In this Maple file, we compute the Lax matrices and the Hamiltonian system for d=1.

```
> restart:
  with(LinearAlgebra):
  d:=1:
```

Formulas for the member of the Painlevé IV hierarchy and symmetry reduction in the (Q_\infty, Q_{0,1},P_\infty, P_{0,1}) variables

Definition of \check{L} in the (Q_\infty, Q_{0,1},P_\infty, P_{0,1}) variables. We omit the check for compactness.

```
> L12:=Q01/lambda+lambda^(2*d-1):
  for k from 0 to 2*d-2 do L12:=L12+ Qinfty[k]*lambda^k: od:
  L12:=L12*omega;
  L11:=P01*Q01/lambda:
  for k from 0 to 2*d-2 do L11:=L11-Pinfty[2*d-2-k]*lambda^k: od:
  L11:=L11- (tinfty[2*d+1]*lambda+tinfty[2*d]-tinfty[2*d+1]*
  Qinfty[2*d-2])*(L12/omega):
  L11:=simplify(L11):
  series(L11,lambda=infinity,20);
```

$$L12 := \left(\frac{Q01}{\lambda} + \lambda + Qinfty_0\right)\omega \tag{1.1}$$

$$-tinfty_3\,\lambda^2 - tinfty_2\,\lambda + \left(Qinfty_0^2 - Q01\right)tinfty_3 - Qinfty_0\,tinfty_2 - Pinfty_0 + \frac{Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)}{\lambda}$$

```
> series(simplify(series((t00^2/lambda^2- L11^2)/(L12/omega),
  lambda=0,3)),lambda=0,1);
  factor(residue((t00^2/lambda^2- L11^2)/(L12/omega),lambda=0));
  ResidueBis:=1/omega*(t00^2/Q01-Q01*(P01-tinfty[2*d]+tinfty[2*
  d+1]*Qinfty[2*d-2])^2);
  simplify( factor(residue((t00^2/lambda^2- L11^2)/L12,lambda=0))
  - ResidueBis);
```

$$-\frac{1}{\lambda}\frac{1}{Q01}\left(\left(Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right) - t00\right)\left(Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right) + t00\right)\right) + \frac{1}{Q01^2}\left(2\,tinfty_3\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)Q01^3 + \left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)\left(-Qinfty_0^2\,tinfty_3 + \left(P01 + tinfty_2\right)Qinfty_0 + 2\,Pinfty_0\right)Q01^2 - t00^2\,Qinfty_0\right) + O(\lambda)$$

$$-\frac{1}{Q01}\left(\left(Q01\,Qinfty_0\,tinfty_3 + P01\,Q01 - Q01\,tinfty_2 - t00\right)\left(Q01\,Qinfty_0\,tinfty_3 + P01\,Q01 - Q01\,tinfty_2 + t00\right)\right) \tag{1.2}$$

$$ResidueBis := \frac{\frac{t00^2}{Q01} - Q01\left(Qinfty_0\, tinfty_3 + P01 - tinfty_2\right)^2}{\omega}\Bigg|_{0}$$

```
> L21infty:=0:
  for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21infty:=
  L21infty+ tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j: od: od:
  Numm:=simplify((L21infty-L11^2)/(L12/omega)):
  L21:=0:
  for j from 1 to 60 do L21:=L21- 1/omega*simplify(residue(Numm*
  lambda^(-j),lambda=infinity))*lambda^(j-1): od:
  L21:=simplify(L21+ (t00^2/Q01-Q01*(Qinfty[2*d-2]*tinfty[2*d+1]+
  P01-tinfty[2*d])^2)/omega/lambda):
```

```
> series(simplify(series(L21,lambda=infinity,2)),lambda=infinity,
  2):
  CoefflambdaL21:=-factor(residue(L21/lambda^2,lambda=infinity));
  Coefflambda0L21:=-factor(residue(L21/lambda,lambda=infinity));
  Coefflambda0L21bis:=1/omega*( 2*tinfty[3]*Q01*P01+2*(tinfty[3]*
  Qinfty[0]-tinfty[2])*Pinfty[0]+4*(tinfty[3]^2*Qinfty[0]-tinfty
  [2]*tinfty[3])*Q01-2*tinfty[3]^2*Qinfty[0]^3+4*tinfty[3]*tinfty
  [2]*Qinfty[0]^2-2*(tinfty[2]^2+tinfty[3]*tinfty[1])*Qinfty[0]
  +2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2] );
  simplify(series(Coefflambda0L21-Coefflambda0L21bis,P01));
  CoefflambdaMinus1L21:=-factor(residue(L21,lambda=infinity)):
```

