

```

> restart:
with(LinearAlgebra):
d:=2;

```

$d := 2$

(1)

We define the Lax matrix using Theorem 2.2

```

> checkL12:=-t[2*d+1]*lambda^(2*d):
for k from 0 to 2*d-1 do checkL12:=checkL12-t[2*d+1]*Qinfy[k]*
lambda^k: od:
checkL12:=checkL12;

```

$$checkL12 := -t_5 \lambda^4 - t_5 Qinfy_3 \lambda^3 - t_5 Qinfy_2 \lambda^2 - t_5 Qinfy_1 \lambda - t_5 Qinfy_0$$

(2)

```

> checkL11:=t00/lambda+t00/Qinfy[0]*lambda^(2*d-1):
for k from 0 to 2*d-1 do checkL11:=checkL11-Pinfy[2*d-1-k]*
lambda^k: od:
for k from 0 to 2*d-2 do for m from 0 to 2*d-2-k do checkL11:=
checkL11-Pinfy[m]*Qinfy[k+1+m]*lambda^k: od: od:
for k from 1 to 2*d-1 do checkL11:=checkL11+t00/Qinfy[0]*Qinfy
[k]*lambda^(k-1): od:
checkL11:=checkL11;

```

checkL22:=-checkL11:

$$checkL11 := \frac{t00}{\lambda} + \frac{t00 \lambda^3}{Qinfy_0} - Pinfy_3 - Pinfy_2 \lambda - Pinfy_1 \lambda^2 - Pinfy_0 \lambda^3 - Pinfy_0 Qinfy_1$$

$$- Pinfy_1 Qinfy_2 - Pinfy_2 Qinfy_3 - Pinfy_0 Qinfy_2 \lambda - Pinfy_1 Qinfy_3 \lambda - Pinfy_0 Qinfy_3 \lambda^2$$

$$+ \frac{t00 Qinfy_1}{Qinfy_0} + \frac{t00 Qinfy_2 \lambda}{Qinfy_0} + \frac{t00 Qinfy_3 \lambda^2}{Qinfy_0}$$

(3)

```

> NumcheckL21poleinfinity:=-checkL11^2/checkL12:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do
NumcheckL21poleinfinity:=NumcheckL21poleinfinity+t[2*d+1-m]*t[j+
m-2*d+1]*lambda^j/checkL12: od: od:
checkL21poleinfinity:=0:
for i from 1 to 100 do
checkL21poleinfinity:=checkL21poleinfinity-(residue
(NumcheckL21poleinfinity*lambda^(-i),lambda=infinity))*lambda^
(i-1): od:

checkL21polezero:=simplify(-(residue(NumcheckL21poleinfinity,
lambda=infinity))/lambda):
checkL21:=simplify(checkL21polezero+checkL21poleinfinity);

```

(4)

$$\begin{aligned}
checkL21 := & \frac{1}{Qinfy_0^2 t_5 \lambda} \left((\lambda t_5^2 - 2 t_5^2 Qinfy_3 + 2 t_5 t_4) Qinfy_0^3 + (-t_5^2 \lambda^5 + t_5 (Qinfy_3 t_5 \right. \\
& - 2 t_4) \lambda^4 + (t_5^2 (-Qinfy_3^2 + Qinfy_2) + (2 t_4 Qinfy_3 - 2 t_3) t_5 - t_4^2 + Pinfty_0^2) \lambda^3 + (t_5^2 (\\
& Qinfy_3^3 - 2 Qinfy_3 Qinfy_2 + Qinfy_1) + (-2 Qinfy_3^2 t_4 + 2 Qinfy_2 t_4 + 2 t_3 Qinfy_3 - 2 t_2) t_5 \\
& + (Pinfty_0^2 + t_4^2) Qinfy_3 + 2 Pinfty_1 Pinfty_0 - 2 t_4 t_3) \lambda^2 + ((-Qinfy_3^4 + 3 Qinfy_2 Qinfy_3^2 \\
& - 2 Qinfy_3 Qinfy_1 - Qinfy_2^2) t_5^2 + (2 Qinfy_3^3 t_4 - 2 t_3 Qinfy_3^2 + (-4 Qinfy_2 t_4 \\
& + 2 t_2) Qinfy_3 + 2 Qinfy_1 t_4 + 2 Qinfy_2 t_3 - 2 t_1) t_5 - t_4^2 Qinfy_3^2 + (2 Pinfty_1 Pinfty_0 \\
& + 2 t_4 t_3) Qinfy_3 + t_4^2 Qinfy_2 + Qinfy_2 Pinfty_0^2 + Pinfty_1^2 + 2 Pinfty_2 Pinfty_0 - 2 t_4 t_2 - t_3^2) \lambda \\
& + (Qinfy_3^5 - 4 Qinfy_2 Qinfy_3^3 + 3 Qinfy_1 Qinfy_3^2 + 3 Qinfy_2^2 Qinfy_3 - 2 Qinfy_1 Qinfy_2) t_5^2 \\
& + (-2 Qinfy_3^4 t_4 + 2 Qinfy_3^3 t_3 + (6 Qinfy_2 t_4 - 2 t_2) Qinfy_3^2 + (-4 Qinfy_1 t_4 - 4 Qinfy_2 t_3 \\
& + 2 t_1) Qinfy_3 - 2 Qinfy_2^2 t_4 + 2 Qinfy_1 t_3 + 2 Qinfy_2 t_2 - 2 t_0) t_5 + Qinfy_3^3 t_4^2 - 2 t_4 t_3 \\
& Qinfy_3^2 + (-2 t_4^2 Qinfy_2 + 2 Pinfty_2 Pinfty_0 + Pinfty_1^2 + 2 t_4 t_2 + t_3^2) Qinfy_3 + t_4^2 Qinfy_1 \\
& + (2 Qinfy_2 t_3 - 2 t_1) t_4 + 2 Pinfty_1 Qinfy_2 Pinfty_0 + Qinfy_1 Pinfty_0^2 + 2 Pinfty_1 Pinfty_2 \\
& + 2 Pinfty_3 Pinfty_0 - 2 t_3 t_2) Qinfy_0^2 - 2 t_0 (Pinfty_0 \lambda^3 + (Pinfty_0 Qinfy_3 + Pinfty_1) \lambda^2 \\
& + (Pinfty_0 Qinfy_2 + Pinfty_1 Qinfy_3 + Pinfty_2) \lambda + Pinfty_1 Qinfy_2 + Pinfty_2 Qinfy_3 \\
& + Pinfty_0 Qinfy_1 + Pinfty_3) Qinfy_0 + t_0^2 (\lambda^3 + \lambda^2 Qinfy_3 + \lambda Qinfy_2 + Qinfy_1) \Big)
\end{aligned} \tag{4}$$

```

> checkLMatrix:=Matrix(2,2,0):
checkLMatrix[1,1]:=checkL11:
checkLMatrix[1,2]:=checkL12:
checkLMatrix[2,1]:=checkL21:
checkLMatrix[2,2]:=checkL22:
checkLMatrix:

```

We define the general auxiliary matrix using Theorem 2.2

```

> checkA12:=0:
for i from 0 to 2*d-1 do checkA12:=checkA12-t[2*d+1]*nu[i]*
lambda^(2*d-i): od:
for k from 1 to 2*d-1 do for i from 0 to k-1 do checkA12:=
checkA12-t[2*d+1]*nu[i]*Qinfy[k]*lambda^(k-i): od: od:
checkA12;

nu[2*d]:=simplify(-residue(lambda^(2*d-1)*checkA12/checkL12,
lambda=infinity));

```

$$\begin{aligned}
& -t_5 v_0 \lambda^4 - t_5 v_0 Qinfy_3 \lambda^3 - t_5 v_1 \lambda^3 - t_5 v_0 Qinfy_2 \lambda^2 - t_5 v_1 Qinfy_3 \lambda^2 - t_5 v_2 \lambda^2 \\
& - t_5 v_0 Qinfy_1 \lambda - t_5 v_1 Qinfy_2 \lambda - t_5 v_2 Qinfy_3 \lambda - t_5 v_3 \lambda \\
& v_4 := -Qinfy_0 v_0 - Qinfy_1 v_1 - Qinfy_2 v_2 - Qinfy_3 v_3
\end{aligned} \tag{5}$$

```

> checkA11:=0:
NumcheckA11:=0:
for i from 0 to 2*d-1 do NumcheckA11:=NumcheckA11+ nu[i]*lambda^
(-i)*checkL11: od:
for i from 1 to 100 do checkA11:=checkA11-(residue (NumcheckA11*
lambda^(-i),lambda=infinity))*lambda^(i-1): od:
checkA11;

```

```
checkA22:=-checkA11:
```

$$\begin{aligned}
& v_0 \left(-Pinfty_3 - Pinfty_0 Qinfy_1 - Pinfty_1 Qinfy_2 - Pinfty_2 Qinfy_3 + \frac{t00 Qinfy_1}{Qinfy_0} \right) + v_1 \left(\right. \\
& \quad \left. -Pinfty_2 - Pinfty_0 Qinfy_2 - Pinfty_1 Qinfy_3 + \frac{t00 Qinfy_2}{Qinfy_0} \right) + v_2 \left(-Pinfty_1 \right. \\
& \quad \left. - Pinfty_0 Qinfy_3 + \frac{t00 Qinfy_3}{Qinfy_0} \right) + v_3 \left(\frac{t00}{Qinfy_0} - Pinfty_0 \right) - \left(-v_0 \left(-Pinfty_2 \right. \right. \\
& \quad \left. \left. - Pinfty_0 Qinfy_2 - Pinfty_1 Qinfy_3 + \frac{t00 Qinfy_2}{Qinfy_0} \right) - v_1 \left(-Pinfty_1 - Pinfty_0 Qinfy_3 \right. \right. \\
& \quad \left. \left. + \frac{t00 Qinfy_3}{Qinfy_0} \right) - v_2 \left(\frac{t00}{Qinfy_0} - Pinfty_0 \right) \right) \lambda - \left(-v_0 \left(-Pinfty_1 - Pinfty_0 Qinfy_3 \right. \right. \\
& \quad \left. \left. + \frac{t00 Qinfy_3}{Qinfy_0} \right) - v_1 \left(\frac{t00}{Qinfy_0} - Pinfty_0 \right) \right) \lambda^2 + v_0 \left(\frac{t00}{Qinfy_0} - Pinfty_0 \right) \lambda^3
\end{aligned} \tag{6}$$

```

> checkA21:=(-t00)/t[2*d+1]/(Qinfy[0])^2*LQinfy[0]/lambda:
QuantitycheckA21:=checkL11/checkL12*(checkL11/checkL12*checkA12
-2*checkA11):
for i from 1 to 100 do checkA21:=checkA21-(residue
(QuantitycheckA21*lambda^(-i),lambda=infinity))*lambda^(i-1): od:
for i from 0 to 100 do checkA21:=checkA21+(residue
(QuantitycheckA21*lambda^(i),lambda=0))*lambda^(-i-1): od:
AdditionalTerm:=t00*(1-t00)/t[2*d+1]/Qinfy[0]^2/lambda*nu[2*d-1]
:
for k from 1 to 2*d-1 do AdditionalTerm:=AdditionalTerm+t00*(1-
t00)/t[2*d+1]/Qinfy[0]^2/lambda*nu[k-1]*Qinfy[k]: od:
checkA21:=checkA21+AdditionalTerm:
NumcheckA21:=0:
for k from 2*d to 4*d do for j from k to 4*d do for m from 0 to
4*d-j do NumcheckA21:=NumcheckA21- t[2*d+1-m]*t[j+m-2*d+1]*nu[j-
k]*lambda^k: od: od: od:
for i from 1 to 100 do checkA21:=checkA21-(residue (NumcheckA21/
(-checkL12)*lambda^(-i),lambda=infinity))*lambda^(i-1): od:
checkA21:=checkA21:
series (checkA21,lambda=infinity):

```

```

> checkAMatrix:=Matrix(2,2,0):
checkAMatrix[1,1]:=checkA11:
checkAMatrix[1,2]:=checkA12:
checkAMatrix[2,1]:=checkA21:
checkAMatrix[2,2]:=checkA22:
checkAMatrix:

```

We define the derivatives of the Lax matrices. The derivative relatively to times and Darboux coordinates is specific to $d=2$ and should be adapted for other values of d .

```

> dcheckAMatrixdlambda:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do dcheckAMatrixdlambda[i,
j]:=diff(checkAMatrix[i,j],lambda): od: od:
> LcheckLMatrix:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do
LcheckLMatrix[i,j]:=diff(checkLMatrix[i,j],t[4])*alpha[4]+diff
(checkLMatrix[i,j],t[3])*alpha[3]+diff(checkLMatrix[i,j],t[2])*
alpha[2]+diff(checkLMatrix[i,j],t[1])*alpha[1]+diff(checkLMatrix
[i,j],Qinfy[0])*LQinfy[0]+diff(checkLMatrix[i,j],Qinfy[1])*
LQinfy[1]+diff(checkLMatrix[i,j],Qinfy[2])*LQinfy[2]+diff
(checkLMatrix[i,j],Qinfy[3])*LQinfy[3]+diff(checkLMatrix[i,j],
Pinfty[0])*LPinfy[0]+ diff(checkLMatrix[i,j],Pinfty[1])*LPinfy
[1]+ diff(checkLMatrix[i,j],Pinfty[2])*LPinfy[2]+ diff
(checkLMatrix[i,j],Pinfty[3])*LPinfy[3]: od: od:

