

In this Maple file, we compute the Lax matrices and the Hamiltonian system for $d=1$.

```
> restart:
with(LinearAlgebra):
d:=1:
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Formulas for the member of the Painlevé IV hierarchy and symmetry reduction in the $(Q_{-\infty}, Q_{\{0,1\}}, P_{-\infty}, P_{\{0,1\}})$ variables

Definition of \check{L} in the $(Q_{-\infty}, Q_{\{0,1\}}, P_{-\infty}, P_{\{0,1\}})$ variables. We omit the check for compactness.

```
> L12:=Q01/lambda+lambda^(2*d-1):
for k from 0 to 2*d-2 do L12:=L12+ Qinfy[k]*lambda^k: od:
L12:=L12*omega:
L11:=P01*Q01/lambda:
for k from 0 to 2*d-2 do L11:=L11-Pinfy[2*d-2-k]*lambda^k: od:
L11:=L11- (tinfy[2*d+1]*lambda+tinfy[2*d]-tinfy[2*d+1]*
Qinfy[2*d-2])*(L12/omega):
L11:=simplify(L11):
series(L11,lambda=infinity,20):
```

$$L12 := \left(\frac{Q01}{\lambda} + \lambda + Qinfy_0 \right) \omega \quad (1.1)$$

$$-tinfy_3 \lambda^2 - tinfty_2 \lambda + (Qinfy_0^2 - Q01) tinfty_3 - Qinfy_0 tinfty_2 - Pinfy_0$$

$$+ \frac{Q01 (Qinfy_0 tinfty_3 + P01 - tinfty_2)}{\lambda}$$

```
> series(simplify(series((t00^2/lambda^2- L11^2)/(L12/omega),
lambda=0,3)),lambda=0,1);
factor(residue((t00^2/lambda^2- L11^2)/(L12/omega),lambda=0));
ResidueBis:=1/omega*(t00^2/Q01-Q01*(P01-tinfy[2*d]+tinfy[2*
d+1]*Qinfy[2*d-2])^2);
simplify(factor(residue((t00^2/lambda^2- L11^2)/L12,lambda=0))
- ResidueBis);
```

$$-\frac{1}{\lambda} \frac{1}{Q01} ((Q01 (Qinfy_0 tinfty_3 + P01 - tinfty_2) - t00) (Q01 (Qinfy_0 tinfty_3 + P01$$

$$- tinfty_2) + t00)) + \frac{1}{Q01^2} (2 tinfty_3 (Qinfy_0 tinfty_3 + P01 - tinfty_2) Q01^3$$

$$+ (Qinfy_0 tinfty_3 + P01 - tinfty_2) (-Qinfy_0^2 tinfty_3 + (P01 + tinfty_2) Qinfy_0$$

$$+ 2 Pinfy_0) Q01^2 - t00^2 Qinfy_0) + O(\lambda)$$

$$- \frac{1}{Q01} ((Q01 Qinfy_0 tinfty_3 + P01 Q01 - Q01 tinfty_2 - t00) (Q01 Qinfy_0 tinfty_3$$

$$+ P01 Q01 - Q01 tinfty_2 + t00)) \quad (1.2)$$

$$ResidueBis := \frac{\frac{t00^2}{Q01} - Q01 (Qinfy_0 tinfy_3 + P01 - tinfy_2)^2}{\omega}$$

```

> L21infty:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21infty:=
L21infty+ tinfy[2*d+1-m]*tinfy[j+m-2*d+1]*lambda^j: od: od:
Numm:=simplify((L21infty-L11^2)/(L12/omega)):
L21:=0:
for j from 1 to 60 do L21:=L21- 1/omega*simplify(residue(Numm*
lambda^(-j),lambda=infinity))*lambda^(j-1): od:
L21:=simplify(L21+ (t00^2/Q01-Q01*(Qinfy[2*d-2]*tinfy[2*d+1]+
P01-tinfy[2*d])^2)/omega/lambda):

```

```

> series(simplify(series(L21,lambda=infinity,2)),lambda=infinity,
2):
CoefflambdaL21:=-factor(residue(L21/lambda^2,lambda=infinity));
Coefflambda0L21:=-factor(residue(L21/lambda,lambda=infinity));
Coefflambda0L21bis:=1/omega*( 2*tinfy[3]*Q01*P01+2*(tinfy[3]*
Qinfy[0]-tinfy[2])*Pinfy[0]+4*(tinfy[3]^2*Qinfy[0]-tinfy
[2]*tinfy[3])*Q01-2*tinfy[3]^2*Qinfy[0]^3+4*tinfy[3]*tinfy
[2]*Qinfy[0]^2-2*(tinfy[2]^2+tinfy[3]*tinfy[1])*Qinfy[0]
+2*tinfy[0]*tinfy[3]+2*tinfy[1]*tinfy[2] );
simplify(series(Coefflambda0L21-Coefflambda0L21bis,P01));
CoefflambdaMinus1L21:=-factor(residue(L21,lambda=infinity)):

```

CoefflambdaL21 :=

$$\frac{2 tinfy_3 (-Qinfy_0^2 tinfy_3 + Q01 tinfy_3 + Qinfy_0 tinfy_2 + Pinfy_0 - tinfy_1)}{\omega}$$

$$Coefflambda0L21 := \frac{1}{\omega} \left(2 \left(-Qinfy_0^3 tinfy_3^2 + 2 Q01 Qinfy_0 tinfy_3^2 + 2 Qinfy_0^2 tinfy_2 tinfy_3 + P01 Q01 tinfy_3 - 2 Q01 tinfy_2 tinfy_3 + Pinfy_0 Qinfy_0 tinfy_3 - Qinfy_0 tinfy_1 tinfy_3 - Qinfy_0 tinfy_2^2 - Pinfy_0 tinfy_2 + tinfy_0 tinfy_3 + tinfy_1 tinfy_2 \right) \right)$$

$$Coefflambda0L21bis := \frac{1}{\omega} \left(2 P01 Q01 tinfy_3 + 2 (Qinfy_0 tinfy_3 - tinfy_2) Pinfy_0 + 4 (Qinfy_0 tinfy_3^2 - tinfy_2 tinfy_3) Q01 - 2 Qinfy_0^3 tinfy_3^2 + 4 Qinfy_0^2 tinfy_2 tinfy_3 - 2 (tinfy_1 tinfy_3 + tinfy_2^2) Qinfy_0 + 2 tinfy_0 tinfy_3 + 2 tinfy_1 tinfy_2 \right)$$

Definition of the auxiliary matrix in the (Q,P) variables.

```

> A12:=0:
for i from 0 to 2*d-1 do A12:=A12+nu[i]*lambda^(2*d-1-i): od:

```

(1.3)

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for k from 0 to 2*d-2 do for i from 0 to k do A12:=A12+Qinfy
[k]*nu[i]*lambda^(k-i) od: od:
A12:=omega*A12;
cinfy0:= 1/2/omega*Lomega+ tinfy[2*d+1]*(Q01*nu[0]+nu[1]*
Qinfy[0]):
A11:=cinfy0:
PolA12:=0:
for i from 0 to 2*d-1 do PolA12:=PolA12+nu[i]*lambda^(-i): od:
PolA12:=PolA12*L11:
for k from 0 to 60 do A11:=A11-residue(PolA12/lambda^(k+1),
lambda=infinity)*lambda^k od:
A11:=A11;
A22:=-A11:

A21:=(-tinfy[2*d+1]*Lomega/omega^2) *lambda- (tinfy[2*d]-
tinfy[2*d+1]*Qinfy[2*d-2])*Lomega/omega^2+ (alpha[2*d]-tinfy
[2*d+1]*LQinfy0)/omega -tinfy[2*d+1]*nu[0]/omega:
L21oper:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do
L21oper:=L21oper+tinfy[2*d+1-m]*tinfy[j+m-2*d+1]*lambda^j od:
od:
L21oper:=L21oper;
A12oper:=0:
for i from 0 to 2*d+1 do A12oper:=A12oper+nu[i]* lambda^(-i):
od:
A12oper:=A12oper;
for k from 0 to 60 do A21:=A21-residue( (L21oper*A12oper/(L12))
/lambda^(k+1), lambda=infinity)*lambda^k od:
for k from 0 to 60 do A21:=A21-residue( ((L11/(L12))* (L11*
A12/L12 -2*A11))/lambda^(k+1), lambda=infinity)*lambda^k od:
nu[2*d]:=- (Q01*nu[0]+Qinfy[0]*nu[1]);
Testnu2:=-nu[0]*Q01:
nu[2*d+1]:=Q01*Qinfy[0]*nu[0]+Qinfy[0]^2*nu[1]-Q01*nu[1]:
Testnu3:=-nu[1]*Q01:
for j from 2 to 2*d do Testnu3:=Testnu3- nu[j]*Qinfy[j-2]: od:
simplify(nu[2*d+1]-Testnu3);
for j from 1 to 2*d-1 do Testnu2:=Testnu2- nu[j]*Qinfy[j-1]:
od:
simplify(nu[2*d]-Testnu2);
A21:=series(simplify(series(simplify(A21), lambda=infinity)),
lambda=infinity);

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$$A12 := \omega (\lambda v_0 + Qinfy_0 v_0 + v_1)$$

(1.4)

$$A11 := \frac{1}{2} \frac{L\omega}{\omega} + \text{tinfty}_3 (Q01 v_0 + Q\text{infty}_0 v_1) + v_0 ((Q\text{infty}_0^2 - Q01) \text{tinfty}_3 - Q\text{infty}_0 \text{tinfty}_2 - P\text{infty}_0) - v_1 \text{tinfty}_2 - (v_0 \text{tinfty}_2 + v_1 \text{tinfty}_3) \lambda - v_0 \text{tinfty}_3 \lambda^2$$

$$L21\text{oper} := \lambda^4 \text{tinfty}_3^2 + 2 \lambda^3 \text{tinfty}_2 \text{tinfty}_3 + 2 \lambda^2 \text{tinfty}_1 \text{tinfty}_3 + \lambda^2 \text{tinfty}_2^2 + 2 \lambda \text{tinfty}_0 \text{tinfty}_3 + 2 \lambda \text{tinfty}_1 \text{tinfty}_2$$

$$A12\text{oper} := v_0 + \frac{v_1}{\lambda} + \frac{v_2}{\lambda^2} + \frac{v_3}{\lambda^3}$$

$$v_2 := -Q01 v_0 - Q\text{infty}_0 v_1$$

$$0$$

$$0$$

$$A21 := \frac{1}{\omega} ((2 Q\text{infty}_0^2 - 2 Q01) v_0 \text{tinfty}_3^2 + (-2 Q\text{infty}_0 \text{tinfty}_2 - 2 P\text{infty}_0 + 2 \text{tinfty}_1) v_0 \text{tinfty}_3) \lambda + \frac{1}{\omega} ((-2 Q\text{infty}_0^3 + 4 Q01 Q\text{infty}_0) v_0 - 2 v_1 (-Q\text{infty}_0^2 + Q01)) \text{tinfty}_3^2 + ((4 Q\text{infty}_0^2 \text{tinfty}_2 + (2 P\text{infty}_0 - 2 \text{tinfty}_1) Q\text{infty}_0 - 4 Q01 \text{tinfty}_2 + 2 \text{tinfty}_0 - 1) v_0 - 2 \text{tinfty}_2 v_1 Q\text{infty}_0 + 2 \text{tinfty}_1 v_1 - LQ\text{infty}_0) \text{tinfty}_3 + 2 \text{tinfty}_2 (-Q\text{infty}_0 \text{tinfty}_2 - P\text{infty}_0 + \text{tinfty}_1) v_0 + \alpha_2$$

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> CoeffLambda1A21:=factor(-residue(A21/lambda^2,lambda=infinity))
;
CoeffLambda0A21:=simplify(-residue(A21/lambda,lambda=infinity))
:
CoeffLambda0A21bis:=1/omega*(-tinfty[3]*LQinfty0 -2*tinfty[3]*
(-Qinfty[0]^2+Q01)*tinfty[3]+Qinfty[0]*tinfty[2]-tinfty[1])*nu
[1] +alpha[2] +4*tinfty[3]*nu[0]*(Qinfty[0]*tinfty[3]-tinfty
[2])*Q01-2*Qinfty[0]^3*nu[0]*tinfty[3]^2+4*Qinfty[0]^2*nu[0]*
tinfty[2]*tinfty[3]+2*nu[0]*(Qinfty[0]*tinfty[3]-tinfty[2])*
Pinfty[0]+nu[0]*(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-
tinfty[3])-2*nu[0]*(tinfty[1]*tinfty[3]+tinfty[2]^2)*Qinfty[0])
;
simplify(series(omega*(CoeffLambda0A21-CoeffLambda0A21bis),
Qinfty[0]));

```

CoeffLambda1A21 :=

$$\frac{2 v_0 \text{tinfty}_3 (-Q\text{infty}_0^2 \text{tinfty}_3 + Q01 \text{tinfty}_3 + Q\text{infty}_0 \text{tinfty}_2 + P\text{infty}_0 - \text{tinfty}_1)}{\omega}$$

$$CoeffLambda0A21bis := \frac{1}{\omega} (-LQ\text{infty}_0 \text{tinfty}_3 - 2 \text{tinfty}_3 ((-Q\text{infty}_0^2 + Q01) \text{tinfty}_3 + Q\text{infty}_0 \text{tinfty}_2 - \text{tinfty}_1) v_1 + \alpha_2 + 4 \text{tinfty}_3 v_0 (Q\text{infty}_0 \text{tinfty}_3 - \text{tinfty}_2) Q01 - 2 Q\text{infty}_0^3 v_0 \text{tinfty}_3^2 + 4 Q\text{infty}_0^2 v_0 \text{tinfty}_2 \text{tinfty}_3 + 2 v_0 (Q\text{infty}_0 \text{tinfty}_3 - \text{tinfty}_2) P\text{infty}_0 + v_0 (2 \text{tinfty}_0 \text{tinfty}_3 + 2 \text{tinfty}_1 \text{tinfty}_2 - \text{tinfty}_3) - 2 v_0 (\text{tinfty}_1 \text{tinfty}_3 +$$

(1.5)

$t_{\infty}^2) Q_{\infty 0}$

0

Compatibility equation and evolutions

```
> L:=Matrix(2,2,0):
L[1,1]:=L11:
L[1,2]:=L12:
L[2,1]:=L21:
L[2,2]:=-L[1,1]:
A:=Matrix(2,2,0):
A[1,1]:=A11:
A[1,2]:=A12:
A[2,1]:=A21:
A[2,2]:=-A[1,1]:

> dAdlambda:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do dAdlambda[i,j]:=diff
(A[i,j],lambda): od: od:
dAdlambda:

> BigLL:=Matrix(2,2,0):
BigLL[1,1]:= diff(L[1,1],Qinfty[0])*LQinfty0+ diff(L[1,1],Q01)*
LQ01+diff(L[1,1],Pinfty[0])*LPinfty0+ diff(L[1,1],P01)*
LP01+diff(L[1,1],tinfy[2])*alpha[2] +diff(L[1,1],tinfy[1])*
alpha[1]+ diff(L[1,1],omega)*Lomega:
BigLL[1,2]:= diff(L[1,2],Qinfty[0])*LQinfty0+ diff(L[1,2],Q01)*
LQ01+diff(L[1,2],Pinfty[0])*LPinfty0+ diff(L[1,2],P01)*LP01+
diff(L[1,2],tinfy[2])*alpha[2] +diff(L[1,2],tinfy[1])*alpha
[1]+ diff(L[1,2],omega)*Lomega:
BigLL[2,1]:= diff(L[2,1],Qinfty[0])*LQinfty0+ diff(L[2,1],Q01)*
LQ01+diff(L[2,1],Pinfty[0])*LPinfty0+ diff(L[2,1],P01)*LP01+
diff(L[2,1],tinfy[2])*alpha[2] +diff(L[2,1],tinfy[1])*alpha
[1]+ diff(L[2,1],omega)*Lomega:
BigLL[2,2]:=-BigLL[1,1]:

> Commutator:=Multiply(A,L)-Multiply(L,A):
CompatibilityEquation:= simplify(dAdlambda-BigLL+Commutator):

> series(CompatibilityEquation[1,2], lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
lambda=infinity));
EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
lambda=infinity));
EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
infinity));
EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
infinity));
```

```

series(simplify(series(CompatibilityEquation[1,1], lambda=
infinity)), lambda=infinity);
EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
lambda=infinity));
EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
lambda=infinity));
EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity));
EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda, lambda=
infinity));
EQ7:=simplify(residue(CompatibilityEquation[1,1], lambda=
infinity));

