tag{2}$$

```
> checkLRed11:=t00/lambda+t00/Qinfty[0]*lambda^(2*d-1):
  for k from 1 to d do checkLRed11:=checkLRed11-Pinfty[2*d-2*k]*
  lambda^(2*k-1): od:
  for k from 1 to d-1 do for m from 0 to d-1-k do checkLRed11:=
  checkLRed11-Pinfty[2*m]*Qinfty[2*k+2*m]*lambda^(2*k-1): od: od:
  for k from 1 to d-1 do checkLRed11:=checkLRed11+t00/Qinfty[0]*
  Qinfty[2*k]*lambda^(2*k-1): od:
  checkLRed11:=checkLRed11;

  checkLRed22:=-checkLRed11:
```

$$checkLRed11 := \frac{t00}{\lambda} + \frac{t00 \, \lambda^3}{Qinfty_0} - Pinfty_2 \, \lambda - Pinfty_0 \, \lambda^3 - Pinfty_0 \, Qinfty_2 \, \lambda + \frac{t00 \, Qinfty_2 \, \lambda}{Qinfty_0} \tag{3}$$

```
> NumcheckLRed21poleinfinity:=-checkLRed11^2/checkLRed12:
  for j from d to 2*d do for m from 0 to 2*d-j do
  NumcheckLRed21poleinfinity:=NumcheckLRed21poleinfinity+t[2*d+1-2*
  m]*t[2*j+2*m-2*d+1]*lambda^(2*j)/checkLRed12: od: od:
  checkLRed21poleinfinity:=0:
  for i from 1 to 100 do  checkLRed21poleinfinity:=
  checkLRed21poleinfinity-(residue(NumcheckLRed21poleinfinity*
  lambda^(-i),lambda=infinity))*lambda^(i-1): od:

  checkLRed21:=checkLRed21poleinfinity;
```

$$checkLRed21 := \frac{1}{t_5} \left( 2 \left( \frac{t00}{Qinfty_0} - Pinfty_0 \right) \left( -Pinfty_2 - Pinfty_0 \, Qinfty_2 + \frac{t00 \, Qinfty_2}{Qinfty_0} \right) - \frac{\left( -Pinfty_0 \, Qinfty_0 + t00 \right)^2 Qinfty_2}{Qinfty_0^2} \right) - 2 \, t_1 - \frac{t_3^2}{t_5} + 2 \, t_3 \, Qinfty_2 - \frac{t_5^2 \, Qinfty_2^2 - t_5^2 \, Qinfty_0}{t_5} - \left( - \frac{\left( \frac{t00}{Qinfty_0} - Pinfty_0 \right)^2}{t_5} + 2 \, t_3 - t_5 \, Qinfty_2 \right) \lambda^2 - t_5 \, \lambda^4 \tag{4}$$

```
> checkLRedMatrix:=Matrix(2,2,0):
  checkLRedMatrix[1,1]:=checkLRed11:
  checkLRedMatrix[1,2]:=checkLRed12:
  checkLRedMatrix[2,1]:=checkLRed21:
  checkLRedMatrix[2,2]:=checkLRed22:
  checkLRedMatrix:
```

We define the general auxiliary matrix using Theorem 4.1

```
> checkARed12:=0:
  for i from 0 to d-1 do checkARed12:=checkARed12-t[2*d+1]*nu[2*
  i+1]*lambda^(2*d-2*i-1): od:
  for k from 1 to d-1 do for i from 0 to k-1 do checkARed12:=
  checkARed12-t[2*d+1]*nu[2*i+1]*Qinfty[2*k]*lambda^(2*k-2*i-1):
  od: od:
  checkARed12;

  nu[2*d]:=simplify(-residue(lambda^(2*d-1)*checkARed12/checkL12,
  lambda=infinity));
```