```

We define the compatibility equations and solve them. This is the main test for the formulas of Theorem 2.2. This part is again specific to $d=2$ and should be adapted for other values of d

```

> CompatibilityEquations:=simplify(
dcheckAMatrixdlambda-LcheckLMatrix+Multiply
(checkAMatrix,checkLMatrix)-Multiply(checkLMatrix,checkAMatrix)
):
> simplify(series(CompatibilityEquations[1,1],lambda=0)):
series(simplify(series(CompatibilityEquations[1,1],lambda=
infinity)),lambda=infinity):
series(simplify(series(CompatibilityEquations[1,2],lambda=
infinity)),lambda=infinity):
series(simplify(series(CompatibilityEquations[2,1],lambda=
infinity)),lambda=infinity):
simplify(residue(CompatibilityEquations[1,1]*lambda,lambda=0));
EQ1:=simplify(residue(CompatibilityEquations[1,1],lambda=0));

simplify(-residue(CompatibilityEquations[1,1]/lambda^5,lambda=
infinity));
EQ2:=-residue(CompatibilityEquations[1,1]/lambda^4,lambda=
infinity);

```

```
EQ3:=-residue(CompatibilityEquations[1,1]/lambda^3,lambda=
infinity);
EQ4:=-residue(CompatibilityEquations[1,1]/lambda^2,lambda=
infinity);
EQ5:=-residue(CompatibilityEquations[1,1]/lambda^1,lambda=
infinity);
```

```
simplify(-residue(CompatibilityEquations[1,2]/lambda^5,lambda=
infinity));
EQ6:=-residue(CompatibilityEquations[1,2]/lambda^4,lambda=
infinity);
EQ7:=-residue(CompatibilityEquations[1,2]/lambda^3,lambda=
infinity);
EQ8:=-residue(CompatibilityEquations[1,2]/lambda^2,lambda=
infinity);
EQ9:=-residue(CompatibilityEquations[1,2]/lambda^1,lambda=
infinity);
```

0

$$EQ1 := -\frac{1}{Q_{infy_0}} \left(2 \left(\left((Pinfty_0 Q_{infy_1} + Pinfty_1 Q_{infy_2} + Pinfty_2 Q_{infy_3} + Pinfty_3) v_0 \right. \right. \right. \\ \left. \left. \left. + (Pinfty_0 v_2 + Pinfty_1 v_1) Q_{infy_3} + (Q_{infy_2} v_1 + v_3) Pinfty_0 + v_1 Pinfty_2 + v_2 Pinfty_1 \right) \right. \right. \\ \left. \left. Q_{infy_0} - \frac{Q_{infy_1} v_0}{2} - \frac{Q_{infy_2} v_1}{2} - \frac{Q_{infy_3} v_2}{2} - \frac{v_3}{2} + \frac{LQ_{infy_0}}{2} \right) t_{00} \right)$$

0

$$EQ2 := \frac{1}{Q_{infy_0}^2} \left(\left(4 Q_{infy_3} v_0 t_5^2 - 4 v_0 t_4 t_5 - 2 v_1 t_5^2 \right) Q_{infy_0}^3 + \left(-Q_{infy_3}^5 v_0 t_5^2 + \left(2 v_0 t_4 t_5 \right. \right. \right. \\ \left. \left. \left. + v_1 t_5^2 \right) Q_{infy_3}^4 + \left(\left(4 Q_{infy_2} v_0 - v_2 \right) t_5^2 + \left(-2 v_0 t_3 - 2 v_1 t_4 \right) t_5 - t_4^2 v_0 \right) Q_{infy_3}^3 + \left(\left(\right. \right. \right. \\ \left. \left. \left. -3 Q_{infy_1} v_0 - 3 Q_{infy_2} v_1 + 3 v_3 \right) t_5^2 + \left(-6 Q_{infy_2} v_0 t_4 + 2 v_0 t_2 + 2 v_1 t_3 + 2 v_2 t_4 \right) t_5 + \right. \right. \\ \left. \left. t_4^2 v_1 + 2 t_4 t_3 v_0 \right) Q_{infy_3}^2 + \left(\left(-3 Q_{infy_2}^2 v_0 + 4 v_1 Q_{infy_1} + 4 v_2 Q_{infy_2} \right) t_5^2 + \left(4 v_0 t_3 \right. \right. \right. \\ \left. \left. \left. + 4 v_1 t_4 \right) Q_{infy_2} + \left(4 Q_{infy_1} t_4 - 2 t_1 \right) v_0 - 2 v_1 t_2 - 2 v_2 t_3 - 4 v_3 t_4 \right) t_5 + 2 Q_{infy_2} v_0 t_4^2 \\ \left. + \left(-2 Pinfty_2 Pinfty_0 - Pinfty_1^2 - 2 t_4 t_2 - t_3^2 \right) v_0 + \left(-2 Pinfty_1 Pinfty_0 - 2 t_4 t_3 \right) v_1 - v_2 \left(\right. \right. \\ \left. \left. Pinfty_0^2 + t_4^2 \right) \right) Q_{infy_3} + \left(v_1 Q_{infy_2}^2 + \left(2 Q_{infy_1} v_0 - 2 v_3 \right) Q_{infy_2} - 2 v_2 Q_{infy_1} \right) t_5^2 + \left(2 \right. \\ \left. Q_{infy_2}^2 v_0 t_4 + \left(-2 v_0 t_2 - 2 v_1 t_3 - 4 v_2 t_4 \right) Q_{infy_2} + \left(-2 Q_{infy_1} t_3 + 2 t_0 \right) v_0 + \left(\right. \right. \\ \left. \left. -4 Q_{infy_1} t_4 + 2 t_1 \right) v_1 + 2 v_2 t_2 + 2 v_3 t_3 \right) t_5 + \left(\left(-2 Pinfty_1 Pinfty_0 - 2 t_4 t_3 \right) v_0 - v_1 \left(\right. \right. \\ \left. \left. Pinfty_0^2 + t_4^2 \right) \right) Q_{infy_2} + \left(-Q_{infy_1} Pinfty_0^2 - t_4^2 Q_{infy_1} - 2 Pinfty_3 Pinfty_0 - 2 Pinfty_1 Pinfty_2 \right. \\ \left. + 2 t_1 t_4 + 2 t_3 t_2 \right) v_0 + \left(-2 Pinfty_2 Pinfty_0 - Pinfty_1^2 + 2 t_4 t_2 + t_3^2 \right) v_1 - 2 v_2 Pinfty_0 Pinfty_1 \right)$$

$$+ 2 v_2 t_3 t_4 - v_3 \text{Pinfty}_0^2 + v_3 t_4^2 + L\text{Pinfty}_0) \text{Qinfty}_0^2 - t00 (\text{Qinfty}_1 v_0 + \text{Qinfty}_2 v_1 \\ + \text{Qinfty}_3 v_2 + v_3) (t00 - 1))$$

$$\text{EQ3} := \frac{1}{\text{Qinfty}_0^2} \left(\left(\text{Qinfty}_3^2 v_0 t_5^2 + 2 \text{Qinfty}_3 v_1 t_5^2 + (2 \text{Qinfty}_2 v_0 - 2 v_2) t_5^2 + (-2 v_0 t_3 \right. \right. \\ \left. \left. - 4 v_1 t_4) t_5 + v_0 (\text{Pinfty}_0^2 - t_4^2) \right) \text{Qinfty}_0^3 + \left(-\text{Qinfty}_3^6 v_0 t_5^2 + 2 \text{Qinfty}_3^5 v_0 t_4 t_5 + 4 \left(\text{Qinfty}_2 \right. \right. \\ \left. \left. t_5^2 - \frac{1}{2} t_3 t_5 - \frac{1}{4} t_4^2 \right) v_0 \text{Qinfty}_3^4 + \left((-3 \text{Qinfty}_1 v_0 + \text{Qinfty}_2 v_1 - v_3) t_5^2 - 6 \left(t_4 \text{Qinfty}_2 \right. \right. \right. \\ \left. \left. - \frac{t_2}{3} \right) v_0 t_5 + 2 t_4 t_3 v_0 \right) \text{Qinfty}_3^3 + \left((-3 \text{Qinfty}_2^2 v_0 - 2 v_1 \text{Qinfty}_1 - 2 v_2 \text{Qinfty}_2) t_5^2 \right. \\ \left. + \left((4 v_0 t_3 - 2 v_1 t_4) \text{Qinfty}_2 + (4 \text{Qinfty}_1 t_4 - 2 t_1) v_0 + 2 v_3 t_4 \right) t_5 + 2 \text{Qinfty}_2 v_0 t_4^2 + \left(\right. \right. \\ \left. \left. -2 \text{Pinfty}_2 \text{Pinfty}_0 - \text{Pinfty}_1^2 - 2 t_4 t_2 - t_3^2 \right) v_0 - 2 v_1 \text{Pinfty}_0 \text{Pinfty}_1 - v_2 \text{Pinfty}_0^2 \right) \text{Qinfty}_3^2 \\ \left. + \left((-2 v_1 \text{Qinfty}_2^2 + (2 \text{Qinfty}_1 v_0 + 4 v_3) \text{Qinfty}_2 + 2 v_2 \text{Qinfty}_1) t_5^2 + (2 \text{Qinfty}_2^2 v_0 t_4 + \left(\right. \right. \right. \\ \left. \left. -2 v_0 t_2 + 2 v_1 t_3 + 4 v_2 t_4) \text{Qinfty}_2 + (-2 \text{Qinfty}_1 t_3 + 2 t_0) v_0 + 4 v_1 \text{Qinfty}_1 t_4 - 2 v_3 t_3 \right) t_5 \right. \\ \left. + \left((-2 \text{Pinfty}_1 \text{Pinfty}_0 - 2 t_4 t_3) v_0 + (-\text{Pinfty}_0^2 + t_4^2) v_1 \right) \text{Qinfty}_2 + (-\text{Qinfty}_1 \text{Pinfty}_0^2 - \right. \\ \left. t_4^2 \text{Qinfty}_1 - 2 \text{Pinfty}_3 \text{Pinfty}_0 - 2 \text{Pinfty}_1 \text{Pinfty}_2 + 2 t_1 t_4 + 2 t_3 t_2) v_0 + (-4 \text{Pinfty}_2 \text{Pinfty}_0 \right. \\ \left. - 2 \text{Pinfty}_1^2) v_1 - 4 v_2 \text{Pinfty}_0 \text{Pinfty}_1 - v_3 \text{Pinfty}_0^2 - v_3 t_4^2 + L\text{Pinfty}_0) \text{Qinfty}_3 \\ \left. + (4 \text{Qinfty}_1 \text{Qinfty}_2 v_1 + 3 \text{Qinfty}_2^2 v_2 - 2 \text{Qinfty}_1 v_3) t_5^2 + (2 v_1 \text{Qinfty}_2^2 t_4 + (-2 v_1 t_2 \right. \right. \\ \left. \left. - 4 v_2 t_3 - 4 v_3 t_4) \text{Qinfty}_2 + (-4 \text{Qinfty}_1 t_3 + 2 t_0) v_1 - 4 v_2 \text{Qinfty}_1 t_4 + 2 v_2 t_1 + 2 v_3 t_2 \right) \right. \\ \left. t_5 + \left((-2 \text{Pinfty}_1 \text{Pinfty}_0 - 2 t_4 t_3) v_1 - 2 t_4^2 v_2 \right) \text{Qinfty}_2 + (-2 t00 \text{Pinfty}_0 - 3 \text{Pinfty}_0) v_0 \right. \\ \left. + (-2 t_4^2 \text{Qinfty}_1 - 2 \text{Pinfty}_3 \text{Pinfty}_0 - 2 \text{Pinfty}_1 \text{Pinfty}_2 + 2 t_1 t_4 + 2 t_3 t_2) v_1 + (2 v_2 t_2 \right. \\ \left. + 2 v_3 t_3) t_4 + (-2 \text{Pinfty}_2 \text{Pinfty}_0 - \text{Pinfty}_1^2 + t_3^2) v_2 + (-2 v_3 \text{Pinfty}_1 + L\text{Qinfty}_3) \text{Pinfty}_0 \right. \\ \left. + L\text{Pinfty}_1) \text{Qinfty}_0^2 + \left((2 v_2 \text{Pinfty}_1 + 2 v_1 \text{Pinfty}_2) \text{Qinfty}_3 + 2 v_1 \text{Pinfty}_1 \text{Qinfty}_2 + (t00 \right. \right. \\ \left. \left. + 3) v_0 + 2 v_1 \text{Pinfty}_3 + 2 v_2 \text{Pinfty}_2 + 2 v_3 \text{Pinfty}_1 - L\text{Qinfty}_3 \right) t00 \text{Qinfty}_0 \right. \\ \left. - t00 \text{Qinfty}_3 (\text{Qinfty}_1 v_0 + \text{Qinfty}_2 v_1 + \text{Qinfty}_3 v_2 + v_3) (t00 - 1)) \right)$$