```

(1.6)

$$\begin{aligned}
EQ0 &:= 0 \\
EQ1 &:= 0 \\
EQ2 &:= (2 P01 Q01 v_0 - 2 Pinfty_0 v_1 + LQinfty0 - v_0) \omega \\
EQ3 &:= (2 P01 Q01 Qinfty_0 v_0 + 2 P01 Q01 v_1 + 2 Q01 Pinfty_0 v_0 + LQ01) \omega \\
EQ4 &:= 0 \\
EQ5 &:= -tinfty_3 (2 P01 Q01 v_0 - 2 Pinfty_0 v_1 + LQinfty0 - v_0) \\
EQ6 &:= \frac{1}{Q01} \left(-2 v_0 tinfty_3^2 Q01^3 + \left((3 Qinfty_0^2 v_0 - 6 Qinfty_0 v_1) tinfty_3^2 + \left((-2 P01 \right. \right. \right. \\
&\quad \left. \left. + 4 tinfty_2) v_1 - 4 \left(Qinfty_0 tinfty_2 - \frac{1}{2} tinfty_1 + \frac{1}{2} Pinfty_0 \right) v_0 \right) tinfty_3 + v_0 (P01 \right. \\
&\quad \left. - tinfty_2)^2 \right) Q01^2 + (4 v_1 Qinfty_0^3 tinfty_3^2 + ((-6 Qinfty_0^2 tinfty_2 + (-2 Pinfty_0 \\
&\quad + 4 tinfty_1) Qinfty_0 - 2 tinfty_0 + 1) v_1 - Qinfty_0 v_0 + Qinfty_0 LQinfty0 - LQ01) tinfty_3 \\
&\quad + 2 tinfty_2 (Qinfty_0 tinfty_2 + Pinfty_0 - tinfty_1) v_1 + v_0 tinfty_2 - tinfty_2 LQinfty0 \\
&\quad \left. - LPinfty_0) Q01 - t00^2 v_0 \right) \\
EQ7 &:= \frac{1}{Q01} \left(((4 Qinfty_0 v_0 - 2 v_1) tinfty_3^2 - 4 tinfty_2 v_0 tinfty_3) Q01^3 + ((-Qinfty_0^3 v_0 \right. \\
&\quad \left. + 3 Qinfty_0^2 v_1) tinfty_3^2 + ((2 P01 + 2 tinfty_2) Qinfty_0^2 + (2 Pinfty_0 - 2 tinfty_1) Qinfty_0 \right. \\
&\quad \left. + 2 tinfty_0 - 1) v_0 + 2 v_1 ((P01 - 2 tinfty_2) Qinfty_0 + tinfty_1) \right) tinfty_3 + ((P01^2 \\
&\quad - 2 P01 tinfty_2 - tinfty_2^2) Qinfty_0 + 2 tinfty_2 (-Pinfty_0 + tinfty_1)) v_0 + P01^2 v_1 \\
&\quad - 2 P01 v_1 tinfty_2 + v_1 tinfty_2^2 + LP01) Q01^2 + LQ01 (Qinfty_0 tinfty_3 + P01 \\
&\quad \left. - tinfty_2) Q01 - t00^2 (Qinfty_0 v_0 + v_1) \right)
\end{aligned}$$

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> LQinfty0:=solve(EQ2, LQinfty0);

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LQ01:=solve(EQ3, LQ01);

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$$LQinfty0 := -2 P01 Q01 v_0 + 2 Pinfty_0 v_1 + v_0$$

(1.7)

$$LQ01 := -2 Q01 (P01 Qinfty_0 v_0 + P01 v_1 + Pinfty_0 v_0)$$

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> simplify(EQ5);

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simplify(EQ6) ;

simplify(EQ7) ;

0

(1.8)

$$\begin{aligned} & \frac{1}{QO1} \left(-2 v_0 \text{tinfty}_3^2 QO1^3 + \left((3 Q\text{infty}_0^2 v_0 - 6 Q\text{infty}_0 v_1) \text{tinfty}_3^2 + \left(4 v_1 \text{tinfty}_2 \right. \right. \right. \\ & \quad \left. \left. - 4 \left(Q\text{infty}_0 \text{tinfty}_2 - \frac{1}{2} \text{tinfty}_1 \right) v_0 \right) \text{tinfty}_3 + v_0 (POI^2 + \text{tinfty}_2^2) \right) QO1^2 + \left(4 v_1 \right. \\ & \quad \left. Q\text{infty}_0^3 \text{tinfty}_3^2 - 6 v_1 \left(Q\text{infty}_0^2 \text{tinfty}_2 - \frac{2}{3} Q\text{infty}_0 \text{tinfty}_1 + \frac{1}{3} \text{tinfty}_0 - \frac{1}{6} \right) \text{tinfty}_3 \right. \\ & \quad \left. + (2 Q\text{infty}_0 \text{tinfty}_2^2 - 2 \text{tinfty}_1 \text{tinfty}_2) v_1 - LP\text{infty}0 \right) QO1 - t00^2 v_0 \Big) \\ & \frac{1}{QO1} \left(\left((4 Q\text{infty}_0 \text{tinfty}_3^2 - 4 \text{tinfty}_2 \text{tinfty}_3) v_0 - 2 \text{tinfty}_3^2 v_1 \right) QO1^3 + \left((-Q\text{infty}_0^3 \text{tinfty}_3^2 \right. \right. \\ & \quad \left. \left. + (2 Q\text{infty}_0^2 \text{tinfty}_2 - 2 Q\text{infty}_0 \text{tinfty}_1 + 2 \text{tinfty}_0 - 1) \text{tinfty}_3 + (-POI^2 - \right. \right. \\ & \quad \left. \left. \text{tinfty}_2^2) Q\text{infty}_0 - 2 POI Pinfty_0 + 2 \text{tinfty}_1 \text{tinfty}_2 \right) v_0 + 3 \text{tinfty}_3^2 v_1 Q\text{infty}_0^2 + 2 v_1 \left(\right. \right. \\ & \quad \left. \left. - 2 Q\text{infty}_0 \text{tinfty}_2 + \text{tinfty}_1 \right) \text{tinfty}_3 + (-POI^2 + \text{tinfty}_2^2) v_1 + LP01 \right) QO1^2 \\ & \quad \left. - t00^2 (Q\text{infty}_0 v_0 + v_1) \right) \end{aligned}$$

> LPinfty0:=solve(EQ6,LPinfty0) ;

LP01:=solve(EQ7,LP01) ;

$$\begin{aligned} LP\text{infty}0 := & \frac{1}{QO1} \left(3 QO1^2 Q\text{infty}_0^2 v_0 \text{tinfty}_3^2 + 4 QO1 Q\text{infty}_0^3 v_1 \text{tinfty}_3^2 - 2 QO1^3 v_0 \text{tinfty}_3^2 \right. \\ & \quad \left. - 4 QO1^2 Q\text{infty}_0 v_0 \text{tinfty}_2 \text{tinfty}_3 - 6 QO1^2 Q\text{infty}_0 v_1 \text{tinfty}_3^2 - 6 QO1 \right. \\ & \quad \left. Q\text{infty}_0^2 v_1 \text{tinfty}_2 \text{tinfty}_3 + POI^2 QO1^2 v_0 + 2 QO1^2 v_0 \text{tinfty}_1 \text{tinfty}_3 + QO1^2 v_0 \text{tinfty}_2^2 \right. \\ & \quad \left. + 4 QO1^2 v_1 \text{tinfty}_2 \text{tinfty}_3 + 4 QO1 Q\text{infty}_0 v_1 \text{tinfty}_1 \text{tinfty}_3 + 2 QO1 Q\text{infty}_0 v_1 \text{tinfty}_2^2 \right. \\ & \quad \left. - 2 QO1 v_1 \text{tinfty}_0 \text{tinfty}_3 - 2 QO1 v_1 \text{tinfty}_1 \text{tinfty}_2 + QO1 v_1 \text{tinfty}_3 - t00^2 v_0 \right) \end{aligned}$$

(1.9)

$$\begin{aligned} LP01 := & \frac{1}{QO1^2} \left(QO1^2 Q\text{infty}_0^3 v_0 \text{tinfty}_3^2 - 4 QO1^3 Q\text{infty}_0 v_0 \text{tinfty}_3^2 - 2 QO1^2 \right. \\ & \quad \left. Q\text{infty}_0^2 v_0 \text{tinfty}_2 \text{tinfty}_3 - 3 QO1^2 Q\text{infty}_0^2 v_1 \text{tinfty}_3^2 + POI^2 QO1^2 Q\text{infty}_0 v_0 \right. \\ & \quad \left. + 4 QO1^3 v_0 \text{tinfty}_2 \text{tinfty}_3 + 2 QO1^3 v_1 \text{tinfty}_3^2 + 2 QO1^2 Q\text{infty}_0 v_0 \text{tinfty}_1 \text{tinfty}_3 \right. \\ & \quad \left. + QO1^2 Q\text{infty}_0 v_0 \text{tinfty}_2^2 + 4 QO1^2 Q\text{infty}_0 v_1 \text{tinfty}_2 \text{tinfty}_3 + POI^2 QO1^2 v_1 \right. \\ & \quad \left. + 2 POI QO1^2 Pinfty_0 v_0 - 2 QO1^2 v_0 \text{tinfty}_0 \text{tinfty}_3 - 2 QO1^2 v_0 \text{tinfty}_1 \text{tinfty}_2 \right. \\ & \quad \left. - 2 QO1^2 v_1 \text{tinfty}_1 \text{tinfty}_3 - QO1^2 v_1 \text{tinfty}_2^2 + QO1^2 v_0 \text{tinfty}_3 + t00^2 Q\text{infty}_0 v_0 + t00^2 v_1 \right) \end{aligned}$$

> Ham:=-2*P01*Q01*Pinfty[0]*nu[0]+Pinfty[0]^2*nu[1]+Pinfty[0]*nu[0]-P01^2*Q01*(Qinfty[0]*nu[0]+nu[1])

-((-2*nu[0]*tinfty[3]^2*Q01^3+(2*nu[0]*tinfty[1]+4*nu[1]*tinfty[2])*tinfty[3]+nu[0]*tinfty[2]^2)*Q01^2-2*nu[1]*((tinfty[0]-1/2)*tinfty[3]+tinfty[1]*tinfty[2])*Q01-t00^2*nu[0])*Qinfty[0]/Q01+(1/2)*(-6*Q01*tinfty[3]^2+4*tinfty[1]*tinfty[3]+2*tinfty[2]^2)*nu[1]-2*Q01*nu[0]*tinfty[2]*tinfty[3])*Qinfty[0]^2+tinfty[3]*(Q01*nu[0]*tinfty[3]-2*nu[1]*tinfty[2])*Qinfty[0]

```

^3+tinfy[3]^2*nu[1]*Qinfy[0]^4)
-(2*Q01^2*nu[0]*tinfy[2]*tinfy[3]+Q01^2*nu[1]*tinfy[3]^2-2*
Q01*nu[0]*tinfy[0]*tinfy[3]-2*Q01*nu[0]*tinfy[1]*tinfy[2]
-2*Q01*nu[1]*tinfy[1]*tinfy[3]-Q01*nu[1]*tinfy[2]^2+nu[0]*
tinfy[3]*Q01-t00^2*nu[1]/Q01);

```

```

simplify(diff(Ham,Pinfty[0])-LQinfy0);
simplify(diff(Ham,P01)-LQ01);
simplify(diff(Ham,Qinfy[0])+LPinfy0);
simplify(diff(Ham,Q01)+LP01);

```

$$Ham := -2 P01 Q01 Pinfty_0 v_0 + Pinfty_0^2 v_1 + v_0 Pinfty_0 - P01^2 Q01 (Qinfy_0 v_0 + v_1) \quad (1.10)$$

$$\begin{aligned}
& - \frac{1}{Q01} \left(\left(-2 v_0 tinfty_3^2 Q01^3 + \left((2 v_0 tinfty_1 + 4 v_1 tinfty_2) tinfty_3 + v_0 tinfty_2^2 \right) Q01^2 \right. \right. \\
& \left. \left. - 2 v_1 \left(\left(tinfty_0 - \frac{1}{2} \right) tinfty_3 + tinfty_1 tinfty_2 \right) Q01 - t00^2 v_0 \right) Qinfy_0 \right) - \left(\frac{1}{2} \left(\right. \right. \\
& \left. \left. - 6 Q01 tinfty_3^2 + 4 tinfty_1 tinfty_3 + 2 tinfty_2^2 \right) v_1 - 2 Q01 v_0 tinfty_2 tinfty_3 \right) Qinfy_0^2 \\
& - tinfty_3 \left(Q01 v_0 tinfty_3 - 2 v_1 tinfty_2 \right) Qinfy_0^3 - tinfty_3^2 v_1 Qinfy_0^4 \\
& - 2 Q01^2 v_0 tinfty_2 tinfty_3 - Q01^2 v_1 tinfty_3^2 + 2 Q01 v_0 tinfty_0 tinfty_3 \\
& + 2 Q01 v_0 tinfty_1 tinfty_2 + 2 Q01 v_1 tinfty_1 tinfty_3 + Q01 v_1 tinfty_2^2 - v_0 tinfty_3 Q01 \\
& + \frac{t00^2 v_1}{Q01}
\end{aligned}$$

0
0
0
0

```

> Hambis := -(Qinfy[0]*nu[0]+nu[1])*Q01*P01^2-2*nu[0]*Q01*P01*
Pinfty[0]+nu[0]*Pinfty[0]+nu[1]*(Pinfty[0])^2+t00^2*(Qinfy[0]*
nu[0]+nu[1])/Q01
+2*nu[0]*tinfy[3]^2*Qinfy[0]*Q01^2 - (Qinfy[0]^3*tinfy[3]^2+
tinfy[2]^2*Qinfy[0])*nu[0]*Q01 - (2*nu[0]*tinfy[2]*tinfy[3]+
nu[1]*tinfy[3]^2)*Q01^2+ 2*tinfy[2]*tinfy[3]*nu[0]*Q01*
Qinfy[0]^2+3*tinfy[3]^2*nu[1]*Qinfy[0]^2+(-2*tinfy[3]*(nu
[0]*tinfy[1]+2*nu[1]*tinfy[2]))*Q01*Qinfy[0]+3*Qinfy[0]^2*
nu[1]*tinfy[3]^2*Q01
-(3*(tinfy[3]^2)+2*tinfy[3]*tinfy[1]+tinfy[2]^2)*nu[1]*
Qinfy[0]^2+nu[1]*(2*tinfy[0]*tinfy[3]+2*tinfy[1]*tinfy[2]-
tinfy[3])*Qinfy[0]+((2*tinfy[1]*nu[1]+nu[0]*(2*tinfy[0]-1))
*tinfy[3]+2*nu[0]*tinfy[1]*tinfy[2]+nu[1]*tinfy[2]^2)*Q01
+2*tinfy[2]*tinfy[3]*nu[1]*(Qinfy[0])^3-tinfy[3]^2*nu[1]*
(Qinfy[0])^4:
simplify(series(simplify(Ham-Hambis),Q01)) ;