$$-t_5\,\nu_1\,\lambda^3 - t_5\,\nu_1\,Qinfty_2\,\lambda - t_5\,\nu_3\,\lambda$$

$$\nu_4 := 0 \tag{5}$$

```
> checkARed11:=0:
  NumcheckARed11:=0:
  for i from 0 to d-1 do NumcheckARed11:=NumcheckARed11+ nu[2*i+1]*
  lambda^(-2*i-1)*checkLRed11: od:
  for i from 1 to 100 do  checkARed11:=checkARed11-(residue
  (NumcheckARed11*lambda^(-i),lambda=infinity))*lambda^(i-1): od:
  checkARed11;

  checkARed22:=-checkARed11:
```

$$\nu_1\left(-Pinfty_2 - Pinfty_0\,Qinfty_2 + \frac{t00\,Qinfty_2}{Qinfty_0}\right) + \nu_3\left(\frac{t00}{Qinfty_0} - Pinfty_0\right) + \nu_1\left(\frac{t00}{Qinfty_0} - Pinfty_0\right)\lambda^2 \tag{6}$$

```
> checkARed21:=(-t00)/t[2*d+1]/(Qinfty[0])^2*LQinfty[0]/lambda:
  QuantitycheckARed21:=checkLRed11/checkLRed12*
  (checkLRed11/checkLRed12*checkARed12-2*checkARed11):
  for i from 1 to 100 do  checkARed21:=checkARed21-(residue
  (QuantitycheckARed21*lambda^(-i),lambda=infinity))*lambda^(i-1):
  od:
  for i from 0 to 100 do  checkARed21:=checkARed21+(residue
  (QuantitycheckARed21*lambda^(i),lambda=0))*lambda^(-i-1): od:
```

```
AdditionalTermRed:=t00*(1-t00)/t[2*d+1]/Qinfty[0]^2/lambda*nu[2*
d-1]:
for k from 1 to d-1 do AdditionalTermRed:=AdditionalTermRed+t00*
(1-t00)/t[2*d+1]/Qinfty[0]^2/lambda*nu[2*k-1]*Qinfty[2*k]: od:
checkARed21:=checkARed21+AdditionalTermRed:
NumcheckARed21:=0:
for k from d to 2*d-1 do for j from k+1 to 2*d do for m from 0 to
2*d-j do NumcheckARed21:=NumcheckARed21- t[2*d+1-2*m]*t[2*j+2*
m-2*d+1]*nu[2*j-2*k-1]*lambda^(2*k+1): od: od: od:
for i from 1 to 100 do  checkARed21:=checkARed21-(residue
(NumcheckARed21/(-checkLRed12)*lambda^(-i),lambda=infinity))*
lambda^(i-1): od:
checkARed21:=checkARed21:
series(checkARed21,lambda=infinity);
```

$$-t_5\,\nu_1\,\lambda^3+\left(\frac{\left(\frac{t00}{Qinfty_0}-Pinfty_0\right)^2\nu_1}{t_5}+\frac{t_5^2\,\nu_1\,Qinfty_2-2\,t_5\,t_3\,\nu_1-t_5^2\,\nu_3}{t_5}\right)\lambda+\frac{1}{\lambda}\left(-\frac{t00\,LQinfty_0}{t_5\,Qinfty_0^2}+\frac{1}{Qinfty_0^4\,t_5}\left(-2\,t00\,Pinfty_0\,Qinfty_0^3\,Qinfty_2\,\nu_1+t00^2\,Qinfty_0^2\,Qinfty_2\,\nu_1-2\,t00\,Pinfty_0\,Qinfty_0^3\,\nu_3-2\,t00\,Pinfty_2\,Qinfty_0^3\,\nu_1+t00^2\,Qinfty_0^2\,\nu_3\right)+\frac{t00\,(1-t00)\,\nu_3}{t_5\,Qinfty_0^2}+\frac{t00\,(1-t00)\,\nu_1\,Qinfty_2}{t_5\,Qinfty_0^2}\right) \tag{7}$$

```
> checkARedMatrix:=Matrix(2,2,0):
checkARedMatrix[1,1]:=checkARed11:
checkARedMatrix[1,2]:=checkARed12:
checkARedMatrix[2,1]:=checkARed21:
checkARedMatrix[2,2]:=checkARed22:
checkARedMatrix:
```