$$\text{EQ4} := \frac{1}{\text{Qinfty}_0^2} \left(\left(\text{Qinfty}_3^3 v_0 t_5^2 + (-2 v_0 t_4 t_5 + v_1 t_5^2) \text{Qinfty}_3^2 + (2 t_5^2 v_2 + 2 t_5 t_3 v_0 + v_0 \left(\right. \right. \right. \\ \left. \left. \text{Pinfty}_0^2 + t_4^2) \right) \text{Qinfty}_3 + (2 \text{Qinfty}_1 v_0 - 2 v_3) t_5^2 + (-2 v_0 t_2 - 2 v_1 t_3 - 4 v_2 t_4) t_5 \right. \\ \left. + (2 \text{Pinfty}_1 \text{Pinfty}_0 - 2 t_4 t_3) v_0 + v_1 (\text{Pinfty}_0^2 - t_4^2) \right) \text{Qinfty}_0^3 + \left(-\text{Qinfty}_3^6 v_1 t_5^2 + \left(- \right. \right. \\ \left. \left. t_5^2 v_0 \text{Qinfty}_2 + 2 t_5 t_4 v_1 \right) \text{Qinfty}_3^5 + \left((5 \text{Qinfty}_2 v_1 + v_3) t_5^2 + (2 \text{Qinfty}_2 v_0 t_4 - 2 v_1 t_3) t_5 - \right. \right. \\ \left. \left. t_4^2 v_1 \right) \text{Qinfty}_3^4 + \left((4 \text{Qinfty}_2^2 v_0 - 2 v_1 \text{Qinfty}_1 + v_2 \text{Qinfty}_2) t_5^2 + \left((-2 v_0 t_3 \right. \right. \right. \\ \left. \left. - 8 v_1 t_4) \text{Qinfty}_2 + 2 v_1 t_2 - 2 v_3 t_4 \right) t_5 - \text{Qinfty}_2 v_0 t_4^2 + 2 t_4 t_3 v_1 \right) \text{Qinfty}_3^3 + \left((-6 v_1 \right. \\ \left. \text{Qinfty}_2^2 + (-3 \text{Qinfty}_1 v_0 - 3 v_3) \text{Qinfty}_2 - 2 v_2 \text{Qinfty}_1) t_5^2 + (-6 \text{Qinfty}_2^2 v_0 t_4 + (2 v_0 t_2 \right. \\ \left. + 6 v_1 t_3 - 2 v_2 t_4) \text{Qinfty}_2 + (2 \text{Qinfty}_1 t_4 - 2 t_1) v_1 + 2 v_3 t_3) t_5 + (2 t_4 t_3 v_0 + 3 \right. \right. \end{array}$$

$$\begin{aligned}
& t_4^2 v_1) Qinfy_2 + (-2 Pinfty_2 Pinfty_0 - Pinfty_1^2 - 2 t_4 t_2 - t_3^2) v_1 - 2 v_2 Pinfty_0 Pinfty_1 + v_3 t_4^2) \\
& Qinfy_3^2 + ((-3 Qinfy_2^3 v_0 + 2 Qinfy_1 Qinfy_2 v_1 - 2 Qinfy_2^2 v_2 + 4 Qinfy_1 v_3) t_5^2 \\
& + ((4 v_0 t_3 + 6 v_1 t_4) Qinfy_2^2 + ((4 Qinfy_1 t_4 - 2 t_1) v_0 - 4 v_1 t_2 + 2 v_2 t_3 \\
& + 4 v_3 t_4) Qinfy_2 + 4 v_2 Qinfy_1 t_4 + 2 v_1 t_0 - 2 v_3 t_2) t_5 + 2 Qinfy_2^2 v_0 t_4^2 + ((\\
& -2 Pinfty_2 Pinfty_0 - Pinfty_1^2 - 2 t_4 t_2 - t_3^2) v_0 + (-4 Pinfty_1 Pinfty_0 - 4 t_4 t_3) v_1 - v_2 Pinfty_0^2 \\
& + t_4^2 v_2) Qinfy_2 - 2 Pinfty_0 (t00 + 1) v_0 + (-2 Pinfty_3 Pinfty_0 - 2 Pinfty_1 Pinfty_2 + 2 t_1 t_4 \\
& + 2 t_3 t_2) v_1 - 2 v_3 t_3 t_4 + (-4 Pinfty_2 Pinfty_0 - 2 Pinfty_1^2) v_2 - 2 Pinfty_0 Pinfty_1 v_3 \\
& + LPinfty_1) Qinfy_3 + (v_1 Qinfy_2^3 + (2 Qinfy_1 v_0 + v_3) Qinfy_2^2 + 4 v_2 Qinfy_1 Qinfy_2 \\
& + 2 v_1 Qinfy_1^2) t_5^2 + (2 Qinfy_2^3 v_0 t_4 + (-2 v_0 t_2 - 2 v_1 t_3 + 2 v_2 t_4) Qinfy_2^2 + ((\\
& -2 Qinfy_1 t_3 + 2 t_0) v_0 + 2 v_1 t_1 - 2 v_2 t_2 - 2 v_3 t_3) Qinfy_2 - 2 v_1 Qinfy_1 t_2 - 4 v_2 Qinfy_1 t_3 \\
& - 4 v_3 Qinfy_1 t_4 + 2 v_2 t_0 + 2 v_3 t_1) t_5 + ((-2 Pinfty_1 Pinfty_0 - 2 t_4 t_3) v_0 - v_1 (Pinfty_0^2 + \\
& t_4^2) Qinfy_2^2 + ((-Qinfy_1 Pinfty_0^2 - t_4^2 Qinfy_1 - 2 Pinfty_3 Pinfty_0 - 2 Pinfty_1 Pinfty_2 + 2 t_1 t_4 \\
& + 2 t_3 t_2) v_0 + (-2 Pinfty_2 Pinfty_0 - Pinfty_1^2 + 2 t_4 t_2 + t_3^2) v_1 - 2 v_2 Pinfty_0 Pinfty_1 \\
& - 2 v_2 t_3 t_4 - v_3 Pinfty_0^2 - v_3 t_4^2 + LPinfty_0) Qinfy_2 - 2 Pinfty_1 (t00 + 1) v_0 + (\\
& -2 t_4 t_3 Qinfy_1 - 2 Pinfty_0 (-Pinfty_1 Qinfy_1 + t00 + 1)) v_1 - 2 Qinfy_1 v_2 t_4^2 + (2 v_2 t_1 \\
& + 2 v_3 t_2) t_4 + (-2 Pinfty_3 Pinfty_0 - 2 Pinfty_1 Pinfty_2 + 2 t_3 t_2) v_2 + v_3 t_3^2 + (-2 v_3 Pinfty_2 \\
& + LQinfy_2) Pinfty_0 - Pinfty_1^2 v_3 + LQinfy_3 Pinfty_1 + LPinfty_2) Qinfy_0^2 + t00 ((2 v_2 Pinfty_1 \\
& + 2 v_1 Pinfty_2) Qinfy_3^2 + (2 v_1 Pinfty_1 Qinfy_2 + (t00 + 2) v_0 + 2 v_1 Pinfty_3 + 4 v_2 Pinfty_2 \\
& + 2 v_3 Pinfty_1) Qinfy_3 + (-2 Pinfty_1 Qinfy_1 + t00 + 2) v_1 + 2 v_2 Pinfty_3 + 2 v_3 Pinfty_2 \\
& - LQinfy_2) Qinfy_0 - t00 Qinfy_2 (Qinfy_1 v_0 + Qinfy_2 v_1 + Qinfy_3 v_2 + v_3) (t00 - 1))
\end{aligned}$$

$$\begin{aligned}
EQ5 := & \frac{1}{Qinfy_0^2} \left(2 Qinfy_0^4 v_0 t_5^2 + (-Qinfy_3^4 v_0 t_5^2 + (2 v_0 t_4 t_5 + v_1 t_5^2) Qinfy_3^3 + ((3 Qinfy_2 v_0 \right. \\
& + v_2) t_5^2 + (-2 v_0 t_3 - 2 v_1 t_4) t_5 - t_4^2 v_0) Qinfy_3^2 + (4 t_5^2 v_3 + (-4 Qinfy_2 v_0 t_4 + 2 v_0 t_2 \\
& + 2 v_1 t_3) t_5 + (2 Pinfty_1 Pinfty_0 + 2 t_4 t_3) v_0 + v_1 (Pinfty_0^2 + t_4^2) Qinfy_3 + (-Qinfy_2^2 v_0 \\
& + 2 v_1 Qinfy_1 + 2 v_2 Qinfy_2) t_5^2 + (2 Qinfy_2 v_0 t_3 - 2 v_0 t_1 - 2 v_1 t_2 - 2 v_2 t_3 - 4 v_3 t_4) t_5 \\
& + v_0 (Pinfty_0^2 + t_4^2) Qinfy_2 + (2 Pinfty_2 Pinfty_0 + Pinfty_1^2 - 2 t_4 t_2 - t_3^2) v_0 \\
& + (2 Pinfty_1 Pinfty_0 - 2 t_4 t_3) v_1 + v_2 Pinfty_0^2 - t_4^2 v_2) Qinfy_3^3 + \left(-Qinfy_3^6 v_2 t_5^2 \right. \\
& \left. - ((Qinfy_1 v_0 + Qinfy_2 v_1 + v_3) t_5 - 2 v_2 t_4) t_5 Qinfy_3^5 + (4 v_2 Qinfy_2 t_5^2 \right.
\end{aligned}$$