```

(1.11)

```

> M:=Matrix(2*d,2*d,0):
  for i from 1 to 2*d do for j from 1 to i do M[i,j]:=tiny[2*
    d+1-(i-j)]: od: od:
  M;
  alphaVector:=Matrix(2*d,1,0):
  for i from 1 to 2*d do alphaVector[i]:=alpha[2*d+1-i]/(2*d+1-i)
    : od:
  alphaVector;
  nuVector:=Multiply(M^(-1),alphaVector);
  for i from 1 to 2*d do nu[i-1]:=nuVector[i,1]: od:
  nu[0]:=nu[0];
  nu[1]:=nu[1];

```

$$\begin{aligned}
 & \begin{bmatrix} \text{tiny}_3 & 0 \\ \text{tiny}_2 & \text{tiny}_3 \end{bmatrix} \\
 & \begin{bmatrix} \frac{1}{2} \alpha_2 \\ \alpha_1 \end{bmatrix} \\
 & \begin{bmatrix} \frac{1}{2} \frac{\alpha_2}{\text{tiny}_3} \\ -\frac{1}{2} \frac{\text{tiny}_2 \alpha_2}{\text{tiny}_3^2} + \frac{\alpha_1}{\text{tiny}_3} \end{bmatrix} \\
 & v_0 := \frac{1}{2} \frac{\alpha_2}{\text{tiny}_3} \\
 & v_1 := -\frac{1}{2} \frac{\text{tiny}_2 \alpha_2}{\text{tiny}_3^2} + \frac{\alpha_1}{\text{tiny}_3}
 \end{aligned} \tag{1.12}$$

```

> Hamfunction:=unapply(simplify(Ham),alpha[1],alpha[2]):
  Hamtiny1:=simplify(Hamfunction(1,0));
  Hamtiny2:=simplify(Hamfunction(0,1));

```

$$\begin{aligned}
 \text{Hamtiny1} &:= \frac{1}{Q01 \text{tiny}_3} \left(-Q0I^3 \text{tiny}_3^2 + (3 Q\text{tiny}_0^2 \text{tiny}_3^2 + (-4 Q\text{tiny}_0 \text{tiny}_2 \right. \\
 & \quad + 2 \text{tiny}_1) \text{tiny}_3 - P0I^2 + \text{tiny}_2^2) Q0I^2 + (-Q\text{tiny}_0^4 \text{tiny}_3^2 + (2 Q\text{tiny}_0^3 \text{tiny}_2 \\
 & \quad - 2 \text{tiny}_1 Q\text{tiny}_0^2 + (2 \text{tiny}_0 - 1) Q\text{tiny}_0) \text{tiny}_3 - Q\text{tiny}_0^2 \text{tiny}_2^2 \\
 & \quad \left. + 2 Q\text{tiny}_0 \text{tiny}_1 \text{tiny}_2 + P\text{tiny}_0^2) Q0I + t00^2 \right) \\
 \text{Hamtiny2} &:= \frac{1}{2} \frac{1}{Q01 \text{tiny}_3^2} \left((2 Q\text{tiny}_0 \text{tiny}_3^3 - \text{tiny}_2 \text{tiny}_3^2) Q0I^3 + (-Q\text{tiny}_0^3 \text{tiny}_3^3 \right.
 \end{aligned} \tag{1.13}$$

$$\begin{aligned}
& + \left(-Q_{infty_0}^2 t_{infty_2} - 2 Q_{infty_0} t_{infty_1} + 2 t_{infty_0} - 1 \right) t_{infty_3}^2 + \left(-P_{01}^2 Q_{infty_0} \right. \\
& + 3 Q_{infty_0} t_{infty_2}^2 - 2 P_{01} P_{infty_0} \left. \right) t_{infty_3} + P_{01}^2 t_{infty_2} - t_{infty_2}^3 \left. \right) Q_{01}^2 + \left(\right. \\
& Q_{infty_0}^4 t_{infty_2} t_{infty_3}^2 + \left(-2 t_{infty_2}^2 Q_{infty_0}^3 + \left(2 t_{infty_1} Q_{infty_0}^2 + \left(-2 t_{infty_0} \right. \right. \right. \\
& + 1 \left. \left. \left. \right) Q_{infty_0} \right) t_{infty_2} + P_{infty_0} \right) t_{infty_3} + Q_{infty_0}^2 t_{infty_2}^3 - 2 Q_{infty_0} t_{infty_1} t_{infty_2}^2 - \\
& P_{infty_0}^2 t_{infty_2} \left. \right) Q_{01} + i00^2 \left(Q_{infty_0} t_{infty_3} - t_{infty_2} \right)
\end{aligned}$$

> Hamtinfy1bis := -Q01*P01^2/tinfy[3]+1/(tinfy[3])*(Pinfy[0])^2
-tinfy[3]*Q01^2 +t00^2/(Q01*tinfy[3]) -Qinfy[0]^4*tinfy[3]
+2*Qinfy[0]^3*tinfy[2] +3*Qinfy[0]^2*tinfy[3]*Q01-4*tinfy
[2]*Qinfy[0]*Q01+Q01*(2*tinfy[1]*tinfy[3]+tinfy[2]^2)
/tinfy[3]-(2*tinfy[1]*tinfy[3]+tinfy[2]^2)*Qinfy[0]
^2/tinfy[3]-Qinfy[0] +2*(tinfy[0]*tinfy[3]+tinfy[1]*tinfy
[2])*Qinfy[0]/tinfy[3];
simplify(series(Hamtinfy1-Hamtinfy1bis, Qinfy[0]));

$$\begin{aligned}
\text{Hamtinfy1bis} := & -\frac{Q_{01} P_{01}^2}{t_{infty_3}} + \frac{P_{infty_0}^2}{t_{infty_3}} - t_{infty_3} Q_{01}^2 + \frac{i00^2}{Q_{01} t_{infty_3}} - Q_{infty_0}^4 t_{infty_3} \quad (1.14) \\
& + 2 Q_{infty_0}^3 t_{infty_2} + 3 Q_{infty_0}^2 t_{infty_3} Q_{01} - 4 t_{infty_2} Q_{infty_0} Q_{01} \\
& + \frac{Q_{01} \left(2 t_{infty_1} t_{infty_3} + t_{infty_2}^2 \right)}{t_{infty_3}} - \frac{\left(2 t_{infty_1} t_{infty_3} + t_{infty_2}^2 \right) Q_{infty_0}^2}{t_{infty_3}} - Q_{infty_0} \\
& + \frac{2 \left(t_{infty_0} t_{infty_3} + t_{infty_1} t_{infty_2} \right) Q_{infty_0}}{t_{infty_3}}
\end{aligned}$$

0

> Hamtinfy2bis := -(1/2)*(Qinfy[0]*tinfy[3]-tinfy[2])*Q01*
P01^2/tinfy[3]^2-Q01*P01*Pinfy[0]/tinfy[3]+(1/2)*Pinfy[0]
/tinfy[3]-(1/2)*Pinfy[0]^2*tinfy[2]/tinfy[3]^2+t00^2*
(Qinfy[0]*tinfy[3]-tinfy[2])/((2*tinfy[3]^2)*Q01)+(Qinfy
[0]*tinfy[3]-(1/2)*tinfy[2])*Q01^2+(1/2)*Qinfy[0]^4*tinfy
[2]
-(1/2)*(Q01*tinfy[3]^2+2*tinfy[2]^2)*Qinfy[0]^3/tinfy[3]+
Q01*(2*tinfy[0]*tinfy[3]^2-tinfy[2]^3-tinfy[3]^2)/(2*tinfy
[3]^2)-(1/2)*tinfy[2]*(Q01*tinfy[3]^2-2*tinfy[1]*tinfy[3]-
tinfy[2]^2)*Qinfy[0]^2/tinfy[3]^2
+3/2*tinfy[2]^2*Qinfy[0]*Q01/tinfy[3] -Q01*Qinfy[0]*tinfy
[1]+((-2*tinfy[0]+1)*tinfy[2]*tinfy[3]-2*tinfy[1]*tinfy[2]
^2)*Qinfy[0]/(2*tinfy[3]^2);
simplify(series(Hamtinfy2-Hamtinfy2bis, Qinfy[0]));

$$\begin{aligned}
\text{Hamtinfy2bis} := & -\frac{1}{2} \frac{\left(Q_{infty_0} t_{infty_3} - t_{infty_2} \right) Q_{01} P_{01}^2}{t_{infty_3}^2} - \frac{Q_{01} P_{01} P_{infty_0}}{t_{infty_3}} \quad (1.15)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{P_{\infty y_0}}{t_{\infty y_3}} - \frac{1}{2} \frac{P_{\infty y_0}^2 t_{\infty y_2}}{t_{\infty y_3}^2} + \frac{1}{2} \frac{t_{00}^2 (Q_{\infty y_0} t_{\infty y_3} - t_{\infty y_2})}{Q_{01} t_{\infty y_3}^2} \\
& + \left(Q_{\infty y_0} t_{\infty y_3} - \frac{1}{2} t_{\infty y_2} \right) Q_{01}^2 + \frac{1}{2} Q_{\infty y_0}^4 t_{\infty y_2} \\
& - \frac{1}{2} \frac{(Q_{01} t_{\infty y_3}^2 + 2 t_{\infty y_2}^2) Q_{\infty y_0}^3}{t_{\infty y_3}} + \frac{1}{2} \frac{Q_{01} (2 t_{\infty y_0} t_{\infty y_3}^2 - t_{\infty y_2}^3 - t_{\infty y_3}^2)}{t_{\infty y_3}^2} \\
& - \frac{1}{2} \frac{t_{\infty y_2} (Q_{01} t_{\infty y_3}^2 - 2 t_{\infty y_1} t_{\infty y_3} - t_{\infty y_2}^2) Q_{\infty y_0}^2}{t_{\infty y_3}^2} + \frac{3}{2} \frac{t_{\infty y_2}^2 Q_{\infty y_0} Q_{01}}{t_{\infty y_3}} \\
& - Q_{01} Q_{\infty y_0} t_{\infty y_1} + \frac{1}{2} \frac{((-2 t_{\infty y_0} + 1) t_{\infty y_2} t_{\infty y_3} - 2 t_{\infty y_1} t_{\infty y_2}^2) Q_{\infty y_0}}{t_{\infty y_3}^2}
\end{aligned}$$

0

We impose the symmetry $\check{L}(-\lambda) = -\sigma_1 \check{L}(\lambda) \sigma_1$

```

> R01:=0:
Rinfy[1]:=0:
tinfy[2]:=0:
tinfy[0]:=0:
alpha[2]:=0:
alpha[1]:=1:

> L12function:=unapply(L12,lambda):
L21function:=unapply(L21,lambda):
L11function:=unapply(L11,lambda):
Qinfy[0]:=-P01/tinfy[3];
Pinfy[0]:=-Q01*tinfy[3]+P01^2/tinfy[3]+tinfy[1]-t00^2/(2*
tinfy[3]*Q01^2);
omega:=-t00/Q01;
Lomega:=t00*LQ01/Q01^2:
simplify(L11function(-lambda)-L11function(lambda));
series(simplify(L12function(-lambda)+L21function(lambda)),
lambda=0);
series(simplify(L21function(-lambda)+L12function(lambda)),
lambda=0);

```

$L11function := \lambda$

(1.16)

$$\rightarrow \frac{-\lambda^3 t_{\infty y_3} + ((Q_{\infty y_0}^2 - Q_{01}) t_{\infty y_3} - P_{\infty y_0}) \lambda + Q_{01} (Q_{\infty y_0} t_{\infty y_3} + P_{01})}{\lambda}$$

$$Q_{\infty y_0} := -\frac{P_{01}}{t_{\infty y_3}}$$

$$P_{\infty y_0} := -Q_{01} t_{\infty y_3} + \frac{P_{01}^2}{t_{\infty y_3}} + t_{\infty y_1} - \frac{1}{2} \frac{t_{00}^2}{t_{\infty y_3} Q_{01}^2}$$

$$\omega := -\frac{t00}{Q01}$$

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

```
> L:=simplify(L) ;
L11:=expand(series(simplify(series(L11,lambda=infinity)),
lambda=infinity)) ;
L12:=expand(series(simplify(series(L12,lambda=infinity)),
lambda=infinity)) ;
L21:=expand(series(simplify(series(L21,lambda=infinity)),
lambda=infinity)) ;
A:=simplify(A) ;
```

$$L11 := -\text{tinfty}_3 \lambda^2 - \text{tinfty}_1 + \frac{1}{2} \frac{t00^2}{\text{tinfty}_3 Q01^2} \quad (1.17)$$

$$L12 := -\frac{t00 \lambda}{Q01} + \frac{t00 P01}{Q01 \text{tinfty}_3} - \frac{t00}{\lambda}$$

$$L21 := -\frac{t00 \lambda}{Q01} - \frac{t00 P01}{Q01 \text{tinfty}_3} - \frac{t00}{\lambda}$$

$$\begin{bmatrix} -\lambda & -\frac{t00}{Q01 \text{tinfty}_3} \\ -\frac{t00}{Q01 \text{tinfty}_3} & \lambda \end{bmatrix}$$

```
> LQinfty0:=expand(LQinfty0) ;
LQ01:=LQ01;
LP01:=expand(simplify(LP01)) ;
```

$$LQinfty0 := -2 Q01 + \frac{2 P01^2}{\text{tinfty}_3^2} + \frac{2 \text{tinfty}_1}{\text{tinfty}_3} - \frac{t00^2}{\text{tinfty}_3^2 Q01^2} \quad (1.18)$$

$$LQ01 := -\frac{2 Q01 P01}{\text{tinfty}_3}$$

$$LP01 := 2 Q01 \text{tinfty}_3 - \frac{2 P01^2}{\text{tinfty}_3} - 2 \text{tinfty}_1 + \frac{t00^2}{\text{tinfty}_3 Q01^2}$$

```
> simplify(LP01-tinfty[3]*(-LQinfty0)) ;
LRinfty0Reduced:=unapply(expand(simplify(-LPinfty0+2*tinfty[3]*
Qinfty[0]*LQinfty0 -tinfty[3]* LQ01)) ,Q01,P01) ;
LSinfty0Reduced:=unapply(expand(simplify(-LQinfty0)) ,Q01,P01) ;
LS01Reduced:=unapply(expand(simplify(LQ01)) ,Q01,P01) ;
LS01Reduced(S01,Sinfty[0]*tinfty[3]) ;
LRinfty0Reduced(S01,Sinfty[0]*tinfty[3]) ;
LSinfty0Reduced(S01,Sinfty[0]*tinfty[3]) ;
```

$$\begin{aligned}
LRinfty0Reduced &:= (Q01, P01) \rightarrow -1 + \frac{2 P01 t00^2}{Q01^2 tinfty_3^2} \\
LSinfty0Reduced &:= (Q01, P01) \rightarrow 2 Q01 - \frac{2 P01^2}{tinfty_3^2} - \frac{2 tinfty_1}{tinfty_3} + \frac{t00^2}{tinfty_3^2 Q01^2} \\
LS01Reduced &:= (Q01, P01) \rightarrow -\frac{2 Q01 P01}{tinfty_3} \\
&\quad -2 S01 Sinfty_0 \\
&\quad -1 + \frac{2 Sinfty_0 t00^2}{S01^2 tinfty_3} \\
&\quad 2 S01 - 2 Sinfty_0^2 - \frac{2 tinfty_1}{tinfty_3} + \frac{t00^2}{tinfty_3^2 S01^2}
\end{aligned}$$

We now obtain the standard FN Hamiltonian system. We take $q = -t00/tinfty[3]/Q01$ and $p = -P01 * t00/2/tinfty[3]/Q01$

```

> Lq:=unapply(t00/tinfty[3]/(Q01)^2*LQ01,Q01,P01,tinfty[3]);
Lp:=unapply(-t00/2/tinfty[3]/Q01*LP01+P01*t00/2/tinfty[3]
/Q01^2*LQ01,Q01,P01,tinfty[3]);
Lqnew:=unapply(expand(simplify(Lq(-t00/tinfty[3]/q,-p/t00*2*
tinfty[3]*(-t00/tinfty[3]/q),tinfty[3]))),tinfty[3]);
Lpnew:=unapply(expand(simplify(Lp(-t00/tinfty[3]/q,-p/t00*2*
tinfty[3]*(-t00/tinfty[3]/q),tinfty[3]))),tinfty[3]);
Hamqp:=unapply(2*p^2/tinfty[3]-(1/8)*q^4*tinfty[3]+(1/2)*q^2*
tinfty[1]+t00*q,tinfty[3]);
simplify(diff(Hamqp(tinfty[3]),p)-Lqnew(tinfty[3]));
series(simplify(diff(Hamqp(tinfty[3]),q)+Lpnew(tinfty[3])),p);