We define the derivatives of the Lax matrices. The derivative relatively to times and Darboux coordinates is specific to d=2 and should be adapted for other values of d.

```
> dcheckARedMatrixdlambda:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do dcheckARedMatrixdlambda
[i,j]:=diff(checkARedMatrix[i,j],lambda): od: od:
> LcheckLRedMatrix:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do
LcheckLRedMatrix[i,j]:=diff(checkLRedMatrix[i,j],t[3])*alpha[3]+
diff(checkLRedMatrix[i,j],t[1])*alpha[1]+diff(checkLRedMatrix[i,
j],Qinfty[0])*LQinfty[0]+diff(checkLRedMatrix[i,j],Qinfty[2])*
LQinfty[2]+diff(checkLRedMatrix[i,j],Pinfty[0])*LPinfty[0]+ diff
```

```
  (checkLRedMatrix[i,j],Pinfty[2])*LPinfty[2] od:  od:
```

We define the compatibility equations and solve them. This is the main test for the formulas of Theorem 4.1. This part is again specific to d=2 and should be adapted for other values of d

```
> CompatibilityEquations:=simplify( dcheckARedMatrixdlambda-
  LcheckLRedMatrix+Multiply(checkARedMatrix,checkLRedMatrix)-
  Multiply(checkLRedMatrix,checkARedMatrix)  ):
> simplify(series(CompatibilityEquations[1,1],lambda=0)):
  series(simplify(series(CompatibilityEquations[1,1],lambda=
  infinity)),lambda=infinity):
  series(simplify(series(CompatibilityEquations[1,2],lambda=
  infinity)),lambda=infinity):
  series(simplify(series(CompatibilityEquations[2,1],lambda=
  infinity)),lambda=infinity):
  simplify(residue(CompatibilityEquations[1,1]*lambda,lambda=0));
  EQ1:=simplify(residue(CompatibilityEquations[1,1],lambda=0));

  simplify(-residue(CompatibilityEquations[1,1]/lambda^5,lambda=
  infinity));
  EQ2:=-residue(CompatibilityEquations[1,1]/lambda^4,lambda=
  infinity);
  EQ3:=-residue(CompatibilityEquations[1,1]/lambda^3,lambda=
  infinity);
  EQ4:=-residue(CompatibilityEquations[1,1]/lambda^2,lambda=
  infinity);
  EQ5:=-residue(CompatibilityEquations[1,1]/lambda^1,lambda=
  infinity);

  simplify(-residue(CompatibilityEquations[1,2]/lambda^5,lambda=
  infinity));
  EQ6:=-residue(CompatibilityEquations[1,2]/lambda^4,lambda=
  infinity);
  EQ7:=-residue(CompatibilityEquations[1,2]/lambda^3,lambda=
  infinity);
  EQ8:=-residue(CompatibilityEquations[1,2]/lambda^2,lambda=
  infinity);
  EQ9:=-residue(CompatibilityEquations[1,2]/lambda^1,lambda=
  infinity);
```

$$0$$

$$EQ1 := -\frac{1}{Qinfty_0}\Big(t00\,\big(2\,Pinfty_0\,Qinfty_0\,Qinfty_2\,\nu_1 + 2\,Pinfty_0\,Qinfty_0\,\nu_3 + 2\,Pinfty_2\,Qinfty_0\,\nu_1 - Qinfty_2\,\nu_1 + LQinfty_0 - \nu_3\big)\Big)$$