$$CoefflambdaL21 := -\frac{2\, tinfty_3\left(-Qinfty_0^2\, tinfty_3 + Q01\, tinfty_3 + Qinfty_0\, tinfty_2 + Pinfty_0 - tinfty_1\right)}{\omega} \tag{1.3}$$

$$Coefflambda0L21 := \frac{1}{\omega}\Big(2\Big(-Qinfty_0^3\, tinfty_3^2 + 2\, Q01\, Qinfty_0\, tinfty_3^2 + 2\, Qinfty_0^2\, tinfty_2\, tinfty_3 + P01\, Q01\, tinfty_3 - 2\, Q01\, tinfty_2\, tinfty_3 + Pinfty_0\, Qinfty_0\, tinfty_3 - Qinfty_0\, tinfty_1\, tinfty_3 - Qinfty_0\, tinfty_2^2 - Pinfty_0\, tinfty_2 + tinfty_0\, tinfty_3 + tinfty_1\, tinfty_2\Big)\Big)$$

$$Coefflambda0L21bis := \frac{1}{\omega}\Big(2\, P01\, Q01\, tinfty_3 + 2\left(Qinfty_0\, tinfty_3 - tinfty_2\right) Pinfty_0 + 4\left(Qinfty_0\, tinfty_3^2 - tinfty_2\, tinfty_3\right) Q01 - 2\, Qinfty_0^3\, tinfty_3^2 + 4\, Qinfty_0^2\, tinfty_2\, tinfty_3 - 2\left(tinfty_1\, tinfty_3 + tinfty_2^2\right) Qinfty_0 + 2\, tinfty_0\, tinfty_3 + 2\, tinfty_1\, tinfty_2\Big)\Big|_{0}$$

Definition of the auxiliary matrix in the (Q,P) variables.

```
> A12:=0:
  for i from 0 to 2*d-1 do A12:=A12+nu[i]*lambda^(2*d-1-i): od:
```

```
for k from 0 to 2*d-2 do for i from 0 to k do A12:=A12+Qinfty
[k]*nu[i]*lambda^(k-i) od: od:
A12:=omega*A12;
cinfty0:= 1/2/omega*Lomega+ tinfty[2*d+1]*(Q01*nu[0]+nu[1]*
Qinfty[0]):
A11:=cinfty0:
PolA12:=0:
for i from 0 to 2*d-1 do PolA12:=PolA12+nu[i]*lambda^(-i): od:
PolA12:=PolA12*L11:
for k from 0 to 60 do A11:=A11-residue(PolA12/lambda^(k+1),
lambda=infinity)*lambda^k od:
A11:=A11;
A22:=-A11:

A21:=(-tinfty[2*d+1]*Lomega/omega^2) *lambda- (tinfty[2*d]-
tinfty[2*d+1]*Qinfty[2*d-2])*Lomega/omega^2+ (alpha[2*d]-tinfty
[2*d+1]*LQinfty0)/omega -tinfty[2*d+1]*nu[0]/omega:
L21oper:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do
L21oper:=L21oper+tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j od:
od:
L21oper:=L21oper;
A12oper:=0:
for i from 0 to 2*d+1 do A12oper:=A12oper+nu[i]* lambda^(-i):
od:
A12oper:=A12oper;
for k from 0 to 60 do A21:=A21-residue( (L21oper*A12oper/(L12))
/lambda^(k+1),lambda=infinity)*lambda^k od:
for k from 0 to 60 do A21:=A21-residue( ((L11/(L12))*(L11*
A12/L12 -2*A11))/lambda^(k+1),lambda=infinity)*lambda^k od:
nu[2*d]:=-(Q01*nu[0]+Qinfty[0]*nu[1]);
Testnu2:=-nu[0]*Q01:
nu[2*d+1]:=Q01*Qinfty[0]*nu[0]+Qinfty[0]^2*nu[1]-Q01*nu[1]:
Testnu3:=-nu[1]*Q01:
for j from 2 to 2*d do Testnu3:=Testnu3- nu[j]*Qinfty[j-2]: od:
simplify(nu[2*d+1]-Testnu3);
for j from 1 to 2*d-1 do Testnu2:=Testnu2- nu[j]*Qinfty[j-1]:
od:
simplify(nu[2*d]-Testnu2);
A21:=series(simplify(series(simplify(A21),lambda=infinity)),
lambda=infinity);
```