```

$$Lq := (Q01, P01, tinfty_3) \rightarrow -\frac{2 t00 P01}{tinfty_3^2 Q01} \quad (1.20)$$

$$\begin{aligned}
Lp &:= (Q01, P01, tinfty_3) \rightarrow \\
&\quad -\frac{1}{2} \frac{t00 \left(2 Q01 tinfty_3 - \frac{2 P01^2}{tinfty_3} - 2 tinfty_1 + \frac{t00^2}{tinfty_3 Q01^2} \right)}{tinfty_3 Q01} - \frac{P01^2 t00}{tinfty_3^2 Q01}
\end{aligned}$$

$$Lqnew := tinfty_3 \rightarrow \frac{4 p}{tinfty_3}$$

$$Lpnew := tinfty_3 \rightarrow \frac{1}{2} tinfty_3 q^3 - tinfty_1 q - t00$$

$$Hamqp := tinfty_3 \rightarrow \frac{2 p^2}{tinfty_3} - \frac{1}{8} q^4 tinfty_3 + \frac{1}{2} q^2 tinfty_1 + t00 q$$

0
0

We now take $Q1 = t00/Q01$ and $P1 = P01/(4*tinfty[3])*t00/Q01 + (t00/Q01)^2/(8*tinfty[3]) - t_{\{infty, 1\}}/4*Q01/t00$ following the proposal of Mazzocco et al.

```

> Rinfty[0] := t00^2/2/tinfty[3]/Q01^2 - tinfty[1];

```

```

Qinfy[0] := (4*P1-Rinfy[0])/omega;
P01theo:=unapply( -tinfy[3]*Qinfy[0],Q01);
P01theo2:=expand(P01theo(t00/Q1));
Sol:=unapply( expand(solve(P01theo2=P01bis,P1)),Q1);
PP1:=P01/(4*tinfy[3])*t00/Q01 +(t00/Q01)^2/(8*tinfy[3])-
tinfy[1]/4;

```

$$\begin{aligned}
 Rinfy_0 &:= -tinfy_1 + \frac{1}{2} \frac{t00^2}{tinfy_3 Q01^2} & (1.21) \\
 Qinfy_0 &:= -\frac{\left(4 P1 + tinfty_1 - \frac{1}{2} \frac{t00^2}{tinfy_3 Q01^2}\right) Q01}{t00} \\
 P01theo &:= Q01 \rightarrow \frac{\left(4 P1 + tinfty_1 - \frac{1}{2} \frac{t00^2}{tinfy_3 Q01^2}\right) Q01 tinfty_3}{t00} \\
 P01theo2 &:= \frac{4 tinfty_3 P1}{Q1} + \frac{tinfy_3 tinfty_1}{Q1} - \frac{1}{2} Q1 \\
 Sol &:= Q1 \rightarrow \frac{1}{4} \frac{P01bis Q1}{tinfy_3} + \frac{1}{8} \frac{Q1^2}{tinfy_3} - \frac{1}{4} tinfty_1 \\
 PP1 &:= \frac{1}{4} \frac{t00 P01}{Q01 tinfty_3} + \frac{1}{8} \frac{t00^2}{tinfy_3 Q01^2} - \frac{1}{4} tinfty_1
 \end{aligned}$$

```

> LQ1:=unapply(-t00/Q01^2*LQ01,Q01,P01,tinfy[3]);
LP1:=unapply(diff(PP1,P01)*LP01+diff(PP1,Q01)*LQ01+diff(PP1,
tinfy[1]),Q01,P01,tinfy[3]);
LQ1new:=unapply(expand(simplify(LQ1(t00/Q1,4*tinfy[3]*P1/Q1
-1/2*Q1+tinfy[3]*tinfy[1]/Q1,tinfy[3])),tinfy[3]);
LP1new:=unapply(expand(simplify(LP1(t00/Q1,4*tinfy[3]*P1/Q1
-1/2*Q1+tinfy[3]*tinfy[1]/Q1,tinfy[3])),tinfy[3]);
int(LQ1new(tinfy[3]),P1);

```

$$\begin{aligned}
 LQ1 &:= (Q01, P01, tinfty_3) \rightarrow \frac{2 t00 P01}{Q01 tinfty_3} & (1.22) \\
 LQ1new &:= tinfty_3 \rightarrow 8 P1 - \frac{Q1^2}{tinfy_3} + 2 tinfty_1 \\
 LP1new &:= tinfty_3 \rightarrow \frac{2 P1 Q1}{tinfy_3} + \frac{1}{2} t00 - \frac{1}{4} \\
 & 4 P1^2 - \frac{Q1^2 P1}{tinfy_3} + 2 tinfty_1 P1
 \end{aligned}$$

```

> Hamnew:=unapply(4*P1^2-Q1^2*P1/tinfy[3]+2*tinfy[1]*P1- ((1/2)
*t00-1/4)*Q1,tinfy[3]);
simplify(diff(Hamnew(tinfy[3]),P1)- LQ1new(tinfy[3]));
series(simplify(diff(Hamnew(tinfy[3]),Q1)+LP1new(tinfy[3])),

```

P1);

$$Hamnew := \text{tinfty}_3 \rightarrow 4 P I^2 - \frac{Q I^2 P I}{\text{tinfty}_3} + 2 \text{tinfty}_1 P I - \left(\frac{1}{2} t00 - \frac{1}{4} \right) Q I \quad (1.23)$$

$$\begin{aligned} > \text{int}(-t00 - qFN * \text{tinfty}[1] + (1/2) * \text{tinfty}[3] * qFN^3, qFN); \\ & -t00 qFN - \frac{1}{2} qFN^2 \text{tinfty}_1 + \frac{1}{8} \text{tinfty}_3 qFN^4 \end{aligned} \quad (1.24)$$

We define $qFN = -t00/\text{tinfty}[3]/Q01$ and $pFN = -t00/2/\text{tinfty}[3]*P01/Q01$

```
> LqFN:=unapply(t00/tinfty[3]*LQ01/Q01^2,Q01,P01);
LpFN:=unapply(expand(simplify(-t00/2/tinfty[3]*
LP01/Q01+t00/2/tinfty[3]*P01/Q01^2*LQ01)),Q01,P01);
LqFNnew:=LqFN(-t00/tinfty[3]/qFN,-pFN^2*tinfty[3]/t00*(-
t00/tinfty[3]/qFN));
LpFNnew:=LpFN(-t00/tinfty[3]/qFN,-pFN^2*tinfty[3]/t00*(-
t00/tinfty[3]/qFN));
HamFNnew:=2*pFN^2/tinfty[3]+t00*qFN+(1/2)*qFN^2*tinfty[1]-(1/8)
*tinfty[3]*qFN^4;
LqFNnew-diff(HamFNnew,pFN);
LpFNnew+diff(HamFNnew,qFN);
ddotqFN=expand(4/tinfty[3]*LpFNnew);
```

$$\begin{aligned} LqFN &:= (Q01, P01) \rightarrow -\frac{2 t00 P01}{\text{tinfty}_3^2 Q01} \\ LpFN &:= (Q01, P01) \rightarrow -t00 + \frac{t00 \text{tinfty}_1}{Q01 \text{tinfty}_3} - \frac{1}{2} \frac{t00^3}{Q01^3 \text{tinfty}_3^2} \\ LqFNnew &:= \frac{4 pFN}{\text{tinfty}_3} \\ LpFNnew &:= -t00 - qFN \text{tinfty}_1 + \frac{1}{2} \text{tinfty}_3 qFN^3 \\ HamFNnew &:= \frac{2 pFN^2}{\text{tinfty}_3} + t00 qFN + \frac{1}{2} qFN^2 \text{tinfty}_1 - \frac{1}{8} \text{tinfty}_3 qFN^4 \\ ddotqFN &= -\frac{4 t00}{\text{tinfty}_3} - \frac{4 qFN \text{tinfty}_1}{\text{tinfty}_3} + 2 qFN^3 \end{aligned} \quad (1.25)$$

We set $x = \mu * \text{tinfty}[1]$ and $q = \nu * qFN$. This gives $d^2/d\text{tinfty}[1]^2 = \mu^2 * d^2/dx^2$ and thus $d^2/d\text{tinfty}[1]^2 qFN = \mu^2/\nu * d^2/dx^2 q$

```
> test:=unapply(expand(nu/mu^2*( -4*t00/tinfty[3]-4*1/nu*q*(x/mu)
/tinfty[3]+2*(1/nu*q)^3)),mu,nu);
```

```
test((-4/tinfy[3])^(1/3), (-4/tinfy[3])^(-1/3));
```

$$test := (\mu, \nu) \rightarrow -\frac{4 \nu t00}{\mu^2 tinfy_3} - \frac{4 q x}{\mu^3 tinfy_3} + \frac{2 q^3}{\nu^2 \mu^2} \quad (1.26)$$

$$2 q^3 + q x + t00$$

Test of FN Lax pair

```
> LFN:=Matrix(2,2,0):
```

```
LFN[1,1]:= -4*lambda^2-x+2*qFN^2:
```

```
LFN[1,2]:=4*qFN*lambda-2*pFN+alphaFN/lambda:
```

```
LFN[2,1]:=4*qFN*lambda+2*pFN+alphaFN/lambda:
```

```
LFN[2,2]:=-LFN[1,1]:
```

```
AFN:=Matrix(2,2,0):
```

```
AFN[1,1]:= -lambda:
```

```
AFN[1,2]:=qFN:
```

```
AFN[2,1]:=qFN:
```

```
AFN[2,2]:=-AFN[1,1]:
```

```
LFN;
```

```
AFN;
```

$$\begin{bmatrix} -4\lambda^2 + 2qFN^2 - x & 4qFN\lambda - 2pFN + \frac{\alpha FN}{\lambda} \\ 4qFN\lambda + 2pFN + \frac{\alpha FN}{\lambda} & 4\lambda^2 - 2qFN^2 + x \end{bmatrix} \quad (2.1)$$

$$\begin{bmatrix} -\lambda & qFN \\ qFN & \lambda \end{bmatrix}$$

```
> dAFNdlambda:=Matrix(2,2,0):
```

```
for i from 1 to 2 do for j from 1 to 2 do dAFNdlambda[i,j]:=
```

```
diff(AFN[i,j],lambda): od: od:
```

```
dAFNdlambda:=
```

```
> BigLLFN:=Matrix(2,2,0):
```

```
BigLLFN[1,1]:= diff(LFN[1,1],qFN)*LqFN+ diff(LFN[1,1],pFN)*
```

```
LpFN+diff(LFN[1,1],x):
```

```
BigLLFN[1,2]:= diff(LFN[1,2],qFN)*LqFN+ diff(LFN[1,2],pFN)*
```

```
LpFN+diff(LFN[1,2],x):
```

```
BigLLFN[2,1]:= diff(LFN[2,1],qFN)*LqFN+ diff(LFN[2,1],pFN)*
```

```
LpFN+diff(LFN[2,1],x):
```

```
BigLLFN[2,2]:=-BigLLFN[1,1]:
```

```
BigLLFN;
```

(2.2)

$$\begin{bmatrix} 4 LqFN qFN - 1 & 4 LqFN \lambda - 2 LpFN \\ 4 LqFN \lambda + 2 LpFN & -4 LqFN qFN + 1 \end{bmatrix} \quad (2.2)$$

```
> CommutatorFN:=Multiply (AFN,LFN)-Multiply (LFN,AFN) :
CompatibilityEquationFN:= simplify (dAFNdlambda-BigLLFN+
CommutatorFN) :
```

```
> LqFN:=pFN;
LpFN:=2*qFN^3-x*qFN+alphaFN;
      LqFN := pFN
      LpFN := 2 qFN^3 - qFN x + alphaFN
```

(2.3)

```
> series (CompatibilityEquationFN[1,2], lambda=infinity) :
EQ0:=simplify (residue (CompatibilityEquationFN[1,2]/lambda^3,
lambda=infinity)) ;
EQ1:=simplify (residue (CompatibilityEquationFN[1,2]/lambda^2,
lambda=infinity)) ;
EQ2:=simplify (residue (CompatibilityEquationFN[1,2]/lambda,
lambda=infinity)) ;
EQ3:=simplify (residue (CompatibilityEquationFN[1,2], lambda=
infinity)) ;
```

```
series (simplify (series (CompatibilityEquationFN[1,1], lambda=
infinity)), lambda=infinity) :
```

```
EQ0:=simplify (residue (CompatibilityEquationFN[1,1]/lambda^4,
lambda=infinity)) ;
EQ4:=simplify (residue (CompatibilityEquationFN[1,1]/lambda^3,
lambda=infinity)) ;
EQ5:=simplify (residue (CompatibilityEquationFN[1,1]/lambda^2,
lambda=infinity)) ;
EQ6:=simplify (residue (CompatibilityEquationFN[1,1]/lambda,
lambda=infinity)) ;
EQ7:=simplify (residue (CompatibilityEquationFN[1,1], lambda=
infinity)) ;
```

```
EQ0 := 0
EQ1 := 0
EQ2 := 0
EQ3 := 0
EQ4 := 0
EQ5 := 0
EQ6 := 0
EQ7 := 0
```

(2.4)

```

> QQ:=Q1*lambda^2;
PP:=P1*lambda^(-2);
TT:=-z*(2*lambda)^(-2);
series(lambda^(-2)*QQ*(1+PP),lambda);
Beven:=series(Q1*lambda^2,lambda,infinity);
series(1/4*(2*lambda)^(2*d+1)*(1+PP-(1+TT)^2/(1+PP))+(2*
lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
Bodd:=(4*P1+z+(1/8)*Q1^2)*lambda;
series(1/4*(2*lambda)^(2*d+1)*(1+PP+(1+TT)^2/(1+PP))-(2*
lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
A:=4*lambda^3+(-z-(1/8)*Q1^2)*lambda;

LMazzocol1:=series(A/lambda,lambda=infinity);
LMazzocol2:=series((Beven+Bodd)/lambda+b0/lambda,lambda=
infinity);
LMazzocol21:=series((Beven-Bodd)/lambda+b0/lambda,lambda=
infinity);

```

$$\begin{aligned}
QQ &:= QI \lambda^2 \\
PP &:= \frac{PI}{\lambda^2} \\
TT &:= -\frac{1}{4} \frac{z}{\lambda^2} \\
&\frac{PI QI}{\lambda^2} + QI \\
Beven &:= QI \lambda^2 \\
&\left(4PI + z + \frac{1}{8} QI^2\right) \lambda + \frac{-\frac{1}{8} z^2 - 2 \left(\frac{1}{2} z + PI\right) PI + \frac{1}{8} QI^2 PI}{\lambda} + O\left(\frac{1}{\lambda^3}\right) \\
Bodd &:= \left(4PI + z + \frac{1}{8} QI^2\right) \lambda \\
&4\lambda^3 + \left(-z - \frac{1}{8} QI^2\right) \lambda + \frac{\frac{1}{8} z^2 + 2 \left(\frac{1}{2} z + PI\right) PI - \frac{1}{8} QI^2 PI}{\lambda} + O\left(\frac{1}{\lambda^3}\right) \\
A &:= 4\lambda^3 + \left(-z - \frac{1}{8} QI^2\right) \lambda \\
LMazzocol1 &:= 4\lambda^2 - z - \frac{1}{8} QI^2 \\
LMazzocol2 &:= QI \lambda + 4PI + z + \frac{1}{8} QI^2 + \frac{b0}{\lambda} \\
LMazzocol21 &:= QI \lambda - 4PI - z - \frac{1}{8} QI^2 + \frac{b0}{\lambda}
\end{aligned}$$