$$0$$

$$EQ2 := \frac{1}{Qinfty_0^2} \Big( -2\, t_5^2\, \nu_1\, Qinfty_0^3 + \Big( \big( t_5^2\, Qinfty_2^2 + \big( -Pinfty_0^2 - 2\, t_5\, t_3 \big)\, Qinfty_2 - 2\, Pinfty_2\, Pinfty_0 + 2\, t_5\, t_1 + t_3^2 \big)\, \nu_1 - 2\, Qinfty_2\, t_5^2\, \nu_3 + \big( -Pinfty_0^2 + 2\, t_5\, t_3 \big)\, \nu_3 + LPinfty_0 \Big)\, Qinfty_0^2 - t00\, \big( Qinfty_2\, \nu_1 + \nu_3 \big)\, (-1 + t00) \Big)$$

$$EQ3 := 0$$

$$EQ4 := \frac{1}{Qinfty_0^2} \Big( \Big( \big( \big( Pinfty_0^2 - 2\, t_5\, t_3 \big)\, \nu_1 - 2\, t_5^2\, \nu_3 \big)\, Qinfty_0^3 + \Big( \Big( \big( Qinfty_2^3\, t_5^2 + \big( -Pinfty_0^2 - 2\, t_5\, t_3 \big)\, Qinfty_2^2 + \big( -2\, Pinfty_2\, Pinfty_0 + 2\, t_5\, t_1 + t_3^2 \big)\, Qinfty_2 - 2\, Pinfty_0\, (t00 + 1) \big)\, \nu_1 + Qinfty_2^2\, t_5^2\, \nu_3 + \big( \big( -Pinfty_0^2 - 2\, t_5\, t_3 \big)\, \nu_3 + LPinfty_0 \big)\, Qinfty_2 + \big( -2\, Pinfty_2\, Pinfty_0 + 2\, t_5\, t_1 + t_3^2 \big)\, \nu_3 + LQinfty_2\, Pinfty_0 + LPinfty_2 \Big)\, Qinfty_0^2 + t00\, \big( (t00 + 2)\, \nu_1 + 2\, Pinfty_2\, \nu_3 - LQinfty_2 \big)\, Qinfty_0 - t00\, Qinfty_2\, \big( Qinfty_2\, \nu_1 + \nu_3 \big)\, (-1 + t00) \Big)$$

$$EQ5 := 0$$

$$0$$

$$EQ6 := 0$$

$$EQ7 := t_5\, \Big( \big( 2\, Pinfty_0\, Qinfty_0 - 3 \big)\, \nu_1 - 2\, Pinfty_2\, \nu_3 + LQinfty_2 \Big)$$

$$EQ8 := 0$$

$$EQ9 := t_5\, \Big( \big( \big( 2\, Pinfty_0\, Qinfty_2 + 2\, Pinfty_2 \big)\, Qinfty_0 - Qinfty_2 \big)\, \nu_1 + 2\, Pinfty_0\, Qinfty_0\, \nu_3 - \nu_3 + LQinfty_0 \Big) \qquad \textbf{(8)}$$

```
> LQinfty[2]:=factor(solve(EQ7,LQinfty[2]));
  LQinfty[0]:=factor(solve(EQ9,LQinfty[0]));
  LPinfty[0]:=factor(solve(EQ2,LPinfty[0]));
  LPinfty[2]:=factor(solve(EQ4,LPinfty[2]));
  EQ1:=simplify(EQ1);
  EQ2:=simplify(EQ2);
  EQ3:=simplify(EQ3);
  EQ4:=simplify(EQ4);
  EQ5:=simplify(EQ5);
  EQ6:=simplify(EQ6);
  EQ7:=simplify(EQ7);
  EQ8:=simplify(EQ8);
  EQ9:=simplify(EQ9);
```

$$LQinfty_2 := -2\, Pinfty_0\, Qinfty_0\, \nu_1 + 2\, Pinfty_2\, \nu_3 + 3\, \nu_1$$