$$\begin{aligned}
& + \left(2 \mathcal{Q}infy_1 v_0 t_4 + 2 \mathcal{Q}infy_2 v_1 t_4 - 2 v_2 t_3 + 2 v_3 t_4 \right) t_5 - t_4^2 v_2 \mathcal{Q}infy_3^4 + \left(\left(4 v_1 \mathcal{Q}infy_2^2 \right. \right. \\
& + \left. \left(4 \mathcal{Q}infy_1 v_0 + 4 v_3 \right) \mathcal{Q}infy_2 - 3 v_2 \mathcal{Q}infy_1 \right) t_5^2 + \left(\left(-2 v_1 t_3 - 6 v_2 t_4 \right) \mathcal{Q}infy_2 \right. \\
& - 2 \mathcal{Q}infy_1 v_0 t_3 + 2 v_2 t_2 - 2 v_3 t_3 \right) t_5 - t_4 \left(\mathcal{Q}infy_1 v_0 t_4 + \mathcal{Q}infy_2 v_1 t_4 - 2 v_2 t_3 + v_3 t_4 \right) \mathcal{Q}infy_3^3 \\
& + \left(\left(-3 \mathcal{Q}infy_1^2 v_0 - 3 \mathcal{Q}infy_1 \mathcal{Q}infy_2 v_1 - 3 \mathcal{Q}infy_2^2 v_2 - 3 \mathcal{Q}infy_1 v_3 \right) t_5^2 + \left(-6 v_1 \mathcal{Q}infy_2^2 t_4 \right. \right. \\
& + \left. \left(-6 \mathcal{Q}infy_1 v_0 t_4 + 2 v_1 t_2 + 4 v_2 t_3 - 6 v_3 t_4 \right) \mathcal{Q}infy_2 + 4 v_2 \mathcal{Q}infy_1 t_4 \right. \\
& + 2 \mathcal{Q}infy_1 v_0 t_2 - 2 v_2 t_1 + 2 v_3 t_2 \left. \right) t_5 + \left(2 t_4 t_3 v_1 + 2 t_4^2 v_2 \right) \mathcal{Q}infy_2 + 2 \mathcal{Q}infy_1 v_0 t_3 t_4 + \left(\right. \\
& - 2 v_2 t_2 + 2 v_3 t_3 \left. \right) t_4 - 2 \left(\mathcal{P}infy_2 \mathcal{P}infy_0 + \frac{\mathcal{P}infy_1^2}{2} + \frac{t_3^2}{2} \right) v_2 \mathcal{Q}infy_3^2 + \left(\left(-3 v_1 \mathcal{Q}infy_2^3 \right. \right. \\
& + \left(-3 \mathcal{Q}infy_1 v_0 - 3 v_3 \right) \mathcal{Q}infy_2^2 + 2 v_2 \mathcal{Q}infy_1 \mathcal{Q}infy_2 \left. \right) t_5^2 + \left(\left(4 v_1 t_3 + 2 v_2 t_4 \right) \mathcal{Q}infy_2^2 \right. \\
& + \left(4 \mathcal{Q}infy_1 v_0 t_3 + \left(4 \mathcal{Q}infy_1 t_4 - 2 t_1 \right) v_1 - 2 v_2 t_2 + 4 v_3 t_3 \right) \mathcal{Q}infy_2 + \left(4 \mathcal{Q}infy_1^2 t_4 \right. \\
& - 2 \mathcal{Q}infy_1 t_1 \left. \right) v_0 - 2 v_2 \mathcal{Q}infy_1 t_3 + 4 v_3 \mathcal{Q}infy_1 t_4 + 2 v_2 t_0 - 2 v_3 t_1 \left. \right) t_5 + 2 \mathcal{Q}infy_2^2 v_1 t_4^2 \\
& + \left(2 \mathcal{Q}infy_1 v_0 t_4^2 + \left(-2 \mathcal{P}infy_2 \mathcal{P}infy_0 - \mathcal{P}infy_1^2 - 2 t_4 t_2 - t_3^2 \right) v_1 - 2 v_2 \mathcal{P}infy_0 \mathcal{P}infy_1 \right. \\
& - 2 v_2 t_3 t_4 + 2 v_3 t_4^2 \left. \right) \mathcal{Q}infy_2 + \left(-2 t_4 t_2 \mathcal{Q}infy_1 + \left(-2 \mathcal{P}infy_2 \mathcal{P}infy_0 - \mathcal{P}infy_1^2 - \right. \right. \\
& t_3^2 \left. \right) \mathcal{Q}infy_1 - 2 t_0 \mathcal{P}infy_1 - \mathcal{P}infy_1 \left. \right) v_0 + \left(-2 t_0 \mathcal{P}infy_0 - \mathcal{P}infy_0 \right) v_1 - \mathcal{Q}infy_1 v_2 t_4^2 \\
& + \left(2 v_2 t_1 - 2 v_3 t_2 \right) t_4 - \mathcal{P}infy_0^2 \mathcal{Q}infy_1 v_2 + \left(-2 \mathcal{P}infy_3 \mathcal{P}infy_0 - 2 \mathcal{P}infy_1 \mathcal{P}infy_2 \right. \\
& + 2 t_3 t_2 \left. \right) v_2 - 2 \mathcal{P}infy_0 \mathcal{P}infy_2 v_3 - \mathcal{P}infy_1^2 v_3 - v_3 t_3^2 + L \mathcal{P}infy_2 \left. \right) \mathcal{Q}infy_3 \\
& + 2 \mathcal{Q}infy_1 \mathcal{Q}infy_2 \left(\mathcal{Q}infy_1 v_0 + \mathcal{Q}infy_2 v_1 + v_3 \right) t_5^2 - 2 \left(-\mathcal{Q}infy_2^2 t_4 + \mathcal{Q}infy_1 t_3 \right. \\
& + \mathcal{Q}infy_2 t_2 - t_0 \left. \right) \left(\mathcal{Q}infy_1 v_0 + \mathcal{Q}infy_2 v_1 + v_3 \right) t_5 - 2 v_1 \left(\mathcal{P}infy_1 \mathcal{P}infy_0 + t_4 t_3 \right) \mathcal{Q}infy_2^2 \\
& + \left(\left(-2 t_4 t_3 \mathcal{Q}infy_1 - 2 \mathcal{P}infy_0 \left(\mathcal{P}infy_1 \mathcal{Q}infy_1 + t_0 + \frac{1}{2} \right) \right) v_0 + \left(-\mathcal{Q}infy_1 \mathcal{P}infy_0^2 - \right. \right. \\
& t_4^2 \mathcal{Q}infy_1 - 2 \mathcal{P}infy_3 \mathcal{P}infy_0 - 2 \mathcal{P}infy_1 \mathcal{P}infy_2 + 2 t_1 t_4 + 2 t_3 t_2 \left. \right) v_1 - 2 \mathcal{P}infy_0 \mathcal{P}infy_1 v_3 \\
& - 2 v_3 t_3 t_4 + L \mathcal{P}infy_1 \left. \right) \mathcal{Q}infy_2 + \left(-t_4^2 \mathcal{Q}infy_1^2 + 2 t_4 t_1 \mathcal{Q}infy_1 - \mathcal{P}infy_0^2 \mathcal{Q}infy_1^2 + \left(\right. \right. \\
& - 2 \mathcal{P}infy_3 \mathcal{P}infy_0 - 2 \mathcal{P}infy_1 \mathcal{P}infy_2 + 2 t_3 t_2 \left. \right) \mathcal{Q}infy_1 - 2 t_0 \mathcal{P}infy_2 - \mathcal{P}infy_2 \left. \right) v_0 + \left(\right. \\
& - 2 t_0 \mathcal{P}infy_1 - \mathcal{P}infy_1 \left. \right) v_1 - \mathcal{Q}infy_1 v_3 t_4^2 + 2 v_3 t_1 t_4 + \left(-v_3 \mathcal{P}infy_0^2 + L \mathcal{P}infy_0 \right) \mathcal{Q}infy_1 \\
& + \left(-2 t_0 \mathcal{P}infy_0 - \mathcal{P}infy_0 \right) v_2 + 2 v_3 t_2 t_3 + \left(-2 v_3 \mathcal{P}infy_3 + L \mathcal{Q}infy_1 \right) \mathcal{P}infy_0
\end{aligned}$$

$$\begin{aligned}
& -2 \text{Pinfty}_1 \text{Pinfty}_2 v_3 + L\text{Qinfty}_2 \text{Pinfty}_1 + L\text{Qinfty}_3 \text{Pinfty}_2 + L\text{Pinfty}_3) \text{Qinfty}_0^2 \\
& + (2 v_2 \text{Pinfty}_2 \text{Qinfty}_3^2 + ((2 v_2 \text{Pinfty}_1 + 2 v_1 \text{Pinfty}_2) \text{Qinfty}_2 + (-2 \text{Pinfty}_1 \text{Qinfty}_1 + t00 \\
& + 1) v_1 + 2 v_2 \text{Pinfty}_3 + 2 v_3 \text{Pinfty}_2) \text{Qinfty}_3 + 2 v_1 \text{Pinfty}_1 \text{Qinfty}_2^2 + ((t00 + 1) v_0 \\
& + 2 v_1 \text{Pinfty}_3 + 2 v_3 \text{Pinfty}_1) \text{Qinfty}_2 - 2 v_1 \text{Pinfty}_2 \text{Qinfty}_1 - 2 v_2 \text{Pinfty}_1 \text{Qinfty}_1 + (t00 \\
& + 1) v_2 + 2 v_3 \text{Pinfty}_3 - L\text{Qinfty}_1) t00 \text{Qinfty}_0 - t00 \text{Qinfty}_1 (\text{Qinfty}_1 v_0 + \text{Qinfty}_2 v_1 \\
& + \text{Qinfty}_3 v_2 + v_3) (t00 - 1)) \\
& 0
\end{aligned}$$

$$\begin{aligned}
EQ6 := & -2 t_5 \left((v_2 \text{Pinfty}_1 + v_1 \text{Pinfty}_2) \text{Qinfty}_3 + (\text{Pinfty}_1 \text{Qinfty}_2 + \text{Pinfty}_3) v_1 + v_3 \text{Pinfty}_1 \right. \\
& \left. + v_2 \text{Pinfty}_2 - v_0 \text{Pinfty}_0 \text{Qinfty}_0 - \frac{L\text{Qinfty}_3}{2} + 2 v_0 \right)
\end{aligned}$$

$$\begin{aligned}
EQ7 := & -2 t_5 \left((v_2 \text{Pinfty}_1 + v_1 \text{Pinfty}_2) \text{Qinfty}_3^2 + \left((\text{Pinfty}_1 \text{Qinfty}_2 + \text{Pinfty}_3) v_1 + \frac{3 v_0}{2} \right. \right. \\
& \left. \left. + v_3 \text{Pinfty}_1 + 2 v_2 \text{Pinfty}_2 - v_0 \text{Pinfty}_0 \text{Qinfty}_0 \right) \text{Qinfty}_3 + \left(-\text{Pinfty}_1 \text{Qinfty}_1 \right. \right. \\
& \left. \left. - \text{Pinfty}_0 \text{Qinfty}_0 + \frac{3}{2} \right) v_1 + v_3 \text{Pinfty}_2 + v_2 \text{Pinfty}_3 - \text{Pinfty}_1 v_0 \text{Qinfty}_0 - \frac{L\text{Qinfty}_2}{2} \right)
\end{aligned}$$

$$\begin{aligned}
EQ8 := & -2 t_5 \left(v_2 \text{Pinfty}_2 \text{Qinfty}_3^2 + ((-\text{Pinfty}_0 \text{Qinfty}_0 - \text{Pinfty}_1 \text{Qinfty}_1 + \text{Pinfty}_2 \text{Qinfty}_2 \right. \\
& \left. + 1) v_1 + \text{Pinfty}_1 \text{Qinfty}_2 v_2 - \text{Pinfty}_1 v_0 \text{Qinfty}_0 + v_3 \text{Pinfty}_2 + v_2 \text{Pinfty}_3) \text{Qinfty}_3 \right. \\
& \left. + (\text{Pinfty}_1 \text{Qinfty}_2^2 - \text{Pinfty}_1 \text{Qinfty}_0 - \text{Pinfty}_2 \text{Qinfty}_1 + \text{Pinfty}_3 \text{Qinfty}_2) v_1 + (-\text{Pinfty}_0 v_2 \right. \\
& \left. - v_0 (\text{Pinfty}_0 \text{Qinfty}_2 + \text{Pinfty}_2)) \text{Qinfty}_0 + (-v_2 \text{Qinfty}_1 + \text{Qinfty}_2 v_3) \text{Pinfty}_1 + v_3 \text{Pinfty}_3 \right. \\
& \left. + \text{Qinfty}_2 v_0 - \frac{L\text{Qinfty}_1}{2} + v_2 \right)
\end{aligned}$$

$$\begin{aligned}
EQ9 := & -2 t_5 \left(\left(-\text{Pinfty}_1 v_1 \text{Qinfty}_0 + (-\text{Pinfty}_0 v_2 - \text{Pinfty}_2 v_0) \text{Qinfty}_0 + \frac{v_2}{2} \right) \text{Qinfty}_3 + \left(\right. \right. \\
& \left. \left. -\text{Pinfty}_0 \text{Qinfty}_2 - \text{Pinfty}_2 \right) \text{Qinfty}_0 + \frac{\text{Qinfty}_2}{2} \right) v_1 + ((-\text{Qinfty}_2 v_0 - v_2) \text{Pinfty}_1 + (\\
& -\text{Pinfty}_0 \text{Qinfty}_1 - \text{Pinfty}_3) v_0 - v_3 \text{Pinfty}_0) \text{Qinfty}_0 - \frac{L\text{Qinfty}_0}{2} + \frac{v_3}{2} + \frac{\text{Qinfty}_1 v_0}{2} \right) \quad (7)
\end{aligned}$$

```

> LQinfty[3]:=factor(solve(EQ6,LQinfty[3])) ;
LQinfty[2]:=factor(solve(EQ7,LQinfty[2])) ;
LQinfty[1]:=factor(solve(EQ8,LQinfty[1])) ;
LQinfty[0]:=factor(solve(EQ9,LQinfty[0])) ;
LPinfty[0]:=factor(solve(EQ2,LPinfty[0])) ;
LPinfty[1]:=factor(solve(EQ3,LPinfty[1])) ;
LPinfty[2]:=factor(solve(EQ4,LPinfty[2])) ;

```

```

LPinfy[3]:=factor(solve(EQ5,LPinfy[3])) ;
EQ1:=simplify(EQ1) ;
EQ2:=simplify(EQ2) ;
EQ3:=simplify(EQ3) ;
EQ4:=simplify(EQ4) ;
EQ5:=simplify(EQ5) ;
EQ6:=simplify(EQ6) ;
EQ7:=simplify(EQ7) ;
EQ8:=simplify(EQ8) ;
EQ9:=simplify(EQ9) ;