(3.1)

```
> DetMazzocco:=series(simplify(-LMazzocco11^2-LMazzocco12*
LMazzocco21),lambda=infinity);
DetL:=series(simplify(-L11^2-L12*L21),lambda=infinity);
```

$$DetMazzocco := -16 \lambda^4 + 8 z \lambda^2 + 16 P I^2 + (Q I^2 + 8 z) P I - 2 Q I b 0 - \frac{b 0^2}{\lambda^2} \quad (3.2)$$

$$DetL := -tinfty_3^2 \lambda^4 - 2 tinfty_1 tinfty_3 \lambda^2 + \frac{1}{4} \frac{1}{tinfty_3^2 Q 0 I^4} (-4 Q 0 I^4 tinfty_1^2 tinfty_3^2 - 8 Q 0 I^3 t 0 0^2 tinfty_3^2 + 4 t 0 0^2 (P 0 I^2 + tinfty_1 tinfty_3) Q 0 I^2 - t 0 0^4) - \frac{t 0 0^2}{\lambda^2}$$

```
> series(simplify(L11),lambda=infinity);
series(simplify(L12),lambda=infinity);
series(simplify(L21),lambda=infinity);
```

$$-tinfty_3 \lambda^2 + \frac{1}{2} \frac{-2 Q 0 I^2 tinfty_1 tinfty_3 + t 0 0^2}{Q 0 I^2 tinfty_3} - \frac{t 0 0 \lambda}{Q 0 I} + \frac{t 0 0 P 0 I}{Q 0 I tinfty_3} - \frac{t 0 0}{\lambda} - \frac{t 0 0 \lambda}{Q 0 I} - \frac{t 0 0 P 0 I}{Q 0 I tinfty_3} - \frac{t 0 0}{\lambda} \quad (3.3)$$

```
> tinfty[3]:=-4;
Q1:=-t00/Q01;
z:=tinfty[1];
b0:=-t00;
P1:=-P01*t00/(16*Q01)-(1/4)*tinfty[1]-t00^2/(32*Q01^2);
simplify(series(simplify(L11-LMazzocco11),Q01));
series(simplify(L12-LMazzocco12),Q01);
series(simplify(L21-LMazzocco21),Q01);
```

$$tinfty_3 := -4$$

$$Q 1 := -\frac{t 0 0}{Q 0 I}$$

$$z := tinfty_1$$

$$b 0 := -t 0 0$$

$$P 1 := -\frac{1}{16} \frac{P 0 I t 0 0}{Q 0 I} - \frac{1}{4} tinfty_1 - \frac{1}{32} \frac{t 0 0^2}{Q 0 I^2}$$

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \quad (3.4)$$

▼ Coordinates (R,S)

```

> restart:
with(LinearAlgebra):
d:=1:

> L12:=Q01/lambda+lambda^(2*d-1):
for k from 0 to 2*d-2 do L12:=L12+ Qinfy[k]*lambda^k: od:
L12:=L12*omega;
L11:=P01*Q01/lambda:
for k from 0 to 2*d-2 do L11:=L11-Pinfy[2*d-2-k]*lambda^k: od:
L11:=L11- (tinfy[2*d+1]*lambda+tinfy[2*d]-tinfy[2*d+1]*
Qinfy[2*d-2])*(L12/omega):
L11:=simplify(L11):
series(L11,lambda=infinity,20);

```

$$L12 := \left(\frac{Q01}{\lambda} + \lambda + Qinfy_0 \right) \omega \quad (4.1)$$

$$-tinfy_3 \lambda^2 - tinfty_2 \lambda + (Qinfy_0^2 - Q01) tinfty_3 - Qinfy_0 tinfty_2 - Pinfy_0$$

$$+ \frac{Q01 (Qinfy_0 tinfty_3 + P01 - tinfty_2)}{\lambda}$$

```

> series(simplify(series((t00^2/lambda^2- L11^2)/(L12/omega),
lambda=0,3)),lambda=0,1);
factor(residue((t00^2/lambda^2- L11^2)/(L12/omega),lambda=0));
ResidueBis:=1/omega*(t00^2/Q01-Q01*(P01-tinfy[2*d]+tinfy[2*
d+1]*Qinfy[2*d-2])^2);
simplify(factor(residue((t00^2/lambda^2- L11^2)/L12,lambda=0))
- ResidueBis);

```

$$-\frac{1}{\lambda} \frac{1}{Q01} ((Q01 (Qinfy_0 tinfty_3 + P01 - tinfty_2) - t00) (Q01 (Qinfy_0 tinfty_3 + P01$$

$$- tinfty_2) + t00)) + \frac{1}{Q01^2} (2 tinfty_3 (Qinfy_0 tinfty_3 + P01 - tinfty_2) Q01^3$$

$$+ (Qinfy_0 tinfty_3 + P01 - tinfty_2) (-Qinfy_0^2 tinfty_3 + (P01 + tinfty_2) Qinfy_0$$

$$+ 2 Pinfy_0) Q01^2 - t00^2 Qinfy_0) + O(\lambda)$$

$$-\frac{1}{Q01} ((Q01 Qinfy_0 tinfty_3 + P01 Q01 - Q01 tinfty_2 - t00) (Q01 Qinfy_0 tinfty_3$$

$$+ P01 Q01 - Q01 tinfty_2 + t00))$$

$$ResidueBis := \frac{\frac{t00^2}{Q01} - Q01 (Qinfy_0 tinfty_3 + P01 - tinfty_2)^2}{\omega}$$

0

```

> L21infy:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21infy:=
L21infy+ tinfty[2*d+1-m]*tinfy[j+m-2*d+1]*lambda^j: od: od:

```

```

Numm:=simplify((L21infty-L11^2)/(L12/omega)):
L21:=0:
for j from 1 to 60 do L21:=L21- 1/omega*simplify(residue(Numm*
lambda^(-j),lambda=infinity))*lambda^(j-1): od:
L21:=simplify(L21+ (t00^2/Q01-Q01*(Qinfty[0]*tinfty[3]+P01-
tinfty[2])^2)/omega/lambda):
> series(simplify(series(L21,lambda=infinity,2)),lambda=infinity,
2):
CoefflambdaL21:=-factor(residue(L21/lambda^2,lambda=infinity));
Coefflambda0L21:=-factor(residue(L21/lambda,lambda=infinity));
Coefflambda0L21bis:=1/omega*( 2*tinfty[3]*Q01*P01+2*(tinfty[3]*
Qinfty[0]-tinfty[2])*Pinfty[0]+4*(tinfty[3]^2*Qinfty[0]-tinfty
[2]*tinfty[3])*Q01-2*tinfty[3]^2*Qinfty[0]^3+4*tinfty[3]*tinfty
[2]*Qinfty[0]^2-2*(tinfty[2]^2+tinfty[3]*tinfty[1])*Qinfty[0]
+2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2] );
simplify(series(Coefflambda0L21-Coefflambda0L21bis,P01));
CoefflambdaMinus1L21:=-factor(residue(L21,lambda=infinity)):

```

CoefflambdaL21 := (4.3)

$$\frac{2 \operatorname{tinfty}_3 \left(-Q_{\operatorname{infty}_0}^2 \operatorname{tinfty}_3 + Q_{01} \operatorname{tinfty}_3 + Q_{\operatorname{infty}_0} \operatorname{tinfty}_2 + P_{\operatorname{infty}_0} - \operatorname{tinfty}_1 \right)}{\omega}$$

$$\operatorname{Coefflambda0L21} := \frac{1}{\omega} \left(2 \left(-Q_{\operatorname{infty}_0}^3 \operatorname{tinfty}_3^2 + 2 Q_{01} Q_{\operatorname{infty}_0} \operatorname{tinfty}_3^2 + 2 Q_{\operatorname{infty}_0}^2 \operatorname{tinfty}_2 \operatorname{tinfty}_3 + P_{01} Q_{01} \operatorname{tinfty}_3 - 2 Q_{01} \operatorname{tinfty}_2 \operatorname{tinfty}_3 + P_{\operatorname{infty}_0} Q_{\operatorname{infty}_0} \operatorname{tinfty}_3 - Q_{\operatorname{infty}_0} \operatorname{tinfty}_1 \operatorname{tinfty}_3 - Q_{\operatorname{infty}_0} \operatorname{tinfty}_2^2 - P_{\operatorname{infty}_0} \operatorname{tinfty}_2 + \operatorname{tinfty}_0 \operatorname{tinfty}_3 + \operatorname{tinfty}_1 \operatorname{tinfty}_2 \right) \right)$$

$$\operatorname{Coefflambda0L21bis} := \frac{1}{\omega} \left(2 P_{01} Q_{01} \operatorname{tinfty}_3 + 2 \left(Q_{\operatorname{infty}_0} \operatorname{tinfty}_3 - \operatorname{tinfty}_2 \right) P_{\operatorname{infty}_0} + 4 \left(Q_{\operatorname{infty}_0} \operatorname{tinfty}_3^2 - \operatorname{tinfty}_2 \operatorname{tinfty}_3 \right) Q_{01} - 2 Q_{\operatorname{infty}_0}^3 \operatorname{tinfty}_3^2 + 4 Q_{\operatorname{infty}_0}^2 \operatorname{tinfty}_2 \operatorname{tinfty}_3 - 2 \left(\operatorname{tinfty}_1 \operatorname{tinfty}_3 + \operatorname{tinfty}_2^2 \right) Q_{\operatorname{infty}_0} + 2 \operatorname{tinfty}_0 \operatorname{tinfty}_3 + 2 \operatorname{tinfty}_1 \operatorname{tinfty}_2 \right)$$

Definition of the auxiliary matrix in the (Q,P) variables.

```

> A12:=0:
for i from 0 to 2*d-1 do A12:=A12+nu[i]*lambda^(2*d-1-i): od:
for k from 0 to 2*d-2 do for i from 0 to k do A12:=A12+Qinfty
[k]*nu[i]*lambda^(k-i) od: od:
omega*nu[0]*lambda^omega*nu[1]*lambda^(2*d-2):
A12:=omega*A12;
cinfty0:= 1/2/omega*Lomega+ tinfty[3]*(Q01*nu[0]+nu[1]*Qinfty
[0]):
A11:=cinfty0:

```

```

PolA12:=0:
for i from 0 to 2*d-1 do PolA12:=PolA12+nu[i]*lambda^(-i): od:
PolA12:=PolA12*L11:
for k from 0 to 60 do A11:=A11-residue(PolA12/lambda^(k+1),
lambda=infinity)*lambda^k od:
A11:=A11;
A22:=-A11:

A21:=(-tinfty[2*d+1]*Lomega/omega^2) *lambda- (tinfty[2*d]-
tinfty[2*d+1]*Qinfty[2*d-2])*Lomega/omega^2+ (alpha[2*d]-tinfty
[2*d+1]*LQinfty0)/omega -tinfty[2*d+1]*nu[0]/omega:
L21oper:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21oper:=
L21oper+tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j od: od:
L21oper:=L21oper;
A12oper:=0:
for i from 0 to 2*d+1 do A12oper:=A12oper+nu[i]* lambda^(-i):
od:
A12oper:=A12oper;
for k from 0 to 60 do A21:=A21-residue( (L21oper*A12oper/(L12))
/lambda^(k+1),lambda=infinity)*lambda^k od:
for k from 0 to 60 do A21:=A21-residue( ((L11/(L12))*(L11*
A12/L12 -2*A11))/lambda^(k+1),lambda=infinity)*lambda^k od:
nu[2]:=- (Q01*nu[0]+Qinfty[0]*nu[1]);
Testnu2:=-nu[0]*Q01:
nu[3]:=Q01*Qinfty[0]*nu[0]+Qinfty[0]^2*nu[1]-Q01*nu[1]:
Testnu3:=-nu[1]*Q01:
for j from 2 to 2*d do Testnu3:=Testnu3- nu[j]*Qinfty[j-2]: od:
simplify(nu[3]-Testnu3);
for j from 1 to 2*d-1 do Testnu2:=Testnu2- nu[j]*Qinfty[j-1]:
od:
simplify(nu[2]-Testnu2);
A21:=series(simplify(series(simplify(A21),lambda=infinity)),
lambda=infinity);

```

$$A12 := \omega (\lambda v_0 + Q_{infty_0} v_0 + v_1) \quad (4.4)$$

$$\begin{aligned}
A11 := & \frac{1}{2} \frac{Lomega}{\omega} + tinfty_3 (Q01 v_0 + Q_{infty_0} v_1) + v_0 ((Q_{infty_0}^2 - Q01) tinfty_3 \\
& - Q_{infty_0} tinfty_2 - P_{infty_0}) - v_1 tinfty_2 - (v_0 tinfty_2 + v_1 tinfty_3) \lambda - v_0 tinfty_3 \lambda^2 \\
L21oper := & \lambda^4 tinfty_3^2 + 2 \lambda^3 tinfty_2 tinfty_3 + 2 \lambda^2 tinfty_1 tinfty_3 + \lambda^2 tinfty_2^2 + 2 \lambda tinfty_0 tinfty_3 \\
& + 2 \lambda tinfty_1 tinfty_2
\end{aligned}$$

$$AI2oper := v_0 + \frac{v_1}{\lambda} + \frac{v_2}{\lambda^2} + \frac{v_3}{\lambda^3}$$

$$v_2 := -Q01 v_0 - Qinfy_0 v_1$$

$$0$$

$$0$$

$$A21 := \frac{1}{\omega} \left(\left((2 Qinfy_0^2 - 2 Q01) v_0 tinfy_3^2 + (-2 Qinfy_0 tinfy_2 - 2 Pinfty_0 + 2 tinfy_1) v_0 tinfy_3 \right) \lambda \right) + \frac{1}{\omega} \left(\left((-2 Qinfy_0^3 + 4 Q01 Qinfy_0) v_0 - 2 v_1 (-Qinfy_0^2 + Q01) \right) tinfy_3^2 + \left((4 Qinfy_0^2 tinfy_2 + (2 Pinfty_0 - 2 tinfy_1) Qinfy_0 - 4 Q01 tinfy_2 + 2 tinfy_0 - 1) v_0 - 2 tinfy_2 v_1 Qinfy_0 + 2 tinfy_1 v_1 - LQinfy_0 \right) tinfy_3 + 2 tinfy_2 (-Qinfy_0 tinfy_2 - Pinfty_0 + tinfy_1) v_0 + \alpha_2 \right)$$

```

> CoeffLambda1A21:=factor(-residue(A21/lambda^2,lambda=infinity))
;
CoeffLambda0A21:=simplify(-residue(A21/lambda,lambda=infinity))
:
CoeffLambda0A21bis:=1/omega*(-tinfy[3]*LQinfy0 -2*tinfy[3]*(-Qinfy[0]^2+Q01)*tinfy[3]+Qinfy[0]*tinfy[2]-tinfy[1])*nu[1] +alpha[2] +4*tinfy[3]*nu[0]*(Qinfy[0]*tinfy[3]-tinfy[2])*Q01-2*Qinfy[0]^3*nu[0]*tinfy[3]^2+4*Qinfy[0]^2*nu[0]*tinfy[2]*tinfy[3]+2*nu[0]*(Qinfy[0]*tinfy[3]-tinfy[2])*Pinfty[0]+nu[0]*(2*tinfy[0]*tinfy[3]+2*tinfy[1]*tinfy[2]-tinfy[3])-2*nu[0]*(tinfy[1]*tinfy[3]+tinfy[2]^2)*Qinfy[0])
;
simplify(series(omega*(CoeffLambda0A21-CoeffLambda0A21bis),Qinfy[0]));