$$LQinfty_0 := -2\, Pinfty_0\, Qinfty_0\, Qinfty_2\, \nu_1 - 2\, Pinfty_0\, Qinfty_0\, \nu_3 - 2\, Pinfty_2\, Qinfty_0\, \nu_1 + Qinfty_2\, \nu_1 + \nu_3$$

$$LPinfty_0 := \frac{1}{Qinfty_0^2} \Big( -Qinfty_0^2\, Qinfty_2^2\, \nu_1\, t_5^2 + Pinfty_0^2\, Qinfty_0^2\, Qinfty_2\, \nu_1 + 2\, t_5^2\, \nu_1\, Qinfty_0^3 + 2\, Qinfty_0^2\, Qinfty_2\, \nu_1\, t_3\, t_5 + 2\, Qinfty_0^2\, Qinfty_2\, \nu_3\, t_5^2 + Pinfty_0^2\, Qinfty_0^2\, \nu_3 + 2\, Pinfty_0\, Pinfty_2\, Qinfty_0^2\, \nu_1 - 2\, Qinfty_0^2\, \nu_1\, t_1\, t_5 - Qinfty_0^2\, \nu_1\, t_3^2 - 2\, Qinfty_0^2\, \nu_3\, t_3\, t_5 + t00^2\, Qinfty_2\, \nu_1 + t00^2\, \nu_3$$

$$- t00\, Qinfty_2\, \nu_1 - t00\, \nu_3\Big)$$

$$LPinfty_2 := -\frac{1}{Qinfty_0}\Big(2\, Qinfty_0^2\, Qinfty_2\, \nu_1\, t_5^2 + 3\, Qinfty_0\, Qinfty_2^2\, \nu_3\, t_5^2 - Pinfty_0^2\, Qinfty_0^2\, \nu_1 - 2\, Qinfty_0^2\, \nu_1\, t_3\, t_5 - 2\, Qinfty_0^2\, \nu_3\, t_5^2 - 4\, Qinfty_0\, Qinfty_2\, \nu_3\, t_3\, t_5 + 2\, Qinfty_0\, \nu_3\, t_1\, t_5 + Qinfty_0\, \nu_3\, t_3^2 + t00^2\, \nu_1 + Pinfty_0\, Qinfty_0\, \nu_1 - t00\, \nu_1\Big)$$

$$EQ1 := 0$$
$$EQ2 := 0$$
$$EQ3 := 0$$
$$EQ4 := 0$$
$$EQ5 := 0$$
$$EQ6 := 0$$
$$EQ7 := 0$$
$$EQ8 := 0$$
$$EQ9 := 0 \qquad \textbf{(9)}$$

```
> series(simplify(series(CompatibilityEquations[2,1],lambda=
  infinity)),lambda=infinity);
```

$$\frac{\left(-6\, \nu_1\, t_5^2 + 2\, t_5\, \alpha_3\right)\lambda^2}{t_5} + \frac{\left(6\, Qinfty_2\, \nu_1 - 2\, \nu_3\right)t_5^2 + \left(-2\, Qinfty_2\, \alpha_3 - 8\, \nu_1\, t_3 + 2\, \alpha_1\right)t_5 + 2\, \alpha_3\, t_3}{t_5} \qquad \textbf{(10)}$$

In order to solve the entry (2,1) of the compatibility equations, we need to express the nu's in terms of the vector \alpha.

```
> MRed:=Matrix(d,d,0):
  for i from 1 to d do for j from 1 to i do
  MRed[i,j]:=t[2*d+1-2*i+2*j]:
  od: od:
  MRed;
  alphaVector:=Matrix(d,1,0):
  for i from 1 to d do alphaVector[i,1]:=alpha[2*d-2*i+1]/(2*d-2*
  i+1): od:
  alphaVector;
  nuVector:=Matrix(d,1,0):
  for i from 1 to d do nuVector[i,1]:=nu[2*i-1]: od:
  nuVector;
```

$$\begin{bmatrix} t_5 & 0 \\ t_3 & t_5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\alpha_3}{3} \\ \alpha_1 \end{bmatrix}$$