```

$$LQinfy_3 := -2 v_0 Pinfty_0 Qinfy_0 + 2 v_1 Pinfty_1 Qinfy_2 + 2 Pinfty_1 Qinfy_3 v_2 \\ + 2 Pinfty_2 Qinfy_3 v_1 + 2 v_3 Pinfty_1 + 2 v_2 Pinfty_2 + 2 v_1 Pinfty_3 + 4 v_0$$

$$LQinfy_2 := -2 Pinfty_0 Qinfy_0 Qinfy_3 v_0 + 2 Pinfty_1 Qinfy_2 Qinfy_3 v_1 + 2 Pinfty_1 Qinfy_3^2 v_2 \\ + 2 Pinfty_2 Qinfy_3^2 v_1 - 2 Pinfty_0 Qinfy_0 v_1 - 2 Pinfty_1 v_0 Qinfy_0 - 2 v_1 Pinfty_1 Qinfy_1 \\ + 2 Pinfty_1 Qinfy_3 v_3 + 4 Pinfty_2 Qinfy_3 v_2 + 2 Pinfty_3 Qinfy_3 v_1 + 2 v_3 Pinfty_2 \\ + 2 v_2 Pinfty_3 + 3 Qinfy_3 v_0 + 3 v_1$$

$$LQinfy_1 := -2 Pinfty_0 Qinfy_0 Qinfy_2 v_0 - 2 Pinfty_0 Qinfy_0 Qinfy_3 v_1 \\ - 2 Pinfty_1 Qinfy_0 Qinfy_3 v_0 - 2 Pinfty_1 Qinfy_1 Qinfy_3 v_1 + 2 v_1 Pinfty_1 Qinfy_2^2 \\ + 2 Pinfty_1 Qinfy_2 Qinfy_3 v_2 + 2 Pinfty_2 Qinfy_2 Qinfy_3 v_1 + 2 v_2 Pinfty_2 Qinfy_3^2 \\ - 2 Pinfty_0 Qinfy_0 v_2 - 2 Pinfty_1 v_1 Qinfy_0 - 2 v_2 Pinfty_1 Qinfy_1 + 2 Pinfty_1 Qinfy_2 v_3 \\ - 2 Pinfty_2 Qinfy_0 v_0 - 2 v_1 Pinfty_2 Qinfy_1 + 2 Pinfty_2 Qinfy_3 v_3 + 2 Pinfty_3 Qinfy_2 v_1 \\ + 2 Pinfty_3 Qinfy_3 v_2 + 2 v_3 Pinfty_3 + 2 Qinfy_2 v_0 + 2 Qinfy_3 v_1 + 2 v_2$$

$$LQinfy_0 := -2 Pinfty_0 Qinfy_0 Qinfy_1 v_0 - 2 Pinfty_0 Qinfy_0 Qinfy_2 v_1 \\ - 2 Pinfty_0 Qinfy_0 Qinfy_3 v_2 - 2 Pinfty_1 Qinfy_0 Qinfy_2 v_0 - 2 Pinfty_1 Qinfy_0 Qinfy_3 v_1 \\ - 2 Pinfty_2 Qinfy_0 Qinfy_3 v_0 - 2 Pinfty_0 Qinfy_0 v_3 - 2 Pinfty_1 Qinfy_0 v_2 \\ - 2 Pinfty_2 Qinfy_0 v_1 - 2 Pinfty_3 Qinfy_0 v_0 + Qinfy_1 v_0 + Qinfy_2 v_1 + Qinfy_3 v_2 + v_3$$

$$LPinfy_0 := \frac{1}{Qinfy_0^2} \left(Qinfy_0^2 Qinfy_3^5 v_0 t_5^2 - 4 Qinfy_0^2 Qinfy_2 Qinfy_3^3 v_0 t_5^2 - 2 Qinfy_0^2 \right. \\ Qinfy_3^4 v_0 t_4 t_5 - Qinfy_0^2 Qinfy_3^4 v_1 t_5^2 + 3 Qinfy_0^2 Qinfy_1 Qinfy_3^2 v_0 t_5^2 + 3 Qinfy_0^2 \\ Qinfy_2^2 Qinfy_3 v_0 t_5^2 + 6 Qinfy_0^2 Qinfy_2 Qinfy_3^2 v_0 t_4 t_5 + 3 Qinfy_0^2 Qinfy_2 Qinfy_3^2 v_1 t_5^2 + 2 \\ Qinfy_0^2 Qinfy_3^3 v_0 t_3 t_5 + Qinfy_0^2 Qinfy_3^3 v_0 t_4^2 + 2 Qinfy_0^2 Qinfy_3^3 v_1 t_4 t_5 + Qinfy_0^2 Qinfy_3^3 v_2 t_5^2 \\ - 4 Qinfy_0^3 Qinfy_3 v_0 t_5^2 - 2 Qinfy_0^2 Qinfy_1 Qinfy_2 v_0 t_5^2 - 4 Qinfy_0^2 Qinfy_1 Qinfy_3 v_0 t_4 t_5 \\ - 4 Qinfy_0^2 Qinfy_1 Qinfy_3 v_1 t_5^2 - 2 Qinfy_0^2 Qinfy_2^2 v_0 t_4 t_5 - Qinfy_0^2 Qinfy_2^2 v_1 t_5^2 - 4 \\ Qinfy_0^2 Qinfy_2 Qinfy_3 v_0 t_3 t_5 - 2 Qinfy_0^2 Qinfy_2 Qinfy_3 v_0 t_4^2 - 4 \\ Qinfy_0^2 Qinfy_2 Qinfy_3 v_1 t_4 t_5 - 4 Qinfy_0^2 Qinfy_2 Qinfy_3 v_2 t_5^2 - 2 Qinfy_0^2 Qinfy_3^2 v_0 t_2 t_5 - 2$$

$$\begin{aligned}
& Q_{infy_0}^2 Q_{infy_3}^2 v_0 t_3 t_4 - 2 Q_{infy_0}^2 Q_{infy_3}^2 v_1 t_3 t_5 - Q_{infy_0}^2 Q_{infy_3}^2 v_1 t_4^2 - 2 Q_{infy_0}^2 \\
& Q_{infy_3}^2 v_2 t_4 t_5 - 3 Q_{infy_0}^2 Q_{infy_3}^2 v_3 t_5^2 + Pinfty_0^2 Q_{infy_0}^2 Q_{infy_1} v_0 + Pinfty_0^2 Q_{infy_0}^2 Q_{infy_2} v_1 \\
& + Pinfty_0^2 Q_{infy_0}^2 Q_{infy_3} v_2 + 2 Pinfty_0 Pinfty_1 Q_{infy_0}^2 Q_{infy_2} v_0 + 2 Pinfty_0 Pinfty_1 \\
& Q_{infy_0}^2 Q_{infy_3} v_1 + 2 Pinfty_0 Pinfty_2 Q_{infy_0}^2 Q_{infy_3} v_0 + Pinfty_1^2 Q_{infy_0}^2 Q_{infy_3} v_0 + 4 \\
& Q_{infy_0}^3 v_0 t_4 t_5 + 2 Q_{infy_0}^3 v_1 t_5^2 + 2 Q_{infy_0}^2 Q_{infy_1} v_0 t_3 t_5 + Q_{infy_0}^2 Q_{infy_1} v_0 t_4^2 + 4 \\
& Q_{infy_0}^2 Q_{infy_1} v_1 t_4 t_5 + 2 Q_{infy_0}^2 Q_{infy_1} v_2 t_5^2 + 2 Q_{infy_0}^2 Q_{infy_2} v_0 t_2 t_5 + 2 \\
& Q_{infy_0}^2 Q_{infy_2} v_0 t_3 t_4 + 2 Q_{infy_0}^2 Q_{infy_2} v_1 t_3 t_5 + Q_{infy_0}^2 Q_{infy_2} v_1 t_4^2 + 4 \\
& Q_{infy_0}^2 Q_{infy_2} v_2 t_4 t_5 + 2 Q_{infy_0}^2 Q_{infy_2} v_3 t_5^2 + 2 Q_{infy_0}^2 Q_{infy_3} v_0 t_1 t_5 + 2 \\
& Q_{infy_0}^2 Q_{infy_3} v_0 t_2 t_4 + Q_{infy_0}^2 Q_{infy_3} v_0 t_3^2 + 2 Q_{infy_0}^2 Q_{infy_3} v_1 t_2 t_5 + 2 \\
& Q_{infy_0}^2 Q_{infy_3} v_1 t_3 t_4 + 2 Q_{infy_0}^2 Q_{infy_3} v_2 t_3 t_5 + Q_{infy_0}^2 Q_{infy_3} v_2 t_4^2 + 4 \\
& Q_{infy_0}^2 Q_{infy_3} v_3 t_4 t_5 + Pinfty_0^2 Q_{infy_0}^2 v_3 + 2 Pinfty_0 Pinfty_1 Q_{infy_0}^2 v_2 + 2 Pinfty_0 Pinfty_2 \\
& Q_{infy_0}^2 v_1 + 2 Pinfty_0 Pinfty_3 Q_{infy_0}^2 v_0 + Pinfty_1^2 Q_{infy_0}^2 v_1 + 2 Pinfty_1 Pinfty_2 Q_{infy_0}^2 v_0 - 2 \\
& Q_{infy_0}^2 v_0 t_0 t_5 - 2 Q_{infy_0}^2 v_0 t_1 t_4 - 2 Q_{infy_0}^2 v_0 t_2 t_3 - 2 Q_{infy_0}^2 v_1 t_1 t_5 - 2 Q_{infy_0}^2 v_1 t_2 t_4 - \\
& Q_{infy_0}^2 v_1 t_3^2 - 2 Q_{infy_0}^2 v_2 t_2 t_5 - 2 Q_{infy_0}^2 v_2 t_3 t_4 - 2 Q_{infy_0}^2 v_3 t_3 t_5 - Q_{infy_0}^2 v_3 t_4^2 \\
& + t00^2 Q_{infy_1} v_0 + t00^2 Q_{infy_2} v_1 + t00^2 Q_{infy_3} v_2 + t00^2 v_3 - t00 Q_{infy_1} v_0 \\
& - t00 Q_{infy_2} v_1 - t00 Q_{infy_3} v_2 - t00 v_3)
\end{aligned}$$

$$\begin{aligned}
LPinfty_1 := & -\frac{1}{Q_{infy_0}} \left(-Q_{infy_0} Q_{infy_3}^5 v_1 t_5^2 + 4 Q_{infy_0} Q_{infy_2} Q_{infy_3}^3 v_1 t_5^2 + 2 Q_{infy_0} \right. \\
& Q_{infy_3}^4 v_1 t_4 t_5 + Q_{infy_0} Q_{infy_3}^4 v_2 t_5^2 - 3 Q_{infy_0}^2 Q_{infy_3}^2 v_0 t_5^2 - 6 Q_{infy_0} Q_{infy_1} Q_{infy_3}^2 v_1 t_5^2 \\
& - 3 Q_{infy_0} Q_{infy_2}^2 Q_{infy_3} v_1 t_5^2 - 6 Q_{infy_0} Q_{infy_2} Q_{infy_3}^2 v_1 t_4 t_5 - 6 Q_{infy_0} Q_{infy_2} \\
& Q_{infy_3}^2 v_2 t_5^2 - 2 Q_{infy_0} Q_{infy_3}^3 v_1 t_3 t_5 - Q_{infy_0} Q_{infy_3}^3 v_1 t_4^2 - 2 Q_{infy_0} Q_{infy_3}^3 v_2 t_4 t_5 \\
& - 4 Q_{infy_0} Q_{infy_3}^3 v_3 t_5^2 + 2 Q_{infy_0}^2 Q_{infy_2} v_0 t_5^2 + 4 Q_{infy_0}^2 Q_{infy_3} v_0 t_4 t_5 + 4 \\
& Q_{infy_0}^2 Q_{infy_3} v_1 t_5^2 + 4 Q_{infy_0} Q_{infy_1} Q_{infy_2} v_1 t_5^2 + 8 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_1 t_4 t_5 \\
& + 4 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_2 t_5^2 + 2 Q_{infy_0} Q_{infy_2}^2 v_1 t_4 t_5 + 3 Q_{infy_0} Q_{infy_2}^2 v_2 t_5^2 \\
& + 4 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_1 t_3 t_5 + 2 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_1 t_4^2 \\
& + 8 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_2 t_4 t_5 + 6 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_3 t_5^2 + 2 Q_{infy_0} Q_{infy_3}^2 v_1 t_2 t_5 \\
& + 2 Q_{infy_0} Q_{infy_3}^2 v_1 t_3 t_4 + 2 Q_{infy_0} Q_{infy_3}^2 v_2 t_3 t_5 + Q_{infy_0} Q_{infy_3}^2 v_2 t_4^2 + 6 Q_{infy_0} \\
& Q_{infy_3}^2 v_3 t_4 t_5 - Pinfty_0^2 Q_{infy_0}^2 v_0 - Pinfty_1^2 Q_{infy_0} Q_{infy_3} v_1 - 2 Q_{infy_0}^2 v_0 t_3 t_5 - \\
& Q_{infy_0}^2 v_0 t_4^2 - 4 Q_{infy_0}^2 v_1 t_4 t_5 - 2 Q_{infy_0}^2 v_2 t_5^2 - 4 Q_{infy_0} Q_{infy_1} v_1 t_3 t_5 \\
& - 2 Q_{infy_0} Q_{infy_1} v_1 t_4^2 - 4 Q_{infy_0} Q_{infy_1} v_2 t_4 t_5 - 2 Q_{infy_0} Q_{infy_1} v_3 t_5^2 \\
& - 2 Q_{infy_0} Q_{infy_2} v_1 t_2 t_5 - 2 Q_{infy_0} Q_{infy_2} v_1 t_3 t_4 - 4 Q_{infy_0} Q_{infy_2} v_2 t_3 t_5 \\
& - 2 Q_{infy_0} Q_{infy_2} v_2 t_4^2 - 4 Q_{infy_0} Q_{infy_2} v_3 t_4 t_5 - 2 Q_{infy_0} Q_{infy_3} v_1 t_1 t_5 \\
& \left. - 2 Q_{infy_0} Q_{infy_3} v_1 t_2 t_4 - Q_{infy_0} Q_{infy_3} v_1 t_3^2 - 2 Q_{infy_0} Q_{infy_3} v_2 t_2 t_5 \right)
\end{aligned}$$