```

$$CoeffLambda1A21 := \frac{2 v_0 tinfy_3 (-Qinfy_0^2 tinfy_3 + Q01 tinfy_3 + Qinfy_0 tinfy_2 + Pinfty_0 - tinfy_1)}{\omega} \tag{4.5}$$

$$CoeffLambda0A21bis := \frac{1}{\omega} \left(-LQinfy_0 tinfy_3 - 2 tinfy_3 \left((-Qinfy_0^2 + Q01) tinfy_3 + Qinfy_0 tinfy_2 - tinfy_1 \right) v_1 + \alpha_2 + 4 tinfy_3 v_0 (Qinfy_0 tinfy_3 - tinfy_2) Q01 - 2 Qinfy_0^3 v_0 tinfy_3^2 + 4 Qinfy_0^2 v_0 tinfy_2 tinfy_3 + 2 v_0 (Qinfy_0 tinfy_3 - tinfy_2) Pinfty_0 + v_0 (2 tinfy_0 tinfy_3 + 2 tinfy_1 tinfy_2 - tinfy_3) - 2 v_0 (tinfy_1 tinfy_3 + tinfy_2^2) Qinfy_0 \right)$$

0

Compatibility equation and evolutions

```

> L:=Matrix(2,2,0):
L[1,1]:=L11:
L[1,2]:=L12:

```

```

L[2,1]:=L21:
L[2,2]:=-L[1,1]:
A:=Matrix(2,2,0):
A[1,1]:=A11:
A[1,2]:=A12:
A[2,1]:=A21:
A[2,2]:=-A[1,1]:
> dAdlamba:=Matrix(2,2,0):
for i from 1 to 2 do for j from 1 to 2 do dAdlamba[i,j]:=diff
(A[i,j],lambda): od: od:
dAdlamba:
> BigLL:=Matrix(2,2,0):
BigLL[1,1]:= diff(L[1,1],Qinfy[0])*LQinfy0+ diff(L[1,1],Q01)*
LQ01+diff(L[1,1],Pinfty[0])*LPinfy0+ diff(L[1,1],P01)*LP01+
diff(L[1,1],tinfty[2])*alpha[2] +diff(L[1,1],tinfty[1])*alpha
[1]+ diff(L[1,1],omega)*Lomega:
BigLL[1,2]:= diff(L[1,2],Qinfy[0])*LQinfy0+ diff(L[1,2],Q01)*
LQ01+diff(L[1,2],Pinfty[0])*LPinfy0+ diff(L[1,2],P01)*LP01+
diff(L[1,2],tinfty[2])*alpha[2] +diff(L[1,2],tinfty[1])*alpha
[1]+ diff(L[1,2],omega)*Lomega:
BigLL[2,1]:= diff(L[2,1],Qinfy[0])*LQinfy0+ diff(L[2,1],Q01)*
LQ01+diff(L[2,1],Pinfty[0])*LPinfy0+ diff(L[2,1],P01)*LP01+
diff(L[2,1],tinfty[2])*alpha[2] +diff(L[2,1],tinfty[1])*alpha
[1]+ diff(L[2,1],omega)*Lomega:
BigLL[2,2]:=-BigLL[1,1]:
> Commutator:=Multiply(A,L)-Multiply(L,A):
CompatibilityEquation:= simplify(dAdlamba-BigLL+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
lambda=infinity));
EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
lambda=infinity));
EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
infinity));
EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
infinity));

series(simplify(series(CompatibilityEquation[1,1], lambda=
infinity)),lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
lambda=infinity));
EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,

```

```

lambda=infinity));
EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity));
EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda,lambda=
infinity));
EQ7:=simplify(residue(CompatibilityEquation[1,1],lambda=
infinity));

```

$$\begin{aligned} EQ0 &:= 0 \\ EQ1 &:= 0 \end{aligned} \quad (4.6)$$

$$EQ2 := (2 P01 Q01 v_0 - 2 Pinfty_0 v_1 + LQinfty0 - v_0) \omega$$

$$EQ3 := (2 P01 Q01 Qinfty_0 v_0 + 2 P01 Q01 v_1 + 2 Q01 Pinfty_0 v_0 + LQ01) \omega$$

$$EQ0 := 0$$

$$EQ4 := 0$$

$$EQ5 := -tinfty_3 (2 P01 Q01 v_0 - 2 Pinfty_0 v_1 + LQinfty0 - v_0)$$

$$\begin{aligned} EQ6 := & \frac{1}{Q01} \left(-2 v_0 tinfty_3^2 Q01^3 + \left((3 Qinfty_0^2 v_0 - 6 Qinfty_0 v_1) tinfty_3^2 + \left((-2 P01 \right. \right. \right. \\ & + 4 tinfty_2) v_1 - 4 v_0 \left(Qinfty_0 tinfty_2 - \frac{1}{2} tinfty_1 + \frac{1}{2} Pinfty_0 \right) \left. \right) tinfty_3 + v_0 (P01 \\ & - tinfty_2)^2 \left. \right) Q01^2 + \left(4 v_1 Qinfty_0^3 tinfty_3^2 + \left((-6 Qinfty_0^2 tinfty_2 + (-2 Pinfty_0 \right. \right. \right. \\ & + 4 tinfty_1) Qinfty_0 - 2 tinfty_0 + 1) v_1 - Qinfty_0 v_0 + Qinfty_0 LQinfty0 - LQ01 \left. \right) tinfty_3 \\ & + 2 tinfty_2 (Qinfty_0 tinfty_2 + Pinfty_0 - tinfty_1) v_1 + v_0 tinfty_2 - tinfty_2 LQinfty0 \\ & \left. - LPinfty_0 \right) Q01 - t00^2 v_0) \end{aligned}$$

$$\begin{aligned} EQ7 := & \frac{1}{Q01} \left(\left((4 Qinfty_0 v_0 - 2 v_1) tinfty_3^2 - 4 tinfty_2 v_0 tinfty_3 \right) Q01^3 + \left((-Qinfty_0^3 v_0 \right. \right. \\ & + 3 Qinfty_0^2 v_1) tinfty_3^2 + \left((2 P01 + 2 tinfty_2) Qinfty_0^2 + (2 Pinfty_0 - 2 tinfty_1) Qinfty_0 \right. \\ & + 2 tinfty_0 - 1) v_0 + 2 \left((P01 - 2 tinfty_2) Qinfty_0 + tinfty_1 \right) v_1 \left. \right) tinfty_3 + \left((P01^2 \right. \\ & - 2 P01 tinfty_2 - tinfty_2^2) Qinfty_0 + 2 tinfty_2 (-Pinfty_0 + tinfty_1) \left. \right) v_0 + P01^2 v_1 \\ & - 2 P01 v_1 tinfty_2 + v_1 tinfty_2^2 + LP01 \left. \right) Q01^2 + LQ01 (Qinfty_0 tinfty_3 + P01 \\ & - tinfty_2) Q01 - t00^2 (Qinfty_0 v_0 + v_1) \end{aligned}$$

```
> LQinfty0:=solve(EQ2,LQinfty0);
```

```
LQ01:=solve(EQ3,LQ01);
```

$$LQinfty0 := -2 P01 Q01 v_0 + 2 Pinfty_0 v_1 + v_0 \quad (4.7)$$

$$LQ01 := -2 Q01 (P01 Qinfty_0 v_0 + P01 v_1 + Pinfty_0 v_0)$$

```
> simplify(EQ5);
```

```
simplify(EQ6);
```

```
simplify(EQ7);
```

$$0$$

(4.8)

$$\frac{1}{Q01} \left(-2 v_0 tinfty_3^2 Q01^3 + \left((3 Qinfty_0^2 v_0 - 6 Qinfty_0 v_1) tinfty_3^2 + \left(4 v_1 tinfty_2 \right. \right. \right.$$

$$\begin{aligned}
& -4 \left(Q_{infty_0} t_{infty_2} - \frac{1}{2} t_{infty_1} \right) v_0 \left) t_{infty_3} + v_0 \left(P0I^2 + t_{infty_2}^2 \right) \right) Q0I^2 + \left(4 v_1 \right. \\
& Q_{infty_0}^3 t_{infty_3}^2 - 6 v_1 \left(Q_{infty_0}^2 t_{infty_2} - \frac{2}{3} Q_{infty_0} t_{infty_1} + \frac{1}{3} t_{infty_0} - \frac{1}{6} \right) t_{infty_3} \\
& \left. + \left(2 Q_{infty_0} t_{infty_2}^2 - 2 t_{infty_1} t_{infty_2} \right) v_1 - LP_{infty_0} \right) Q0I - t00^2 v_0 \left) \right. \\
& \frac{1}{Q0I} \left(\left(\left(4 Q_{infty_0} t_{infty_3}^2 - 4 t_{infty_2} t_{infty_3} \right) v_0 - 2 t_{infty_3}^2 v_1 \right) Q0I^3 + \left(-Q_{infty_0}^3 t_{infty_3}^2 \right. \right. \\
& \left. \left. + \left(2 Q_{infty_0}^2 t_{infty_2} - 2 Q_{infty_0} t_{infty_1} + 2 t_{infty_0} - 1 \right) t_{infty_3} + \left(-P0I^2 - \right. \right. \right. \\
& \left. \left. t_{infty_2}^2 \right) Q_{infty_0} - 2 P0I P_{infty_0} + 2 t_{infty_1} t_{infty_2} \right) v_0 + 3 t_{infty_3}^2 v_1 Q_{infty_0}^2 + 2 v_1 \left(\right. \\
& \left. -2 Q_{infty_0} t_{infty_2} + t_{infty_1} \right) t_{infty_3} + \left(-P0I^2 + t_{infty_2}^2 \right) v_1 + LP0I \right) Q0I^2 \\
& \left. - t00^2 \left(Q_{infty_0} v_0 + v_1 \right) \right)
\end{aligned}$$

> LP_{infty0} := solve(EQ6, LP_{infty0}) ;

LP01 := solve(EQ7, LP01) ;

$$\begin{aligned}
LP_{infty_0} := & \frac{1}{Q0I} \left(3 Q0I^2 Q_{infty_0}^2 v_0 t_{infty_3}^2 + 4 Q0I Q_{infty_0}^3 v_1 t_{infty_3}^2 - 2 Q0I^3 v_0 t_{infty_3}^2 \right. \\
& - 4 Q0I^2 Q_{infty_0} v_0 t_{infty_2} t_{infty_3} - 6 Q0I^2 Q_{infty_0} v_1 t_{infty_3}^2 - 6 Q0I \\
& Q_{infty_0}^2 v_1 t_{infty_2} t_{infty_3} + P0I^2 Q0I^2 v_0 + 2 Q0I^2 v_0 t_{infty_1} t_{infty_3} + Q0I^2 v_0 t_{infty_2}^2 \\
& + 4 Q0I^2 v_1 t_{infty_2} t_{infty_3} + 4 Q0I Q_{infty_0} v_1 t_{infty_1} t_{infty_3} + 2 Q0I Q_{infty_0} v_1 t_{infty_2}^2 \\
& \left. - 2 Q0I v_1 t_{infty_0} t_{infty_3} - 2 Q0I v_1 t_{infty_1} t_{infty_2} + Q0I v_1 t_{infty_3} - t00^2 v_0 \right)
\end{aligned}$$

(4.9)

$$\begin{aligned}
LP01 := & \frac{1}{Q0I^2} \left(Q0I^2 Q_{infty_0}^3 v_0 t_{infty_3}^2 - 4 Q0I^3 Q_{infty_0} v_0 t_{infty_3}^2 - 2 Q0I^2 \right. \\
& Q_{infty_0}^2 v_0 t_{infty_2} t_{infty_3} - 3 Q0I^2 Q_{infty_0}^2 v_1 t_{infty_3}^2 + P0I^2 Q0I^2 Q_{infty_0} v_0 \\
& + 4 Q0I^3 v_0 t_{infty_2} t_{infty_3} + 2 Q0I^3 v_1 t_{infty_3}^2 + 2 Q0I^2 Q_{infty_0} v_0 t_{infty_1} t_{infty_3} \\
& + Q0I^2 Q_{infty_0} v_0 t_{infty_2}^2 + 4 Q0I^2 Q_{infty_0} v_1 t_{infty_2} t_{infty_3} + P0I^2 Q0I^2 v_1 \\
& + 2 P0I Q0I^2 P_{infty_0} v_0 - 2 Q0I^2 v_0 t_{infty_0} t_{infty_3} - 2 Q0I^2 v_0 t_{infty_1} t_{infty_2} \\
& \left. - 2 Q0I^2 v_1 t_{infty_1} t_{infty_3} - Q0I^2 v_1 t_{infty_2}^2 + Q0I^2 v_0 t_{infty_3} + t00^2 Q_{infty_0} v_0 + t00^2 v_1 \right)
\end{aligned}$$

> Ham := -2*P01*Q01*P_{infty}[0]*nu[0]+P_{infty}[0]^2*nu[1]+P_{infty}[0]*nu[0]-P01^2*Q01*(Q_{infty}[0]*nu[0]+nu[1])
-((-2*nu[0]*t_{infty}[3]^2*Q01^3+(2*nu[0]*t_{infty}[1]+4*nu[1]*t_{infty}[2])*t_{infty}[3]+nu[0]*t_{infty}[2]^2)*Q01^2-2*nu[1]*((t_{infty}[0]-1/2)*t_{infty}[3]+t_{infty}[1]*t_{infty}[2])*Q01-t00^2*nu[0])*Q_{infty}[0]/Q01+((1/2)*(-6*Q01*t_{infty}[3]^2+4*t_{infty}[1]*t_{infty}[3]+2*t_{infty}[2]^2)*nu[1]-2*Q01*nu[0]*t_{infty}[2]*t_{infty}[3])*Q_{infty}[0]^2+t_{infty}[3]*(Q01*nu[0]*t_{infty}[3]-2*nu[1]*t_{infty}[2])*Q_{infty}[0]^3+t_{infty}[3]^2*nu[1]*Q_{infty}[0]^4)
-(2*Q01^2*nu[0]*t_{infty}[2]*t_{infty}[3]+Q01^2*nu[1]*t_{infty}[3]^2-2*Q01*nu[0]*t_{infty}[0]*t_{infty}[3]-2*Q01*nu[0]*t_{infty}[1]*t_{infty}[2]-2*Q01*nu[1]*t_{infty}[1]*t_{infty}[3]-Q01*nu[1]*t_{infty}[2]^2+nu[0]*

`tinfty[3]*Q01-t00^2*nu[1]/Q01);`

`simplify(diff(Ham,Pinfy[0])-LQinfy0);`

`simplify(diff(Ham,P01)-LQ01);`

`simplify(diff(Ham,Qinfy[0])+LPinfy0);`

`simplify(diff(Ham,Q01)+LP01);`

$$Ham := -2 P01 Q01 Pinfy_0 v_0 + Pinfy_0^2 v_1 + v_0 Pinfy_0 - P01^2 Q01 (Qinfy_0 v_0 + v_1) \quad (4.10)$$

$$- \frac{1}{Q01} \left(\left(-2 v_0 tinfty_3^2 Q01^3 + \left((2 v_0 tinfty_1 + 4 v_1 tinfty_2) tinfty_3 + v_0 tinfty_2^2 \right) Q01^2 \right. \right.$$

$$\left. - 2 v_1 \left(\left(tinfty_0 - \frac{1}{2} \right) tinfty_3 + tinfty_1 tinfty_2 \right) Q01 - t00^2 v_0 \right) Qinfy_0 - \left(\frac{1}{2} \left(\right. \right.$$

$$\left. - 6 Q01 tinfty_3^2 + 4 tinfty_1 tinfty_3 + 2 tinfty_2^2 \right) v_1 - 2 Q01 v_0 tinfty_2 tinfty_3 \left. \right) Qinfy_0^2$$

$$- tinfty_3 \left(Q01 v_0 tinfty_3 - 2 v_1 tinfty_2 \right) Qinfy_0^3 - tinfty_3^2 v_1 Qinfy_0^4$$

$$- 2 Q01^2 v_0 tinfty_2 tinfty_3 - Q01^2 v_1 tinfty_3^2 + 2 Q01 v_0 tinfty_0 tinfty_3$$

$$+ 2 Q01 v_0 tinfty_1 tinfty_2 + 2 Q01 v_1 tinfty_1 tinfty_3 + Q01 v_1 tinfty_2^2 - v_0 tinfty_3 Q01$$

$$+ \frac{t00^2 v_1}{Q01}$$

0
0
0
0

`> Qinfy[0]:=-Sinfy[0];`

`Q01:=S01;`

`P01:=R01+tinfty[3]*Sinfy[0];`

`Pinfy[0]:=-Rinfy[0]+tinfty[3]*Sinfy[0]^2-tinfty[3]*S01;`

$$Qinfy_0 := -Sinfy_0$$

$$Q01 := S01$$

$$P01 := Sinfy_0 tinfty_3 + R01$$

$$Pinfy_0 := Sinfy_0^2 tinfty_3 - S01 tinfty_3 - Rinfy_0$$

(4.11)