$$A12 := \omega\left(\lambda\,\nu_0 + Qinfty_0\,\nu_0 + \nu_1\right) \tag{1.4}$$

$$A11 := \frac{1}{2}\,\frac{Lomega}{\omega} + tinfty_3\left(Q01\,\nu_0 + Qinfty_0\,\nu_1\right) + \nu_0\left(\left(Qinfty_0^2 - Q01\right)tinfty_3 - Qinfty_0\,tinfty_2 - Pinfty_0\right) - \nu_1\,tinfty_2 - \left(\nu_0\,tinfty_2 + \nu_1\,tinfty_3\right)\lambda - \nu_0\,tinfty_3\,\lambda^2$$

$$L21oper := \lambda^4\,tinfty_3^2 + 2\,\lambda^3\,tinfty_2\,tinfty_3 + 2\,\lambda^2\,tinfty_1\,tinfty_3 + \lambda^2\,tinfty_2^2 + 2\,\lambda\,tinfty_0\,tinfty_3 + 2\,\lambda\,tinfty_1\,tinfty_2$$

$$A12oper := \nu_0 + \frac{\nu_1}{\lambda} + \frac{\nu_2}{\lambda^2} + \frac{\nu_3}{\lambda^3}$$

$$\nu_2 := -Q01\,\nu_0 - Qinfty_0\,\nu_1$$

$$0$$

$$0$$

$$A21 := \frac{1}{\omega}\left(\left(\left(2\,Qinfty_0^2 - 2\,Q01\right)\nu_0\,tinfty_3^2 + \left(-2\,Qinfty_0\,tinfty_2 - 2\,Pinfty_0 + 2\,tinfty_1\right)\nu_0\,tinfty_3\right)\lambda\right) + \frac{1}{\omega}\left(\left(\left(-2\,Qinfty_0^3 + 4\,Q01\,Qinfty_0\right)\nu_0 - 2\,\nu_1\left(-Qinfty_0^2 + Q01\right)\right)tinfty_3^2 + \left(\left(4\,Qinfty_0^2\,tinfty_2 + \left(2\,Pinfty_0 - 2\,tinfty_1\right)Qinfty_0 - 4\,Q01\,tinfty_2 + 2\,tinfty_0 - 1\right)\nu_0 - 2\,tinfty_2\,\nu_1\,Qinfty_0 + 2\,tinfty_1\,\nu_1 - LQinfty0\right)tinfty_3 + 2\,tinfty_2\left(-Qinfty_0\,tinfty_2 - Pinfty_0 + tinfty_1\right)\nu_0 + \alpha_2\right)$$

```
> CoeffLambda1A21:=factor(-residue(A21/lambda^2,lambda=infinity))
  ;
  CoeffLambda0A21:=simplify(-residue(A21/lambda,lambda=infinity))
  :
  CoeffLambda0A21bis:=1/omega*(-tinfty[3]*LQinfty0 -2*tinfty[3]*(
  (-Qinfty[0]^2+Q01)*tinfty[3]+Qinfty[0]*tinfty[2]-tinfty[1])*nu
  [1]  +alpha[2] +4*tinfty[3]*nu[0]*(Qinfty[0]*tinfty[3]-tinfty
  [2])*Q01-2*Qinfty[0]^3*nu[0]*tinfty[3]^2+4*Qinfty[0]^2*nu[0]*
  tinfty[2]*tinfty[3]+2*nu[0]*(Qinfty[0]*tinfty[3]-tinfty[2])*
  Pinfty[0]+nu[0]*(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-
  tinfty[3])-2*nu[0]*(tinfty[1]*tinfty[3]+tinfty[2]^2)*Qinfty[0])
  ;
  simplify(series(omega*(CoeffLambda0A21-CoeffLambda0A21bis),
  Qinfty[0]));
```