$$\begin{aligned}
& -2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_2 t_3 t_4 - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_3 t_3 t_5 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_3 t_4^2 - \\
& \operatorname{Pinfty}_1^2 \operatorname{Qinfy}_0 v_2 - 2 \operatorname{Pinfty}_1 \operatorname{Pinfty}_2 \operatorname{Qinfy}_0 v_1 + 2 \operatorname{Qinfy}_0 v_1 t_0 t_5 + 2 \operatorname{Qinfy}_0 v_1 t_1 t_4 \\
& + 2 \operatorname{Qinfy}_0 v_1 t_2 t_3 + 2 \operatorname{Qinfy}_0 v_2 t_1 t_5 + 2 \operatorname{Qinfy}_0 v_2 t_2 t_4 + \operatorname{Qinfy}_0 v_2 t_3^2 + 2 \operatorname{Qinfy}_0 v_3 t_2 t_5 \\
& + 2 \operatorname{Qinfy}_0 v_3 t_3 t_4 + t00^2 v_0 + v_0 \operatorname{Pinfty}_0 \operatorname{Qinfy}_0 - t00 v_0)
\end{aligned}$$

$$\begin{aligned}
L\operatorname{Pinfty}_2 := & -\frac{1}{\operatorname{Qinfy}_0} \left(-\operatorname{Qinfy}_0 \operatorname{Qinfy}_3^5 v_2 t_5^2 + 4 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3^3 v_0 t_5^2 + 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \right. \\
& \operatorname{Qinfy}_3^3 v_1 t_5^2 + 8 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3^3 v_2 t_5^2 + 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^4 v_2 t_4 t_5 + 5 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^4 v_3 \\
& t_5^2 - 6 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3 v_0 t_5^2 - 6 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3^2 v_0 t_4 t_5 - 3 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3^2 v_1 t_5^2 \\
& - 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3 v_1 t_5^2 - 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3^2 v_1 t_4 t_5 - 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \\
& \operatorname{Qinfy}_3^2 v_2 t_5^2 - 9 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2^2 \operatorname{Qinfy}_3 v_2 t_5^2 - 12 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3^2 v_2 t_4 t_5 \\
& - 12 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3^2 v_3 t_5^2 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^3 v_2 t_3 t_5 - \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^3 v_2 t_4^2 \\
& - 8 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^3 v_3 t_4 t_5 + 2 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_1 v_0 t_5^2 + 4 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_2 v_0 t_4 t_5 + 2 \\
& \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_2 v_1 t_5^2 + 4 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3 v_0 t_3 t_5 + 2 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3 v_0 t_4^2 + 4 \\
& \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3 v_1 t_4 t_5 + 4 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3 v_2 t_5^2 + 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1^2 v_1 t_5^2 \\
& + 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_2 v_1 t_4 t_5 + 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_2 v_2 t_5^2 \\
& + 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3 v_1 t_3 t_5 + 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3 v_1 t_4^2 \\
& + 8 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3 v_2 t_4 t_5 + 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3 v_3 t_5^2 + 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2^2 v_2 t_4 t_5 \\
& + 3 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2^2 v_3 t_5^2 + 8 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3 v_2 t_3 t_5 + 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3 v_2 t_4^2 \\
& + 12 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 \operatorname{Qinfy}_3 v_3 t_4 t_5 + 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^2 v_2 t_2 t_5 + 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^2 v_2 t_3 t_4 \\
& + 6 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^2 v_3 t_3 t_5 + 3 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3^2 v_3 t_4^2 - \operatorname{Pinfty}_0^2 \operatorname{Qinfy}_0^2 v_1 - 2 \operatorname{Pinfty}_0 \operatorname{Pinfty}_1 \\
& \operatorname{Qinfy}_0^2 v_0 + 2 \operatorname{Pinfty}_1^2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 v_1 + \operatorname{Pinfty}_1^2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_2 \\
& + 2 \operatorname{Pinfty}_1 \operatorname{Pinfty}_2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_1 - 2 \operatorname{Qinfy}_0^2 v_0 t_2 t_5 - 2 \operatorname{Qinfy}_0^2 v_0 t_3 t_4 - 2 \operatorname{Qinfy}_0^2 v_1 t_3 t_5 \\
& - \operatorname{Qinfy}_0^2 v_1 t_4^2 - 4 \operatorname{Qinfy}_0^2 v_2 t_4 t_5 - 2 \operatorname{Qinfy}_0^2 v_3 t_5^2 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 v_1 t_2 t_5 \\
& - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 v_1 t_3 t_4 - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 v_2 t_3 t_5 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 v_2 t_4^2 \\
& - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 v_3 t_4 t_5 - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 v_2 t_2 t_5 - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 v_2 t_3 t_4 \\
& - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 v_3 t_3 t_5 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_2 v_3 t_4^2 - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_2 t_1 t_5 \\
& - 2 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_2 t_2 t_4 - \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_2 t_3^2 - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_3 t_2 t_5 \\
& - 4 \operatorname{Qinfy}_0 \operatorname{Qinfy}_3 v_3 t_3 t_4 + \operatorname{Pinfty}_1^2 \operatorname{Qinfy}_0 v_3 + 2 \operatorname{Pinfty}_1 \operatorname{Pinfty}_3 \operatorname{Qinfy}_0 v_1 \\
& + 2 \operatorname{Qinfy}_0 v_2 t_0 t_5 + 2 \operatorname{Qinfy}_0 v_2 t_1 t_4 + 2 \operatorname{Qinfy}_0 v_2 t_2 t_3 + 2 \operatorname{Qinfy}_0 v_3 t_1 t_5 \\
& + 2 \operatorname{Qinfy}_0 v_3 t_2 t_4 + \operatorname{Qinfy}_0 v_3 t_3^2 + t00^2 v_1 + \operatorname{Pinfty}_0 \operatorname{Qinfy}_0 v_1 + 2 \operatorname{Pinfty}_1 v_0 \operatorname{Qinfy}_0 \\
& \left. - t00 v_1 \right)
\end{aligned}$$

$$L\operatorname{Pinfty}_3 := -\frac{1}{\operatorname{Qinfy}_0} \left(-5 \operatorname{Qinfy}_0^2 \operatorname{Qinfy}_3^4 v_0 t_5^2 - 5 \operatorname{Qinfy}_0 \operatorname{Qinfy}_1 \operatorname{Qinfy}_3^4 v_1 t_5^2 \right.$$

$$\begin{aligned}
& -5 Q_{infy_0} Q_{infy_2} Q_{infy_3}^4 v_2 t_5^2 - 6 Q_{infy_0} Q_{infy_3}^5 v_3 t_5^2 + 12 Q_{infy_0}^2 Q_{infy_2} Q_{infy_3}^2 v_0 t_5^2 \\
& + 8 Q_{infy_0}^2 Q_{infy_3}^3 v_0 t_4 t_5 + 4 Q_{infy_0}^2 Q_{infy_3}^3 v_1 t_5^2 + 12 Q_{infy_0} Q_{infy_1} Q_{infy_2} Q_{infy_3}^2 v_1 t_5^2 \\
& + 8 Q_{infy_0} Q_{infy_1} Q_{infy_3}^3 v_1 t_4 t_5 + 4 Q_{infy_0} Q_{infy_1} Q_{infy_3}^3 v_2 t_5^2 + 12 Q_{infy_0} Q_{infy_2}^2 \\
& Q_{infy_3}^2 v_2 t_5^2 + 8 Q_{infy_0} Q_{infy_2} Q_{infy_3}^3 v_2 t_4 t_5 + 20 Q_{infy_0} Q_{infy_2} Q_{infy_3}^3 v_3 t_5^2 + 10 Q_{infy_0} \\
& Q_{infy_3}^4 v_3 t_4 t_5 - 6 Q_{infy_0}^2 Q_{infy_1} Q_{infy_3} v_0 t_5^2 - 3 Q_{infy_0}^2 Q_{infy_2}^2 v_0 t_5^2 - 12 \\
& Q_{infy_0}^2 Q_{infy_2} Q_{infy_3} v_0 t_4 t_5 - 6 Q_{infy_0}^2 Q_{infy_2} Q_{infy_3} v_1 t_5^2 - 6 Q_{infy_0}^2 Q_{infy_3}^2 v_0 t_3 t_5 - 3 \\
& Q_{infy_0}^2 Q_{infy_3}^2 v_0 t_4^2 - 6 Q_{infy_0}^2 Q_{infy_3}^2 v_1 t_4 t_5 - 3 Q_{infy_0}^2 Q_{infy_3}^2 v_2 t_5^2 - 6 Q_{infy_0} \\
& Q_{infy_1}^2 Q_{infy_3} v_1 t_5^2 - 3 Q_{infy_0} Q_{infy_1} Q_{infy_2}^2 v_1 t_5^2 \\
& - 12 Q_{infy_0} Q_{infy_1} Q_{infy_2} Q_{infy_3} v_1 t_4 t_5 - 12 Q_{infy_0} Q_{infy_1} Q_{infy_2} Q_{infy_3} v_2 t_5^2 \\
& - 6 Q_{infy_0} Q_{infy_1} Q_{infy_3}^2 v_1 t_3 t_5 - 3 Q_{infy_0} Q_{infy_1} Q_{infy_3}^2 v_1 t_4^2 - 6 Q_{infy_0} Q_{infy_1} \\
& Q_{infy_3}^2 v_2 t_4 t_5 - 12 Q_{infy_0} Q_{infy_1} Q_{infy_3}^2 v_3 t_5^2 - 3 Q_{infy_0} Q_{infy_2}^3 v_2 t_5^2 - 12 Q_{infy_0} \\
& Q_{infy_2}^2 Q_{infy_3} v_2 t_4 t_5 - 12 Q_{infy_0} Q_{infy_2}^2 Q_{infy_3} v_3 t_5^2 - 6 Q_{infy_0} Q_{infy_2} Q_{infy_3}^2 v_2 t_3 t_5 \\
& - 3 Q_{infy_0} Q_{infy_2} Q_{infy_3}^2 v_2 t_4^2 - 24 Q_{infy_0} Q_{infy_2} Q_{infy_3}^2 v_3 t_4 t_5 - 8 Q_{infy_0} Q_{infy_3}^3 v_3 t_3 t_5 \\
& - 4 Q_{infy_0} Q_{infy_3}^3 v_3 t_4^2 + 2 Q_{infy_0}^3 v_0 t_5^2 + 4 Q_{infy_0}^2 Q_{infy_1} v_0 t_4 t_5 + 4 Q_{infy_0}^2 Q_{infy_1} v_1 t_5^2 \\
& + 4 Q_{infy_0}^2 Q_{infy_2} v_0 t_3 t_5 + 2 Q_{infy_0}^2 Q_{infy_2} v_0 t_4^2 + 4 Q_{infy_0}^2 Q_{infy_2} v_1 t_4 t_5 + 4 \\
& Q_{infy_0}^2 Q_{infy_2} v_2 t_5^2 + 4 Q_{infy_0}^2 Q_{infy_3} v_0 t_2 t_5 + 4 Q_{infy_0}^2 Q_{infy_3} v_0 t_3 t_4 + 4 \\
& Q_{infy_0}^2 Q_{infy_3} v_1 t_3 t_5 + 2 Q_{infy_0}^2 Q_{infy_3} v_1 t_4^2 + 4 Q_{infy_0}^2 Q_{infy_3} v_2 t_4 t_5 + 6 \\
& Q_{infy_0}^2 Q_{infy_3} v_3 t_5^2 + 4 Q_{infy_0} Q_{infy_1}^2 v_1 t_4 t_5 + 2 Q_{infy_0} Q_{infy_1}^2 v_2 t_5^2 \\
& + 4 Q_{infy_0} Q_{infy_1} Q_{infy_2} v_1 t_3 t_5 + 2 Q_{infy_0} Q_{infy_1} Q_{infy_2} v_1 t_4^2 \\
& + 8 Q_{infy_0} Q_{infy_1} Q_{infy_2} v_2 t_4 t_5 + 6 Q_{infy_0} Q_{infy_1} Q_{infy_2} v_3 t_5^2 \\
& + 4 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_1 t_2 t_5 + 4 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_1 t_3 t_4 \\
& + 4 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_2 t_3 t_5 + 2 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_2 t_4^2 \\
& + 12 Q_{infy_0} Q_{infy_1} Q_{infy_3} v_3 t_4 t_5 + 4 Q_{infy_0} Q_{infy_2}^2 v_2 t_3 t_5 + 2 Q_{infy_0} Q_{infy_2}^2 v_2 t_4^2 \\
& + 6 Q_{infy_0} Q_{infy_2}^2 v_3 t_4 t_5 + 4 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_2 t_2 t_5 \\
& + 4 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_2 t_3 t_4 + 12 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_3 t_3 t_5 \\
& + 6 Q_{infy_0} Q_{infy_2} Q_{infy_3} v_3 t_4^2 + 6 Q_{infy_0} Q_{infy_3}^2 v_3 t_2 t_5 + 6 Q_{infy_0} Q_{infy_3}^2 v_3 t_3 t_4 - \\
& P_{infy_0}^2 Q_{infy_0}^2 v_2 - 2 P_{infy_0} P_{infy_1} Q_{infy_0}^2 v_1 - 2 P_{infy_0} P_{infy_2} Q_{infy_0}^2 v_0 - P_{infy_1}^2 \\
& Q_{infy_0}^2 v_0 - P_{infy_1}^2 Q_{infy_0} Q_{infy_1} v_1 + P_{infy_1}^2 Q_{infy_0} Q_{infy_2} v_2 \\
& + 2 P_{infy_1} P_{infy_2} Q_{infy_0} Q_{infy_2} v_1 + 4 P_{infy_1} P_{infy_2} Q_{infy_0} Q_{infy_3} v_2 + 2 \\
& P_{infy_2}^2 Q_{infy_0} Q_{infy_3} v_1 - 2 Q_{infy_0}^2 v_0 t_1 t_5 - 2 Q_{infy_0}^2 v_0 t_2 t_4 - Q_{infy_0}^2 v_0 t_3^2 - 2 \\
& Q_{infy_0}^2 v_1 t_2 t_5 - 2 Q_{infy_0}^2 v_1 t_3 t_4 - 2 Q_{infy_0}^2 v_2 t_3 t_5 - Q_{infy_0}^2 v_2 t_4^2 - 4 Q_{infy_0}^2 v_3 t_4 t_5 \\
& - 2 Q_{infy_0} Q_{infy_1} v_1 t_1 t_5 - 2 Q_{infy_0} Q_{infy_1} v_1 t_2 t_4 - Q_{infy_0} Q_{infy_1} v_1 t_3^2
\end{aligned}$$