`> simplify(series(L11,lambda=infinity));`

`simplify(L11-(-tinfty[3]*lambda^2+Rinfy[0] +R01*S01/lambda-tinfty[2]*L12/omega));`

`simplify(series(L12,lambda=infinity));`

`ConstantTermL21:=simplify(-omega*residue(L21/lambda,lambda=infinity));`

`ConstantTermL21bis:=2*tinfty[3]*R01*S01+2*(Sinfy[0]*tinfty[3]+tinfty[2])*Rinfy[0]-2*S01*tinfty[2]*tinfty[3]+2*Sinfy[0]^2*tinfty[2]*tinfty[3] +2*(tinfty[1]*tinfty[3]+tinfty[2]^2)*Sinfy[0]+2*(tinfty[0]*tinfty[3]+tinfty[1]*tinfty[2]);`

`ConstantTermL21bis;`

`series(simplify(ConstantTermL21-ConstantTermL21bis),Sinfy[0]);`

$$\frac{-\lambda^3 \text{tiny}_3 - \lambda^2 \text{tiny}_2 + (\text{Sinfty}_0 \text{tiny}_2 + \text{Rinfty}_0) \lambda + \text{S01} (\text{R01} - \text{tiny}_2)}{\lambda} \quad (4.12)$$

$$\frac{\omega (\lambda^2 - \lambda \text{Sinfty}_0 + \text{S01})}{\lambda}$$

$$2 \text{R01} \text{S01} \text{tiny}_3 + 2 (\text{Sinfty}_0 \text{tiny}_3 + \text{tiny}_2) \text{Rinfty}_0 - 2 \text{S01} \text{tiny}_2 \text{tiny}_3 + 2 \text{Sinfty}_0^2 \text{tiny}_2 \text{tiny}_3 + 2 (\text{tiny}_1 \text{tiny}_3 + \text{tiny}_2^2) \text{Sinfty}_0 + 2 \text{tiny}_0 \text{tiny}_3 + 2 \text{tiny}_1 \text{tiny}_2$$

0

```
> series (A12, lambda=infinity) ;
series (A11, lambda=infinity) ;
series (Lomega/ (2*omega)+tiny[3] * (S01*nu[0]-Sinfty[0]*nu[1]) +
nu[0] * ((Sinfty[0]^2-S01) *tiny[3]+Sinfty[0]*tiny[2]-Sinfty
[0]^2*tiny[3]+S01*tiny[3]+Rinfty[0]) -nu[1]*tiny[2], nu[0])
;
```

$$v_0 \omega \lambda + \omega (-\text{Sinfty}_0 v_0 + v_1) \quad (4.13)$$

$$-v_0 \text{tiny}_3 \lambda^2 + (-v_0 \text{tiny}_2 - v_1 \text{tiny}_3) \lambda + \frac{1}{2} \frac{\text{Lomega}}{\omega} + \text{tiny}_3 (\text{S01} v_0 - \text{Sinfty}_0 v_1) + v_0 ((\text{Sinfty}_0^2 - \text{S01}) \text{tiny}_3 + \text{Sinfty}_0 \text{tiny}_2 - \text{tiny}_3 \text{Sinfty}_0^2 + \text{tiny}_3 \text{S01} + \text{Rinfty}_0) - v_1 \text{tiny}_2$$

$$\frac{1}{2} \frac{\text{Lomega}}{\omega} - \text{Sinfty}_0 v_1 \text{tiny}_3 - v_1 \text{tiny}_2 + (2 \text{tiny}_3 \text{S01} + (\text{Sinfty}_0^2 - \text{S01}) \text{tiny}_3 + \text{Sinfty}_0 \text{tiny}_2 - \text{tiny}_3 \text{Sinfty}_0^2 + \text{Rinfty}_0) v_0$$

```
> series (simplify (A21), lambda=infinity) ;
ConstantTermA21:=simplify (-omega*residue (A21/lambda, lambda=
infinity)) :
ConstantTermA21bis:=alpha[2]+2*tiny[3] * (tiny[2]*Sinfty[0]+
Rinfty[0]+tiny[1]) *nu[1]
+nu[0] * (2*tiny[3]*R01*S01+2*(tiny[3]*Sinfty[0]+tiny[2]) *
Rinfty[0] -2*tiny[2]*tiny[3]*S01+2*tiny[2]*tiny[3] *
(Sinfty[0])^2+2*(tiny[1]*tiny[3]+(tiny[2])^2)*Sinfty[0]
+2*((tiny[0]-1)*tiny[3]+tiny[1]*tiny[2])) :
series (simplify (ConstantTermA21-ConstantTermA21bis), Sinfty[0]) ;
ConstantTermA21bis ;
```

$$\frac{(2 \text{Sinfty}_0 \text{tiny}_2 + 2 \text{Rinfty}_0 + 2 \text{tiny}_1) v_0 \text{tiny}_3 \lambda}{\omega} + \frac{1}{\omega} \left(\left((2 \text{Sinfty}_0^2 - 2 \text{S01}) \text{tiny}_2 \right. \right. \quad (4.14)$$

$$\left. + (2 \text{Rinfty}_0 + 2 \text{tiny}_1) \text{Sinfty}_0 + 2 \text{S01} \text{R01} + 2 \text{tiny}_0 - 2 \right) v_0 + 2 v_1 (\text{Sinfty}_0 \text{tiny}_2 + \text{Rinfty}_0 + \text{tiny}_1) \text{tiny}_3 + 2 \text{tiny}_2 (\text{Sinfty}_0 \text{tiny}_2 + \text{Rinfty}_0 + \text{tiny}_1) v_0 + \alpha_2$$

0

$$\alpha_2 + 2 \text{tinfty}_3 (\text{Sinfty}_0 \text{tinfty}_2 + \text{Rinfty}_0 + \text{tinfty}_1) v_1 + v_0 (2 \text{R01} \text{S01} \text{tinfty}_3$$

$$+ 2 (\text{Sinfty}_0 \text{tinfty}_3 + \text{tinfty}_2) \text{Rinfty}_0 - 2 \text{S01} \text{tinfty}_2 \text{tinfty}_3 + 2 \text{Sinfty}_0^2 \text{tinfty}_2 \text{tinfty}_3$$

$$+ 2 (\text{tinfty}_1 \text{tinfty}_3 + \text{tinfty}_2^2) \text{Sinfty}_0 + 2 (\text{tinfty}_0 - 1) \text{tinfty}_3 + 2 \text{tinfty}_1 \text{tinfty}_2)$$

```

> BigLLRS:=Matrix(2,2,0):
BigLLRS[1,1]:= diff(L[1,1],Sinfty[0])*LSinfty0+ diff(L[1,1],
S01)*LS01+diff(L[1,1],Rinfty[0])*LRinfty0+ diff(L[1,1],R01)*
LR01+diff(L[1,1],tinfty[2])*alpha[2] +diff(L[1,1],tinfty[1])*
alpha[1]+ diff(L[1,1],omega)*Lomega:
BigLLRS[1,2]:= diff(L[1,2],Sinfty[0])*LSinfty0+ diff(L[1,2],
S01)*LS01+diff(L[1,2],Rinfty[0])*LRinfty0+ diff(L[1,2],R01)*
LR01+diff(L[1,2],tinfty[2])*alpha[2] +diff(L[1,2],tinfty[1])*
alpha[1]+ diff(L[1,2],omega)*Lomega:
BigLLRS[2,1]:= diff(L[2,1],Sinfty[0])*LSinfty0+ diff(L[2,1],
S01)*LS01+diff(L[2,1],Rinfty[0])*LRinfty0+ diff(L[2,1],R01)*
LR01+diff(L[2,1],tinfty[2])*alpha[2] +diff(L[2,1],tinfty[1])*
alpha[1]+ diff(L[2,1],omega)*Lomega:
BigLLRS[2,2]:=-BigLL[1,1]:
> A:=simplify(A):
L:=simplify(L):
> Commutator:=Multiply(A,L)-Multiply(L,A):
CompatibilityEquation:= simplify(dAdlambda-BigLLRS+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
lambda=infinity));
EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
lambda=infinity));
EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
infinity));
EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
infinity));

series(simplify(series(CompatibilityEquation[1,1], lambda=
infinity)),lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
lambda=infinity));
EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
lambda=infinity));
EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity));
EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda,lambda=
infinity));

```

EQ7:=simplify(residue(CompatibilityEquation[1,1],lambda=infinity));

$$\begin{aligned} EQ0 &:= 0 \\ EQ1 &:= 0 \end{aligned} \quad (4.15)$$

$$EQ2 := 2 \omega \left(\left((Sinfy_0 \text{tinfty}_3 + R01) v_0 + v_1 \text{tinfty}_3 \right) S01 + \left(-Sinfy_0^2 \text{tinfty}_3 + Rinfy_0 \right) v_1 - \frac{1}{2} v_0 - \frac{1}{2} L Sinfy_0 \right)$$

$$EQ3 := -2 \left(S01^2 v_0 \text{tinfty}_3 + \left((R01 Sinfy_0 + Rinfy_0) v_0 - (Sinfy_0 \text{tinfty}_3 + R01) v_1 \right) S01 - \frac{1}{2} L S01 \right) \omega$$

$$\begin{aligned} EQ0 &:= 0 \\ EQ4 &:= 0 \\ EQ5 &:= 0 \end{aligned}$$

$$EQ6 := \frac{1}{S01} \left(\left(\left((2 Sinfy_0 v_0 + 2 v_1) \text{tinfty}_2 - 2 R01 v_1 + 2 v_0 (Rinfy_0 + \text{tinfty}_1) \right) \text{tinfty}_3 + v_0 (R01 - \text{tinfty}_2)^2 \right) S01^2 + \left((-4 \text{tinfty}_2 v_1 Sinfy_0^2 + (-4 \text{tinfty}_1 - 4 Rinfy_0) Sinfy_0 - 2 \text{tinfty}_0 + 1) v_1 + 2 Sinfy_0 v_0 \right) \text{tinfty}_3 - 2 v_1 Sinfy_0 \text{tinfty}_2^2 + \left((-2 Rinfy_0 - 2 \text{tinfty}_1) v_1 + v_0 + L Sinfy_0 \right) \text{tinfty}_2 + L Rinfy_0 \right) S01 - t00^2 v_0$$

$$EQ7 := \frac{1}{S01} \left(2 \text{tinfty}_3 v_0 (R01 - \text{tinfty}_2) S01^3 + \left((Sinfy_0 \text{tinfty}_2^2 + (2 Sinfy_0^2 \text{tinfty}_3 + 2 R01 Sinfy_0 + 2 Rinfy_0 + 2 \text{tinfty}_1) \text{tinfty}_2 + ((2 Rinfy_0 + 2 \text{tinfty}_1) Sinfy_0 + 2 \text{tinfty}_0 - 2) \text{tinfty}_3 - R01^2 Sinfy_0 \right) v_0 + v_1 \text{tinfty}_2^2 - 2 v_1 (-Sinfy_0 \text{tinfty}_3 + R01) \text{tinfty}_2 + 2 v_1 (Rinfy_0 + \text{tinfty}_1) \text{tinfty}_3 + R01^2 v_1 + L R01 \right) S01^2 + (R01 - \text{tinfty}_2) L S01 S01 + t00^2 (Sinfy_0 v_0 - v_1)$$

> L Sinfy0:=solve(EQ2, L Sinfy0);
LS01:=solve(EQ3, LS01);

$$L Sinfy_0 := 2 S01 Sinfy_0 v_0 \text{tinfty}_3 - 2 Sinfy_0^2 v_1 \text{tinfty}_3 + 2 R01 S01 v_0 + 2 S01 v_1 \text{tinfty}_3 + 2 Rinfy_0 v_1 - v_0 \quad (4.16)$$

$$L S01 := 2 S01 \left(R01 Sinfy_0 v_0 + S01 v_0 \text{tinfty}_3 - Sinfy_0 v_1 \text{tinfty}_3 - R01 v_1 + Rinfy_0 v_0 \right)$$

> simplify(EQ5);
simplify(EQ6);
simplify(EQ7);

$$0 \quad (4.17)$$

$$\frac{1}{S01} \left(\left((-2 R01 + 4 \text{tinfty}_2) v_1 + 2 v_0 (2 Sinfy_0 \text{tinfty}_2 + Rinfy_0 + \text{tinfty}_1) \right) \text{tinfty}_3 + v_0 (R01^2 + \text{tinfty}_2^2) \right) S01^2 + \left((-6 Sinfy_0^2 \text{tinfty}_2 + (-4 \text{tinfty}_1 - 4 Rinfy_0) Sinfy_0 - 2 \text{tinfty}_0 + 1) v_1 + 2 Sinfy_0 v_0 \right) \text{tinfty}_3 + \left(-2 Sinfy_0 \text{tinfty}_2^2 - 2 \text{tinfty}_1 \text{tinfty}_2 \right) v_1 + L Rinfy_0 \right) S01 - t00^2 v_0$$

$$\frac{1}{S01} \left(4 \text{tinfty}_3 v_0 (R01 - \text{tinfty}_2) S01^3 + \left((2 \text{Sinfty}_0^2 \text{tinfty}_2 + (2 \text{Rinfty}_0 + 2 \text{tinfty}_1) \text{Sinfty}_0 + 2 \text{tinfty}_0 - 2) \text{tinfty}_3 + (R01^2 + \text{tinfty}_2^2) \text{Sinfty}_0 + 2 R01 \text{Rinfty}_0 + 2 \text{tinfty}_1 \text{tinfty}_2 \right) v_0 - 2 v_1 \left((R01 - 2 \text{tinfty}_2) \text{Sinfty}_0 - \text{tinfty}_1 - \text{Rinfty}_0 \right) \text{tinfty}_3 + \left(-R01^2 + \text{tinfty}_2^2 \right) v_1 + LR01 \right) S01^2 + t00^2 \left(\text{Sinfty}_0 v_0 - v_1 \right)$$

> LRinfty0:=solve(EQ6,LRinfty0);