$$\begin{bmatrix} \nu_1 \\ \nu_3 \end{bmatrix} \tag{11}$$

```
> nuVectorbis:=Multiply(MRed^(-1),alphaVector);
  for k from 0 to d-1 do nu[2*k+1]:=nuVectorbis[k+1,1]: od:
  nu[1]:=nu[1];
  nu[3]:=nu[3];
```

$$nuVectorbis := \begin{bmatrix} \frac{\alpha_3}{3\, t_5} \\ -\frac{t_3\, \alpha_3}{3\, t_5^2} + \frac{\alpha_1}{t_5} \end{bmatrix}$$

$$\nu_1 := \frac{\alpha_3}{3\, t_5}$$

$$\nu_3 := -\frac{t_3\, \alpha_3}{3\, t_5^2} + \frac{\alpha_1}{t_5} \tag{12}$$

```
> simplify(CompatibilityEquations);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

This verifies that the relation between nu's and alpha's is correct. We check now that the formulas for the Hamiltonians are correct.

```
> for j from 0 to d-1 do
  Hinfty[2*j]:=simplify(-residue(lambda^(-2*j-1)*(checkLRed11^2+
  checkLRed12*checkLRed21+checkLRed12*diff(checkLRed11/checkLRed12,
  lambda)),lambda=infinity)): od:
  Hinfty[0]:=Hinfty[0];
  Hinfty[2]:=Hinfty[2];
```

$$Hinfty_0 := \frac{1}{Qinfty_0} \Big( -Qinfty_0^3\, t_5^2 + \big( t_5^2\, Qinfty_2^2 + \big( -Pinfty_0^2 - 2\, t_5\, t_3 \big)\, Qinfty_2 - 2\, Pinfty_2\, Pinfty_0 + 2\, t_5\, t_1 + t_3^2 \big)\, Qinfty_0^2 + \big( Pinfty_0\, Qinfty_2 + 3\, Pinfty_2 \big)\, Qinfty_0 + t00\, Qinfty_2\, (-1 + t00) \Big)$$

$$Hinfty_2 := \frac{1}{Qinfty_0} \Big( \big( -2\, Qinfty_2\, t_5^2 - Pinfty_0^2 + 2\, t_5\, t_3 \big)\, Qinfty_0^2 + \big( Qinfty_2^3\, t_5^2 - 2\, t_3\, Qinfty_2^2\, t_5 + \big( 2\, t_5\, t_1 + t_3^2 \big)\, Qinfty_2 + Pinfty_2^2 + Pinfty_0 \big)\, Qinfty_0 + t00^2 - t00 \Big) \tag{14}$$

```
> HamGeneralRed:=0:
  for k from 0 to d-1 do HamGeneralRed:=HamGeneralRed+nu[2*k+1]*
  Hinfty[2*k]: od:
  HamGeneralRed:=simplify(HamGeneralRed);
```

(15)

$$HamGeneralRed := \frac{1}{3\, t_5^2\, Qinfty_0} \Big( -Qinfty_0^3\, t_5^3\, \alpha_3 + \big( Qinfty_2 \left( Qinfty_2\, \alpha_3 - 6\, \alpha_1 \right) t_5^3 + \big( 6\, \alpha_1\, t_3 + 2\, \alpha_3\, t_1 \big) t_5^2 + \big( \left( -Qinfty_2\, Pinfty_0^2 - 2\, Pinfty_2\, Pinfty_0 - t_3^2 \right) \alpha_3 - 3\, Pinfty_0^2\, \alpha_1 \big) t_5 + t_3\, Pinfty_0^2\, \alpha_3 \big) Qinfty_0^2 + \big( 3\, Qinfty_2^3\, \alpha_1\, t_5^3 - \left( Qinfty_2^2\, t_3\, \alpha_3 + 6\, \alpha_1\, \left( t_3\, Qinfty_2 - t_1 \right) \right) Qinfty_2\, t_5^2 + \big( \left( 2\, Qinfty_2^2\, t_3^2 + \left( -2\, t_1\, t_3 + Pinfty_0 \right) Qinfty_2 + 3\, Pinfty_2 \right) \alpha_3 + 3\, \alpha_1\, \left( Qinfty_2\, t_3^2 + Pinfty_2^2 + Pinfty_0 \right) \big) t_5 - t_3\, \alpha_3\, \left( Qinfty_2\, t_3^2 + Pinfty_2^2 + Pinfty_0 \right) \big) Qinfty_0 + (-1 + t00)\, t00\, \left( \left( Qinfty_2\, \alpha_3 + 3\, \alpha_1 \right) t_5 - \alpha_3\, t_3 \right) \Big) \tag{15}$$