$$\begin{aligned}
& -2 \text{Qinfy}_0 \text{Qinfy}_1 v_2 t_2 t_5 - 2 \text{Qinfy}_0 \text{Qinfy}_1 v_2 t_3 t_4 - 4 \text{Qinfy}_0 \text{Qinfy}_1 v_3 t_3 t_5 \\
& -2 \text{Qinfy}_0 \text{Qinfy}_1 v_3 t_4^2 - 2 \text{Qinfy}_0 \text{Qinfy}_2 v_2 t_1 t_5 - 2 \text{Qinfy}_0 \text{Qinfy}_2 v_2 t_2 t_4 \\
& - \text{Qinfy}_0 \text{Qinfy}_2 v_2 t_3^2 - 4 \text{Qinfy}_0 \text{Qinfy}_2 v_3 t_2 t_5 - 4 \text{Qinfy}_0 \text{Qinfy}_2 v_3 t_3 t_4 \\
& -4 \text{Qinfy}_0 \text{Qinfy}_3 v_3 t_1 t_5 - 4 \text{Qinfy}_0 \text{Qinfy}_3 v_3 t_2 t_4 - 2 \text{Qinfy}_0 \text{Qinfy}_3 v_3 t_3^2 \\
& + 2 \text{Pinfty}_1 \text{Pinfty}_2 \text{Qinfy}_0 v_3 + 2 \text{Pinfty}_1 \text{Pinfty}_3 \text{Qinfy}_0 v_2 + 2 \text{Pinfty}_2^2 \text{Qinfy}_0 v_2 \\
& + 2 \text{Pinfty}_2 \text{Pinfty}_3 \text{Qinfy}_0 v_1 + 2 \text{Qinfy}_0 v_3 t_0 t_5 + 2 \text{Qinfy}_0 v_3 t_1 t_4 + 2 \text{Qinfy}_0 v_3 t_2 t_3 \\
& + t00^2 v_2 + \text{Pinfty}_0 \text{Qinfy}_0 v_2 + 2 \text{Pinfty}_1 v_1 \text{Qinfy}_0 + 3 \text{Pinfty}_2 \text{Qinfy}_0 v_0 - t00 v_2)
\end{aligned}$$

$$EQ1 := 0$$

$$EQ2 := 0$$

$$EQ3 := 0$$

$$EQ4 := 0$$

$$EQ5 := 0$$

$$EQ6 := 0$$

$$EQ7 := 0$$

$$EQ8 := 0$$

$$EQ9 := 0$$

(8)

> series(simplify(series(CompatibilityEquations[2,1],lambda=infinity)),lambda=infinity);

$$\frac{(-8 t_5^2 v_0 + 2 \alpha_4 t_5) \lambda^3}{t_5}$$

(9)

$$\begin{aligned}
& + \frac{\left((8 \text{Qinfy}_3 v_0 - 6 v_1) t_5^2 + (-2 \text{Qinfy}_3 \alpha_4 - 14 v_0 t_4 + 2 \alpha_3) t_5 + 2 \alpha_4 t_4 \right) \lambda^2}{t_5} \\
& + \frac{1}{t_5} \left(\left((-8 \text{Qinfy}_3^2 v_0 + 8 \text{Qinfy}_2 v_0 + 6 \text{Qinfy}_3 v_1 - 4 v_2) t_5^2 + (2 \text{Qinfy}_3^2 \alpha_4 + (14 v_0 t_4 - 2 \alpha_3) \text{Qinfy}_3 - 2 \text{Qinfy}_2 \alpha_4 - 12 v_0 t_3 - 10 v_1 t_4 + 2 \alpha_2) t_5 - 2 \text{Qinfy}_3 \alpha_4 t_4 - 6 t_4^2 v_0 \right. \right. \\
& + 2 \alpha_3 t_4 + 2 \alpha_4 t_3) \lambda \left. \right) + \frac{1}{t_5} \left((8 \text{Qinfy}_3^3 v_0 - 6 v_1 \text{Qinfy}_3^2 + (-16 \text{Qinfy}_2 v_0 \right. \\
& + 4 v_2) \text{Qinfy}_3 + 8 \text{Qinfy}_1 v_0 + 6 \text{Qinfy}_2 v_1 - 2 v_3) t_5^2 + (-2 \text{Qinfy}_3^3 \alpha_4 + (-14 v_0 t_4 + 2 \alpha_3) \text{Qinfy}_3^2 + (4 \text{Qinfy}_2 \alpha_4 + 12 v_0 t_3 + 10 v_1 t_4 - 2 \alpha_2) \text{Qinfy}_3 + (14 \text{Qinfy}_2 v_0 - 6 v_2) t_4 - 2 \text{Qinfy}_2 \alpha_3 - 2 \alpha_4 \text{Qinfy}_1 - 10 v_0 t_2 - 8 v_1 t_3 + 2 \alpha_1) t_5 + 2 \text{Qinfy}_3^2 \alpha_4 t_4 \\
& + (6 t_4^2 v_0 - 2 \alpha_3 t_4 - 2 \alpha_4 t_3) \text{Qinfy}_3 - 4 t_4^2 v_1 + (-2 \text{Qinfy}_2 \alpha_4 - 10 v_0 t_3 + 2 \alpha_2) t_4 \\
& + 2 \alpha_3 t_3 + 2 \alpha_4 t_2) + \frac{1}{t_5 \lambda} \left((-8 v_0 \text{Qinfy}_3^4 + 6 \text{Qinfy}_3^3 v_1 + (24 \text{Qinfy}_2 v_0 - 4 v_2) \right. \\
& \text{Qinfy}_3^2 + (-16 \text{Qinfy}_1 v_0 - 12 \text{Qinfy}_2 v_1 + 2 v_3) \text{Qinfy}_3 + (-8 \text{Qinfy}_2^2 + 8 \text{Qinfy}_0) v_0 \\
& + 6 v_1 \text{Qinfy}_1 + 4 v_2 \text{Qinfy}_2) t_5^2 + (2 \text{Qinfy}_3^4 \alpha_4 + (14 v_0 t_4 - 2 \alpha_3) \text{Qinfy}_3^3 + (-6 \text{Qinfy}_2 \alpha_4 - 12 v_0 t_3 - 10 v_1 t_4 + 2 \alpha_2) \text{Qinfy}_3^2 + ((-28 \text{Qinfy}_2 v_0 + 6 v_2) t_4 \\
& + 4 \text{Qinfy}_2 \alpha_3 + 4 \alpha_4 \text{Qinfy}_1 + 10 v_0 t_2 + 8 v_1 t_3 - 2 \alpha_1) \text{Qinfy}_3 + (14 \text{Qinfy}_1 v_0
\end{aligned}$$

$$\begin{aligned}
& + 10 \, Qinfy_2 v_1 - 2 v_3) t_4 + (12 \, Qinfy_2 t_3 - 8 t_1) v_0 + (2 \, Qinfy_2^2 - 2 \, Qinfy_0) \alpha_4 \\
& - 2 \, Qinfy_2 \alpha_2 - 2 \alpha_3 \, Qinfy_1 - 6 v_1 t_2 - 4 v_2 t_3) t_5 - 2 \, Qinfy_3^3 \alpha_4 t_4 + (-6 t_4^2 v_0 + 2 \alpha_3 t_4 \\
& + 2 \alpha_4 t_3) \, Qinfy_3^2 + (4 t_4^2 v_1 + (4 \, Qinfy_2 \alpha_4 + 10 v_0 t_3 - 2 \alpha_2) t_4 - 2 \alpha_3 t_3 \\
& - 2 \alpha_4 t_2) \, Qinfy_3 + (6 \, Qinfy_2 v_0 - 2 v_2) t_4^2 + (-2 \alpha_4 \, Qinfy_1 - 2 \, Qinfy_2 \alpha_3 - 8 v_0 t_2 \\
& - 6 v_1 t_3 + 2 \alpha_1) t_4 - 4 t_3^2 v_0 + (-2 \, Qinfy_2 t_3 + 2 t_1) \alpha_4 + 2 \alpha_2 t_3 + 2 \alpha_3 t_2)
\end{aligned}$$

In order to solve the entry (2,1) of the compatibility equations, we need to express the nu's in terms of the vector \alpha.

```

> M:=Matrix(2*d,2*d,0):
  for i from 1 to 2*d do for j from 1 to i do
    M[i,j]:=t[2*d+1-i+j]:
  od: od:
M;
alphaVector:=Matrix(2*d,1,0):
for i from 1 to 2*d do alphaVector[i,1]:=alpha[2*d-i+1]/(2*d-i+1)
: od:
alphaVector;
nuVector:=Matrix(2*d,1,0):
for i from 1 to 2*d do nuVector[i,1]:=nu[i-1]: od:
nuVector;

```

$$\begin{bmatrix} t_5 & 0 & 0 & 0 \\ t_4 & t_5 & 0 & 0 \\ t_3 & t_4 & t_5 & 0 \\ t_2 & t_3 & t_4 & t_5 \end{bmatrix} \begin{bmatrix} \frac{\alpha_4}{4} \\ \frac{\alpha_3}{3} \\ \frac{\alpha_2}{2} \\ \alpha_1 \end{bmatrix}$$

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (10)$$

```
> nuVectorbis:=Multiply(M^(-1),alphaVector):
for k from 0 to 2*d-1 do nu[k]:=nuVectorbis[k+1,1]: od:
nu[0]:=nu[0];
nu[1]:=nu[1];
nu[2]:=nu[2];
nu[3]:=nu[3];
```

$$\begin{aligned} v_0 &:= \frac{\alpha_4}{4 t_5} \\ v_1 &:= -\frac{t_4 \alpha_4}{4 t_5^2} + \frac{\alpha_3}{3 t_5} \\ v_2 &:= -\frac{(t_3 t_5 - t_4^2) \alpha_4}{4 t_5^3} - \frac{t_4 \alpha_3}{3 t_5^2} + \frac{\alpha_2}{2 t_5} \\ v_3 &:= -\frac{(t_2 t_5^2 - 2 t_4 t_3 t_5 + t_4^3) \alpha_4}{4 t_5^4} - \frac{(t_3 t_5 - t_4^2) \alpha_3}{3 t_5^3} - \frac{t_4 \alpha_2}{2 t_5^2} + \frac{\alpha_1}{t_5} \end{aligned} \quad (11)$$

```
> simplify(CompatibilityEquations);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (12)$$

This verifies that the relation between nu's and alpha's is correct. We check now that the formulas for the Hamiltonians are correct.