LR01:=solve(EQ7,LR01);

$$LRinfty0 := -\frac{1}{S01} \left(4 S01^2 \text{Sinfty}_0 v_0 \text{tinfty}_2 \text{tinfty}_3 - 6 S01 \text{Sinfty}_0^2 v_1 \text{tinfty}_2 \text{tinfty}_3 \right. \quad (4.18)$$

$$+ R01^2 S01^2 v_0 - 2 R01 S01^2 v_1 \text{tinfty}_3 + 2 S01^2 \text{Rinfty}_0 v_0 \text{tinfty}_3$$

$$+ 2 S01^2 v_0 \text{tinfty}_1 \text{tinfty}_3 + S01^2 v_0 \text{tinfty}_2^2 + 4 S01^2 v_1 \text{tinfty}_2 \text{tinfty}_3$$

$$- 4 S01 \text{Rinfty}_0 \text{Sinfty}_0 v_1 \text{tinfty}_3 - 4 S01 \text{Sinfty}_0 v_1 \text{tinfty}_1 \text{tinfty}_3 - 2 S01 \text{Sinfty}_0 v_1 \text{tinfty}_2^2$$

$$+ 2 S01 \text{Sinfty}_0 v_0 \text{tinfty}_3 - 2 S01 v_1 \text{tinfty}_0 \text{tinfty}_3 - 2 S01 v_1 \text{tinfty}_1 \text{tinfty}_2$$

$$+ S01 v_1 \text{tinfty}_3 - t00^2 v_0)$$

$$LR01 := -\frac{1}{S01^2} \left(2 S01^2 \text{Sinfty}_0^2 v_0 \text{tinfty}_2 \text{tinfty}_3 + R01^2 S01^2 \text{Sinfty}_0 v_0 \right.$$

$$+ 4 R01 S01^3 v_0 \text{tinfty}_3 - 2 R01 S01^2 \text{Sinfty}_0 v_1 \text{tinfty}_3 - 4 S01^3 v_0 \text{tinfty}_2 \text{tinfty}_3$$

$$+ 2 S01^2 \text{Rinfty}_0 \text{Sinfty}_0 v_0 \text{tinfty}_3 + 2 S01^2 \text{Sinfty}_0 v_0 \text{tinfty}_1 \text{tinfty}_3 + S01^2 \text{Sinfty}_0 v_0 \text{tinfty}_2^2$$

$$+ 4 S01^2 \text{Sinfty}_0 v_1 \text{tinfty}_2 \text{tinfty}_3 - R01^2 S01^2 v_1 + 2 R01 S01^2 \text{Rinfty}_0 v_0$$

$$+ 2 S01^2 \text{Rinfty}_0 v_1 \text{tinfty}_3 + 2 S01^2 v_0 \text{tinfty}_0 \text{tinfty}_3 + 2 S01^2 v_0 \text{tinfty}_1 \text{tinfty}_2$$

$$+ 2 S01^2 v_1 \text{tinfty}_1 \text{tinfty}_3 + S01^2 v_1 \text{tinfty}_2^2 - 2 S01^2 v_0 \text{tinfty}_3 + t00^2 \text{Sinfty}_0 v_0 - t00^2 v_1)$$

> HamRS:=-((2*Sinfty[0]^2-2*S01)*nu[1]-2*S01*nu[0]*Sinfty[0])*
tinfty[3]*Rinfty[0]-2*R01*S01*nu[0]*Rinfty[0]-Rinfty[0]^2*nu[1]
+Rinfty[0]*nu[0]) -(-2*S01*(tinfty[3]*(S01*nu[0]-Sinfty[0]*nu
[1])*R01+(1/2)*R01^2*(Sinfty[0]*nu[0]-nu[1])) -(-2*S01*Sinfty
[0]^2*nu[0]*tinfty[2]*tinfty[3]+2*Sinfty[0]^3*nu[1]*tinfty[2]*
tinfty[3]-2*S01*Sinfty[0]*nu[0]*tinfty[1]*tinfty[3]-S01*Sinfty
[0]*nu[0]*tinfty[2]^2-4*S01*Sinfty[0]*nu[1]*tinfty[2]*tinfty[3]
+2*Sinfty[0]^2*nu[1]*tinfty[1]*tinfty[3]+nu[1]*Sinfty[0]^2*
tinfty[2]^2-Sinfty[0]^2*nu[0]*tinfty[3]+2*Sinfty[0]*nu[1]*
tinfty[0]*tinfty[3]+2*nu[1]*tinfty[2]*Sinfty[0]*tinfty[1]-
Sinfty[0]*tinfty[3]*nu[1]+t00^2*Sinfty[0]*nu[0]/S01) -(2*S01^2*
nu[0]*tinfty[2]*tinfty[3]-2*S01*nu[0]*tinfty[0]*tinfty[3]-2*
S01*nu[0]*tinfty[1]*tinfty[2]-2*S01*nu[1]*tinfty[1]*tinfty[3]-
S01*nu[1]*tinfty[2]^2+2*S01*nu[0]*tinfty[3]-t00^2*nu[1]/S01);
simplify(diff(HamRS,Rinfty[0])-LSinfty0);
simplify(diff(HamRS,R01)-LS01);
simplify(diff(HamRS,Sinfty[0])+LRinfty0);
simplify(diff(HamRS,S01)+LR01);

$$\begin{aligned}
HamRS := & - \left((2 \text{Sinfty}_0^2 - 2 \text{SOI}) v_1 - 2 \text{SOI} v_0 \text{Sinfty}_0 \right) \text{tinfty}_3 \text{Rinfty}_0 \\
& + 2 \text{ROI} \text{SOI} v_0 \text{Rinfty}_0 + \text{Rinfty}_0^2 v_1 - \text{Rinfty}_0 v_0 + 2 \text{SOI} \left(\text{tinfty}_3 (\text{SOI} v_0 \right. \\
& \left. - \text{Sinfty}_0 v_1) \text{ROI} + \frac{1}{2} \text{ROI}^2 (\text{Sinfty}_0 v_0 - v_1) \right) + 2 \text{SOI} \text{Sinfty}_0^2 v_0 \text{tinfty}_2 \text{tinfty}_3 - 2 \\
& \text{Sinfty}_0^3 v_1 \text{tinfty}_2 \text{tinfty}_3 + 2 \text{SOI} \text{Sinfty}_0 v_0 \text{tinfty}_1 \text{tinfty}_3 + \text{SOI} \text{Sinfty}_0 v_0 \text{tinfty}_2^2 \\
& + 4 \text{SOI} \text{Sinfty}_0 v_1 \text{tinfty}_2 \text{tinfty}_3 - 2 \text{Sinfty}_0^2 v_1 \text{tinfty}_1 \text{tinfty}_3 - v_1 \text{Sinfty}_0^2 \text{tinfty}_2^2 + \\
& \text{Sinfty}_0^2 v_0 \text{tinfty}_3 - 2 \text{Sinfty}_0 v_1 \text{tinfty}_0 \text{tinfty}_3 - 2 v_1 \text{tinfty}_2 \text{Sinfty}_0 \text{tinfty}_1 + \text{Sinfty}_0 v_1 \text{tinfty}_3 \\
& - \frac{t00^2 \text{Sinfty}_0 v_0}{\text{SOI}} - 2 \text{SOI}^2 v_0 \text{tinfty}_2 \text{tinfty}_3 + 2 \text{SOI} v_0 \text{tinfty}_0 \text{tinfty}_3 \\
& + 2 \text{SOI} v_0 \text{tinfty}_1 \text{tinfty}_2 + 2 \text{SOI} v_1 \text{tinfty}_1 \text{tinfty}_3 + \text{SOI} v_1 \text{tinfty}_2^2 - 2 \text{SOI} v_0 \text{tinfty}_3 \\
& + \frac{t00^2 v_1}{\text{SOI}}
\end{aligned}$$

0
0
0
0

```

> HamRS:=simplify(HamRS):
M:=Matrix(2*d,2*d,0):
for i from 1 to 2*d do for j from 1 to i do M[i,j]:=tinfty[2*
d+1-(i-j)]: od: od:
M;
alphaVector:=Matrix(2*d,1,0):
for i from 1 to 2*d do alphaVector[i]:=alpha[2*d+1-i]/(2*d+1-i)
: od:
alphaVector;
nuVector:=Multiply(M^(-1),alphaVector);
for i from 1 to 2*d do nu[i-1]:=nuVector[i,1]: od:
nu[0]:=nu[0];
nu[1]:=nu[1];

```

$$\begin{bmatrix} \text{tinfty}_3 & 0 \\ \text{tinfty}_2 & \text{tinfty}_3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} \alpha_2 \\ \alpha_1 \end{bmatrix}$$

(4.20)

$$\begin{bmatrix} \frac{1}{2} \frac{\alpha_2}{\text{tinfty}_3} \\ -\frac{1}{2} \frac{\text{tinfty}_2 \alpha_2}{\text{tinfty}_3^2} + \frac{\alpha_1}{\text{tinfty}_3} \end{bmatrix}$$

$$v_0 := \frac{1}{2} \frac{\alpha_2}{\text{tinfty}_3}$$

$$v_1 := -\frac{1}{2} \frac{\text{tinfty}_2 \alpha_2}{\text{tinfty}_3^2} + \frac{\alpha_1}{\text{tinfty}_3}$$

> HamRSfunction:=unapply(simplify(HamRS), alpha[1], alpha[2]):
HamRStinfty1:=simplify(HamRSfunction(1,0));
HamRStinfty2:=simplify(HamRSfunction(0,1));

$$\text{HamRStinfty1} := \frac{1}{\text{tinfty}_3 \text{SOI}} \left(\left((-2 \text{ROI} + 4 \text{tinfty}_2) \text{Sinfty}_0 + 2 \text{Rinfty}_0 + 2 \text{tinfty}_1 \right) \text{tinfty}_3 \right. \quad (4.21)$$

$$\left. - \text{ROI}^2 + \text{tinfty}_2^2 \right) \text{SOI}^2 + \left(-2 \left(\text{Sinfty}_0^2 \text{tinfty}_2 + (\text{Rinfty}_0 + \text{tinfty}_1) \text{Sinfty}_0 + \text{tinfty}_0 \right. \right.$$

$$\left. - \frac{1}{2} \right) \text{Sinfty}_0 \text{tinfty}_3 - \text{Sinfty}_0^2 \text{tinfty}_2^2 - 2 \text{Sinfty}_0 \text{tinfty}_1 \text{tinfty}_2 + \text{Rinfty}_0^2 \left. \right) \text{SOI} + t00^2$$

$$\text{HamRStinfty2} := \frac{1}{2} \frac{1}{\text{tinfty}_3^2 \text{SOI}} \left(2 \text{tinfty}_3^2 (\text{ROI} - \text{tinfty}_2) \text{SOI}^3 + \left((2 \text{Sinfty}_0^2 \text{tinfty}_2 \right. \right.$$

$$\left. + (2 \text{Rinfty}_0 + 2 \text{tinfty}_1) \text{Sinfty}_0 + 2 \text{tinfty}_0 - 2 \right) \text{tinfty}_3^2 + (\text{ROI} - \text{tinfty}_2) (\text{ROI} \text{Sinfty}_0$$

$$\left. + 3 \text{Sinfty}_0 \text{tinfty}_2 + 2 \text{Rinfty}_0) \text{tinfty}_3 + \text{ROI}^2 \text{tinfty}_2 - \text{tinfty}_2^3 \right) \text{SOI}^2 + \left(\text{Sinfty}_0^2 \text{tinfty}_3^2 \right.$$

$$\left. + \left(2 \text{tinfty}_2^2 \text{Sinfty}_0^3 + 2 \text{Sinfty}_0 \left((\text{Rinfty}_0 + \text{tinfty}_1) \text{Sinfty}_0 + \text{tinfty}_0 - \frac{1}{2} \right) \text{tinfty}_2 \right. \right.$$

$$\left. - \text{Rinfty}_0 \right) \text{tinfty}_3 + \text{Sinfty}_0^2 \text{tinfty}_2^3 + 2 \text{Sinfty}_0 \text{tinfty}_1 \text{tinfty}_2^2 - \text{Rinfty}_0^2 \text{tinfty}_2 \left. \right) \text{SOI}$$

$$- t00^2 (\text{Sinfty}_0 \text{tinfty}_3 + \text{tinfty}_2)$$

> HamRStinfty1bis:=t00^2/(SOI*tinfty[3]) -SOI*ROI^2/tinfty[3]-2*
SOI*ROI*Sinfty[0]+2*(SOI-Sinfty[0]^2)*Rinfty[0]+Rinfty[0]
^2/tinfty[3]+4*tinfty[2]*Sinfty[0]*SOI+(2*tinfty[1]*tinfty[3]+
tinfty[2]^2)*SOI/tinfty[3]-2*Sinfty[0]^3*tinfty[2]-(2*tinfty[1]
*tinfty[3]+tinfty[2]^2)*Sinfty[0]^2/tinfty[3]
-(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-tinfty[3])*Sinfty
[0]/tinfty[3];
simplify(series(HamRStinfty1-HamRStinfty1bis, Sinfty[0]));

$$\text{HamRStinfty1bis} := \frac{t00^2}{\text{tinfty}_3 \text{SOI}} - \frac{\text{SOI} \text{ROI}^2}{\text{tinfty}_3} - 2 \text{SOI} \text{ROI} \text{Sinfty}_0 + 2 \left(-\text{Sinfty}_0^2 \right. \quad (4.22)$$

$$\begin{aligned}
& + S01) Rinfy_0 + \frac{Rinfy_0^2}{tinfy_3} + 4 \tinfy_2 Sinfy_0 S01 + \frac{(2 \tinfy_1 \tinfy_3 + \tinfy_2^2) S01}{tinfy_3} - 2 \\
& Sinfy_0^3 \tinfy_2 - \frac{(2 \tinfy_1 \tinfy_3 + \tinfy_2^2) Sinfy_0^2}{tinfy_3} \\
& - \frac{(2 \tinfy_0 \tinfy_3 + 2 \tinfy_1 \tinfy_2 - \tinfy_3) Sinfy_0}{tinfy_3}
\end{aligned}$$

0

```

> HamRStinfy2bis := -(t00)^2*(Sinfy[0]*tinfy[3]+tinfy[2])/((2*
tinfy[3]^2)*S01)+(1/2)*(tinfy[3]*Sinfy[0]+tinfy[2])*S01*
R01^2/tinfy[3]^2+S01*(tinfy[3]*S01+Sinfy[0]*tinfy[2]+Rinfy
[0])*R01/tinfy[3] - (1/2)*Rinfy[0]^2*tinfy[2]/tinfy[3]^2+
(1/2)*((2*Sinfy[0]^2-2*S01)*tinfy[2]+2*S01*Sinfy[0]*tinfy
[3]-1)*Rinfy[0]/tinfy[3] -S01^2*tinfy[2]+(tinfy[2]^2)/
(tinfy[3])*(Sinfy[0])^3+(1/2)*((2*Sinfy[0]^2*tinfy[2]+2*
Sinfy[0]*tinfy[1]+2*tinfy[0]-2)*tinfy[3]^2-3*Sinfy[0]*
tinfy[2]^2*tinfy[3]-tinfy[2]^3)*S01/tinfy[3]^2+(tinfy[2]*
(2*tinfy[0]-1)*tinfy[3]+2*tinfy[1]*tinfy[2]^2)*Sinfy[0]/
(2*tinfy[3]^2)+(1/2)*(2*tinfy[1]*tinfy[2]*tinfy[3]+tinfy
[2]^3+tinfy[3]^2)*Sinfy[0]^2/tinfy[3]^2;
simplify(series(HamRStinfy2-HamRStinfy2bis, Sinfy[0]));

```

$$HamRStinfy2bis := -\frac{1}{2} \frac{t00^2 (Sinfy_0 \tinfy_3 + \tinfy_2)}{tinfy_3^2 S01} \tag{4.23}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{(Sinfy_0 \tinfy_3 + \tinfy_2) S01 R01^2}{tinfy_3^2} \\
& + \frac{S01 (S01 \tinfy_3 + Sinfy_0 \tinfy_2 + Rinfy_0) R01}{tinfy_3} - \frac{1}{2} \frac{Rinfy_0^2 \tinfy_2}{tinfy_3^2} \\
& + \frac{1}{2} \frac{((2 Sinfy_0^2 - 2 S01) \tinfy_2 + 2 S01 Sinfy_0 \tinfy_3 - 1) Rinfy_0}{tinfy_3} - S01^2 \tinfy_2 \\
& + \frac{tinfy_2^2 Sinfy_0^3}{tinfy_3} + \frac{1}{2} \frac{1}{tinfy_3^2} \left((2 Sinfy_0^2 \tinfy_2 + 2 Sinfy_0 \tinfy_1 + 2 \tinfy_0 \right. \\
& \left. - 2) \tinfy_3^2 - 3 Sinfy_0 \tinfy_2^2 \tinfy_3 - \tinfy_3^3 \right) S01 \\
& + \frac{1}{2} \frac{(\tinfy_2 (2 \tinfy_0 - 1) \tinfy_3 + 2 \tinfy_1 \tinfy_2^2) Sinfy_0}{tinfy_3^2} \\
& + \frac{1}{2} \frac{(2 \tinfy_1 \tinfy_2 \tinfy_3 + \tinfy_2^3 + \tinfy_3^2) Sinfy_0^2}{tinfy_3^2}
\end{aligned}$$

0