(1.5)

$$CoeffLambda1A21 := -\frac{2\,\nu_0\,tinfty_3\left(-Qinfty_0^2\,tinfty_3 + Q01\,tinfty_3 + Qinfty_0\,tinfty_2 + Pinfty_0 - tinfty_1\right)}{\omega}$$

$$CoeffLambda0A21bis := \frac{1}{\omega}\left(-LQinfty0\,tinfty_3 - 2\,tinfty_3\left(\left(-Qinfty_0^2 + Q01\right)tinfty_3 + Qinfty_0\,tinfty_2 - tinfty_1\right)\nu_1 + \alpha_2 + 4\,tinfty_3\,\nu_0\left(Qinfty_0\,tinfty_3 - tinfty_2\right)Q01 - 2\,Qinfty_0^3\,\nu_0\,tinfty_3^2 + 4\,Qinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3 + 2\,\nu_0\left(Qinfty_0\,tinfty_3 - tinfty_2\right)Pinfty_0 + \nu_0\left(2\,tinfty_0\,tinfty_3 + 2\,tinfty_1\,tinfty_2 - tinfty_3\right) - 2\,\nu_0\left(tinfty_1\,tinfty_3 +\right.\right.$$

$tinfty_2^2\big)\ Qinfty_0\big)$

0

Compatibility equation and evolutions

```
> L:=Matrix(2,2,0):
  L[1,1]:=L11:
  L[1,2]:=L12:
  L[2,1]:=L21:
  L[2,2]:=-L[1,1]:
  A:=Matrix(2,2,0):
  A[1,1]:=A11:
  A[1,2]:=A12:
  A[2,1]:=A21:
  A[2,2]:=-A[1,1]:
> dAdlambda:=Matrix(2,2,0):
  for i from 1 to 2 do for j from 1 to 2 do dAdlambda[i,j]:=diff
  (A[i,j],lambda): od: od:
  dAdlambda:
> BigLL:=Matrix(2,2,0):
  BigLL[1,1]:= diff(L[1,1],Qinfty[0])*LQinfty0+ diff(L[1,1],Q01)*
  LQ01+diff(L[1,1],Pinfty[0])*LPinfty0+ diff(L[1,1],P01)*
  LP01+diff(L[1,1],tinfty[2])*alpha[2] +diff(L[1,1],tinfty[1])*
  alpha[1]+ diff(L[1,1],omega)*Lomega:
  BigLL[1,2]:= diff(L[1,2],Qinfty[0])*LQinfty0+ diff(L[1,2],Q01)*
  LQ01+diff(L[1,2],Pinfty[0])*LPinfty0+ diff(L[1,2],P01)*LP01+
  diff(L[1,2],tinfty[2])*alpha[2] +diff(L[1,2],tinfty[1])*alpha
  [1]+ diff(L[1,2],omega)*Lomega:
  BigLL[2,1]:= diff(L[2,1],Qinfty[0])*LQinfty0+ diff(L[2,1],Q01)*
  LQ01+diff(L[2,1],Pinfty[0])*LPinfty0+ diff(L[2,1],P01)*LP01+
  diff(L[2,1],tinfty[2])*alpha[2] +diff(L[2,1],tinfty[1])*alpha
  [1]+ diff(L[2,1],omega)*Lomega:
  BigLL[2,2]:=-BigLL[1,1]:
> Commutator:=Multiply(A,L)-Multiply(L,A):
  CompatibilityEquation:= simplify(dAdlambda-BigLL+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
  lambda=infinity));
  EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
  lambda=infinity));
  EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
  infinity));
  EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
  infinity));
```

```
series(simplify(series(CompatibilityEquation[1,1], lambda=
infinity)),lambda=infinity):
EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
lambda=infinity));
EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
lambda=infinity));
EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity));
EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda,lambda=
infinity));
EQ7:=simplify(residue(CompatibilityEquation[1,1],lambda=
infinity));
```