```
> H01:=simplify(residue( checkL11^2+checkL12*checkL21+checkL12*diff
(checkL11/checkL12,lambda) , lambda=0) );
for j from 0 to 2*d-2 do
Hinfty[j]:=simplify(-residue(lambda^(-j-1)*(checkL11^2+checkL12*
checkL21+checkL12*diff(checkL11/checkL12,lambda)) , lambda=
infinity)): od:
Hinfty[0]:=Hinfty[0];
Hinfty[1]:=Hinfty[1];
Hinfty[2]:=Hinfty[2];
```

$$\begin{aligned} H01 := & \frac{1}{Qinfy_0} \left((2 Qinfy_3 t_5^2 - 2 t_4 t_5) Qinfy_0^3 + (-Qinfy_3^5 t_5^2 + 2 t_4 t_5 Qinfy_3^4 + (4 Qinfy_2 t_5^2 \right. \\ & - 2 t_3 t_5 - t_4^2) Qinfy_3^3 + (-3 t_5^2 Qinfy_1 + (-6 t_4 Qinfy_2 + 2 t_2) t_5 + 2 t_4 t_3) Qinfy_3^2 + (-3 \\ & Qinfy_2^2 t_5^2 + (4 Qinfy_1 t_4 + 4 Qinfy_2 t_3 - 2 t_1) t_5 + 2 t_4^2 Qinfy_2 - Pinfty_1^2 - 2 Pinfty_2 Pinfty_0 \end{aligned}$$

$$\begin{aligned}
& -2 t_4 t_2 - t_3^2) Qinfy_3 + 2 Qinfy_1 Qinfy_2 t_5^2 + (2 Qinfy_2^2 t_4 - 2 Qinfy_1 t_3 - 2 Qinfy_2 t_2 \\
& + 2 t_0) t_5 - t_4^2 Qinfy_1 + (-2 Qinfy_2 t_3 + 2 t_1) t_4 - 2 Pinfty_1 Qinfy_2 Pinfty_0 - Qinfy_1 Pinfty_0^2 \\
& - 2 Pinfty_1 Pinfty_2 - 2 Pinfty_3 Pinfty_0 + 2 t_3 t_2) Qinfy_0^2 + t00 (t00 - 1) Qinfy_1)
\end{aligned}$$

$$\begin{aligned}
Hinfy_0 := & \frac{1}{Qinfy_0} \left(-t_5^2 Qinfy_0^3 + (Qinfy_3^4 t_5^2 - 2 t_4 t_5 Qinfy_3^3 + (-3 Qinfy_2 t_5^2 + 2 t_3 t_5 + t_4^2) \right. \\
& Qinfy_3^2 + (4 t_5^2 Qinfy_1 + (4 t_4 Qinfy_2 - 2 t_2) t_5 - 2 Pinfty_1 Pinfty_0 - 2 t_4 t_3) Qinfy_3 \\
& - 4 t_4 Qinfy_1 t_5 + Qinfy_2^2 t_5^2 + (-2 Qinfy_2 t_3 + 2 t_1) t_5 + (-Pinfty_0^2 - t_4^2) Qinfy_2 - Pinfty_1^2 \\
& - 2 Pinfty_2 Pinfty_0 + 2 t_4 t_2 + t_3^2) Qinfy_0^2 + \left(-Qinfy_1 Qinfy_3^5 t_5^2 + 2 t_4 t_5 Qinfy_1 Qinfy_3^4 \right. \\
& + 4 \left(Qinfy_2 t_5^2 - \frac{1}{2} t_3 t_5 - \frac{1}{4} t_4^2 \right) Qinfy_1 Qinfy_3^3 + (-3 Qinfy_1^2 t_5^2 + ((-6 t_4 Qinfy_2 \\
& + 2 t_2) t_5 + 2 t_4 t_3) Qinfy_1 + Pinfty_2^2) Qinfy_3^2 + (4 t_4 t_5 Qinfy_1^2 + (-3 Qinfy_2^2 t_5^2 \\
& + (4 Qinfy_2 t_3 - 2 t_1) t_5 + 2 t_4^2 Qinfy_2 - Pinfty_1^2 - 2 t_4 t_2 - t_3^2) Qinfy_1 \\
& + 2 Pinfty_1 Pinfty_2 Qinfy_2 + 2 Pinfty_2 Pinfty_3 + 2 Pinfty_1) Qinfy_3 + (2 Qinfy_2 t_5^2 - 2 t_3 t_5 \\
& - t_4^2) Qinfy_1^2 + ((2 Qinfy_2^2 t_4 - 2 Qinfy_2 t_2 + 2 t_0) t_5 - 2 Qinfy_2 t_3 t_4 - 2 Pinfty_1 Pinfty_2 \\
& + 2 t_1 t_4 + 2 t_3 t_2) Qinfy_1 + Pinfty_1^2 Qinfy_2^2 + (2 Pinfty_1 Pinfty_3 + Pinfty_0) Qinfy_2 + Pinfty_3^2 \\
& \left. + 3 Pinfty_2) Qinfy_0 + t00 Qinfy_2 (t00 - 1) \right)
\end{aligned}$$

$$\begin{aligned}
Hinfy_1 := & \frac{1}{Qinfy_0} \left((-Qinfy_3^3 t_5^2 + 2 Qinfy_3^2 t_4 t_5 + (4 Qinfy_2 t_5^2 - Pinfty_0^2 - 2 t_3 t_5 - t_4^2) Qinfy_3 \right. \\
& - 2 t_5^2 Qinfy_1 + (-4 t_4 Qinfy_2 + 2 t_2) t_5 - 2 Pinfty_1 Pinfty_0 + 2 t_4 t_3) Qinfy_0^2 + (-Qinfy_2 \\
& Qinfy_3^5 t_5^2 + t_5 (Qinfy_1 t_5 + 2 t_4 Qinfy_2) Qinfy_3^4 + (4 Qinfy_2^2 t_5^2 + (-2 Qinfy_1 t_4 \\
& - 2 Qinfy_2 t_3) t_5 - t_4^2 Qinfy_2) Qinfy_3^3 + (-6 Qinfy_1 Qinfy_2 t_5^2 + (-6 Qinfy_2^2 t_4 \\
& + 2 Qinfy_1 t_3 + 2 Qinfy_2 t_2) t_5 + t_4^2 Qinfy_1 + 2 Qinfy_2 t_3 t_4 + 2 Pinfty_1 Pinfty_2) Qinfy_3^2 \\
& + ((-3 Qinfy_2^3 + 2 Qinfy_1^2) t_5^2 + (4 Qinfy_2^2 t_3 + (8 Qinfy_1 t_4 - 2 t_1) Qinfy_2 \\
& - 2 Qinfy_1 t_2) t_5 + 2 Qinfy_2^2 t_4^2 + (Pinfty_1^2 - 2 t_4 t_2 - t_3^2) Qinfy_2 - 2 t_4 t_3 Qinfy_1 \\
& + 2 Pinfty_1 Pinfty_3 + 2 Pinfty_2^2 + Pinfty_0) Qinfy_3 + 3 Qinfy_1 Qinfy_2^2 t_5^2 + (2 Qinfy_2^3 t_4 \\
& - 2 t_2 Qinfy_2^2 + (-4 Qinfy_1 t_3 + 2 t_0) Qinfy_2 - 2 Qinfy_1^2 t_4 + 2 Qinfy_1 t_1) t_5 - 2 t_3 t_4 \\
& Qinfy_2^2 + (-2 t_4^2 Qinfy_1 + 2 t_1 t_4 + 2 t_3 t_2) Qinfy_2 + 2 t_4 t_2 Qinfy_1 + (-Pinfty_1^2 + t_3^2) Qinfy_1 \\
& \left. + 2 Pinfty_2 Pinfty_3 + 2 Pinfty_1) Qinfy_0 + t00 Qinfy_3 (t00 - 1) \right)
\end{aligned}$$

$$\begin{aligned}
Hinfy_2 := & \frac{1}{Qinfy_0} \left((3 Qinfy_3^2 t_5^2 - 2 Qinfy_2 t_5^2 - 4 Qinfy_3 t_4 t_5 - Pinfty_0^2 + 2 t_3 t_5 + t_4^2) Qinfy_0^2 \right. \\
& + (-Qinfy_3^6 t_5^2 + 2 t_4 t_5 Qinfy_3^5 + (5 Qinfy_2 t_5^2 - 2 t_3 t_5 - t_4^2) Qinfy_3^4 + (-4 t_5^2 Qinfy_1 + (-8 t_4 Qinfy_2 \\
& + 2 t_2) t_5 + 2 t_4 t_3) Qinfy_3^3 + (-6 Qinfy_2^2 t_5^2 + (6 Qinfy_1 t_4 + 6 Qinfy_2 t_3 \\
& - 2 t_1) t_5 + 3 t_4^2 Qinfy_2 - 2 t_4 t_2 - t_3^2) Qinfy_3^2 + (6 Qinfy_1 Qinfy_2 t_5^2 + (6 Qinfy_2^2 t_4 \\
& - 4 Qinfy_1 t_3 - 4 Qinfy_2 t_2 + 2 t_0) t_5 - 2 t_4^2 Qinfy_1 - 4 Qinfy_2 t_3 t_4 + 2 Pinfty_1 Pinfty_2 \\
& \left. + 2 Pinfty_2 Pinfty_3 + 2 Pinfty_1) Qinfy_0 + t00 Qinfy_3 (t00 - 1) \right) \quad (13)
\end{aligned}$$

$$\begin{aligned}
& + 2 t_1 t_4 + 2 t_3 t_2) Q_{\text{infty}} y_3 + (Q_{\text{infty}} y_2^3 - Q_{\text{infty}} y_1^2) t_5^2 + (-2 Q_{\text{infty}} y_2^2 t_3 + (-4 Q_{\text{infty}} y_1 t_4 \\
& + 2 t_1) Q_{\text{infty}} y_2 + 2 Q_{\text{infty}} y_1 t_2) t_5 - Q_{\text{infty}} y_2^2 t_4^2 + (P_{\text{infty}} y_1^2 + 2 t_4 t_2 + t_3^2) Q_{\text{infty}} y_2 \\
& + 2 t_4 t_3 Q_{\text{infty}} y_1 + 2 P_{\text{infty}} y_1 P_{\text{infty}} y_3 + P_{\text{infty}} y_2^2 + P_{\text{infty}} y_0) Q_{\text{infty}} y_0 + t_{00}^2 - t_{00}
\end{aligned}$$

```

> HamGeneral:=0:
for k from 0 to 2*d-2 do HamGeneral:=HamGeneral+nu[k+1]*Hinfty[k]
: od:
HamGeneral:=HamGeneral+nu[0]*(H01+2*d*Pinfty[2*d-1]):
for k from 0 to 2*d-2 do HamGeneral:=HamGeneral+nu[0]*(k+1)*
Qinfty[k+1]*Pinfty[k]: od:
HamGeneral:=simplify(HamGeneral):

```

```

> simplify(LQinfty[0]-diff(HamGeneral,Pinfty[0]));
simplify(LQinfty[1]-diff(HamGeneral,Pinfty[1]));
simplify(LQinfty[2]-diff(HamGeneral,Pinfty[2]));
simplify(LQinfty[3]-diff(HamGeneral,Pinfty[3]));
simplify(LPPinfty[0]+diff(HamGeneral,Qinfty[0]));
simplify(LPPinfty[1]+diff(HamGeneral,Qinfty[1]));
simplify(LPPinfty[2]+diff(HamGeneral,Qinfty[2]));
simplify(LPPinfty[3]+diff(HamGeneral,Qinfty[3]));

```

0
0
0
0
0
0
0
0
0

(14)

```

> HamGeneralFunction:=unapply(HamGeneral,alpha[1],alpha[2],alpha
[3],alpha[4]):
Hamt1:=simplify(HamGeneralFunction(1,0,0,0)):
Hamt2:=simplify(HamGeneralFunction(0,1,0,0)):
Hamt3:=simplify(HamGeneralFunction(0,0,1,0)):
Hamt4:=simplify(HamGeneralFunction(0,0,0,1)):
VectorHam:=Matrix(2*d,1,0):
VectorHam[1,1]:=1*Hamt1:
VectorHam[2,1]:=2*Hamt2:
VectorHam[3,1]:=3*Hamt3:
VectorHam[4,1]:=4*Hamt4:
HVector:=Matrix(2*d,1,0):
for k from 1 to 2*d-1 do
HVector[k,1]:=Hinfty[2*d-1-k]: od:
HVector[2*d,1]:=H01+2*d*Pinfty[2*d-1]:
for k from 0 to 2*d-2 do HVector[2*d,1]:=HVector[2*d,1]+(k+1)*

```

```

Qinfy[k+1]*Pinfy[k]: od:
HVector[2*d,1]:=simplify(HVector[2*d,1]):

simplify(VectorHam-Multiply(M^(-1),HVector));

```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(15)