```
> simplify(LQinfty[0]-diff(HamGeneralRed,Pinfty[0]));
  simplify(LQinfty[2]-diff(HamGeneralRed,Pinfty[2]));
  simplify(LPinfty[0]+diff(HamGeneralRed,Qinfty[0]));
  simplify(LPinfty[2]+diff(HamGeneralRed,Qinfty[2]));
```

$$0$$
$$0$$
$$0$$
$$0 \tag{16}$$

```
> HamGeneralRedFunction:=unapply(HamGeneralRed,alpha[1],alpha[3]):
  HamRedt1:=simplify(HamGeneralRedFunction(1,0)):
  HamRedt3:=simplify(HamGeneralRedFunction(0,1)):
  VectorHamRed:=Matrix(d,1,0):
  VectorHamRed[1,1]:=1*HamRedt1:
  VectorHamRed[2,1]:=3*HamRedt3:
  VectorHamRed:
  HVectorRed:=Matrix(d,1,0):
  for k from 1 to d do
  HVectorRed[k,1]:=Hinfty[2*d-2*k]: od:

  simplify(VectorHamRed-Multiply(MRed^(-1),HVectorRed));
```

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{17}$$

```
> series(simplify(checkLRed11),lambda);
  series(simplify(checkLRed12),lambda);
  simplify(series(simplify(checkLRed21),lambda));
```

$$t00\, \lambda^{-1} + \frac{\left( -Pinfty_0\, Qinfty_2 - Pinfty_2 \right) Qinfty_0 + t00\, Qinfty_2}{Qinfty_0}\, \lambda + \frac{-Pinfty_0\, Qinfty_0 + t00}{Qinfty_0}\, \lambda^3$$

$$-Qinfty_0\, t_5 - t_5\, Qinfty_2\, \lambda^2 - t_5\, \lambda^4$$

$$\frac{1}{t_5\, Qinfty_0^2} \Big( Qinfty_0^3\, t_5^2 + \big( -t_5^2\, Qinfty_2^2 + \left( Pinfty_0^2 + 2\, t_5\, t_3 \right) Qinfty_2 + 2\, Pinfty_2\, Pinfty_0 - 2\, t_5\, t_1 - t_3^2 \big) Qinfty_0^2 - 2\, t00\, \left( Pinfty_0\, Qinfty_2 + Pinfty_2 \right) Qinfty_0 + t00^2\, Qinfty_2 \Big) \tag{18}$$

$$+\frac{\left(Qinfty_2\, t_5^2+Pinfty_0^2-2\, t_5\, t_3\right) Qinfty_0^2-2\, Pinfty_0\, t00\, Qinfty_0+t00^2}{t_5\, Qinfty_0^2}\, \lambda^2-t_5\, \lambda^4$$

```
> solve({nu1 = alpha[3]/(3*t[5]),nu3 = -t[3]*alpha[3]/(3*t[5]^2)+
  alpha[1]/t[5]},{alpha[1],alpha[3]});
  alpha[1] := nu1*t[3]+nu3*t[5];
  alpha[3] := 3*nu1*t[5];
  simplify(nu[1]-nu1);
  simplify(nu[3]-nu3);
```

$$\left\{\alpha_1=\nu 1\, t_3+\nu 3\, t_5,\ \alpha_3=3\ \nu 1\, t_5\right\}$$

$$\alpha_1 := \nu 1\, t_3+\nu 3\, t_5$$

$$\alpha_3 := 3\ \nu 1\, t_5$$

$$0$$

$$0 \tag{19}$$

```
> simplify(series(simplify(checkARed11),lambda=0));
  simplify(series(simplify(checkARed12),lambda=0));
  simplify(series(simplify(checkARed21),lambda=0));
```

$$\frac{\left(\left(-Pinfty_0\, Qinfty_2-Pinfty_2\right) \nu 1-\nu 3\, Pinfty_0\right) Qinfty_0+\left(Qinfty_2\, \nu 1+\nu 3\right) t00}{Qinfty_0}+\frac{\nu 1\ \left(-Pinfty_0\, Qinfty_0+t00\right)}{Qinfty_0}\, \lambda^2$$

$$-t_5\left(Qinfty_2\, \nu 1+\nu 3\right)\lambda-\nu 1\, t_5\, \lambda^3$$

$$\frac{\left(\left(Qinfty_2\, t_5^2+Pinfty_0^2-2\, t_5\, t_3\right) \nu 1-\nu 3\, t_5^2\right) Qinfty_0^2-2\ \nu 1\ t00\, Pinfty_0\, Qinfty_0+\nu 1\ t00^2}{t_5\, Qinfty_0^2}\, \lambda-\nu 1\, t_5\, \lambda^3 \tag{20}$$

```
> HamRedt3bis:=t00*(Qinfty[2]*t[5]-t[3])*(-1+t00)/(3*t[5]^2)/Qinfty
  [0]-(1/3)*t[5]*Qinfty[0]^2-(1/3)*Qinfty[0]*(Qinfty[2]*t[5]-t[3])*
  Pinfty[0]^2/t[5]^2 -(1/3)*Pinfty[2]^2*t[3]/t[5]^2-(1/3)*Qinfty[2]
  ^3*t[3]+(1/3)*(Qinfty[0]*t[5]^2+2*t[3]^2)*Qinfty[2]^2/t[5]+(1/3)*
  (-2*Pinfty[0]*Pinfty[2]+2*t[1]*t[5]-t[3]^2)*Qinfty[0]/t[5]+
  (Pinfty[2])/(t[5])+(1/3)*(Qinfty[2]*t[5]-t[3])*Pinfty[0]/t[5]^2-
  Qinfty[2]*t[3]*(2*t[1]*t[5]+t[3]^2)/(3*t[5]^2);
  series(simplify(HamRedt3-HamRedt3bis),Pinfty[0]);
```

$$HamRedt3bis := \frac{t00\ \left(t_5\, Qinfty_2-t_3\right)\ (-1+t00)}{3\, t_5^2\, Qinfty_0}-\frac{t_5\, Qinfty_0^2}{3}$$

$$-\frac{Qinfty_0\ \left(t_5\, Qinfty_2-t_3\right) Pinfty_0^2}{3\, t_5^2}-\frac{Pinfty_2^2\, t_3}{3\, t_5^2}-\frac{Qinfty_2^3\, t_3}{3}$$

$$+\frac{\left(Qinfty_0\,t_5^2+2\,t_3^2\right)Qinfty_2^2}{3\,t_5}+\frac{\left(-2\,Pinfty_2\,Pinfty_0+2\,t_5\,t_1-t_3^2\right)Qinfty_0}{3\,t_5}+\frac{Pinfty_2}{t_5}$$

$$+\frac{\left(t_5\,Qinfty_2-t_3\right)Pinfty_0}{3\,t_5^2}-\frac{Qinfty_2\,t_3\left(2\,t_5\,t_1+t_3^2\right)}{3\,t_5^2}$$

$$0 \tag{21}$$

> ?