$$EQ0 := 0$$
$$EQ1 := 0$$
$$EQ2 := \left(2\,P01\,Q01\,\nu_0 - 2\,Pinfty_0\,\nu_1 + LQinfty0 - \nu_0\right)\omega$$
$$EQ3 := \left(2\,P01\,Q01\,Qinfty_0\,\nu_0 + 2\,P01\,Q01\,\nu_1 + 2\,Q01\,Pinfty_0\,\nu_0 + LQ01\right)\omega$$
$$EQ0 := 0$$
$$EQ4 := 0$$
$$EQ5 := -tinfty_3\left(2\,P01\,Q01\,\nu_0 - 2\,Pinfty_0\,\nu_1 + LQinfty0 - \nu_0\right)$$
$$EQ6 := \frac{1}{Q01}\left(-2\,\nu_0\,tinfty_3^2\,Q01^3 + \left(\left(3\,Qinfty_0^2\,\nu_0 - 6\,Qinfty_0\,\nu_1\right)tinfty_3^2 + \left(\left(-2\,P01 + 4\,tinfty_2\right)\nu_1 - 4\left(Qinfty_0\,tinfty_2 - \frac{1}{2}\,tinfty_1 + \frac{1}{2}\,Pinfty_0\right)\nu_0\right)tinfty_3 + \nu_0\left(P01 - tinfty_2\right)^2\right)Q01^2 + \left(4\,\nu_1\,Qinfty_0^3\,tinfty_3^2 + \left(\left(-6\,Qinfty_0^2\,tinfty_2 + \left(-2\,Pinfty_0 + 4\,tinfty_1\right)Qinfty_0 - 2\,tinfty_0 + 1\right)\nu_1 - Qinfty_0\,\nu_0 + Qinfty_0\,LQinfty0 - LQ01\right)tinfty_3 + 2\,tinfty_2\left(Qinfty_0\,tinfty_2 + Pinfty_0 - tinfty_1\right)\nu_1 + \nu_0\,tinfty_2 - tinfty_2\,LQinfty0 - LPinfty0\right)Q01 - t00^2\,\nu_0\right)$$
$$EQ7 := \frac{1}{Q01}\left(\left(\left(4\,Qinfty_0\,\nu_0 - 2\,\nu_1\right)tinfty_3^2 - 4\,tinfty_2\,\nu_0\,tinfty_3\right)Q01^3 + \left(\left(-Qinfty_0^3\,\nu_0 + 3\,Qinfty_0^2\,\nu_1\right)tinfty_3^2 + \left(\left(\left(2\,P01 + 2\,tinfty_2\right)Qinfty_0^2 + \left(2\,Pinfty_0 - 2\,tinfty_1\right)Qinfty_0 + 2\,tinfty_0 - 1\right)\nu_0 + 2\,\nu_1\left(\left(P01 - 2\,tinfty_2\right)Qinfty_0 + tinfty_1\right)\right)tinfty_3 + \left(\left(P01^2 - 2\,P01\,tinfty_2 - tinfty_2^2\right)Qinfty_0 + 2\,tinfty_2\left(-Pinfty_0 + tinfty_1\right)\right)\nu_0 + P01^2\,\nu_1 - 2\,P01\,\nu_1\,tinfty_2 + \nu_1\,tinfty_2^2 + LP01\right)Q01^2 + LQ01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)Q01 - t00^2\left(Qinfty_0\,\nu_0 + \nu_1\right)\right)$$

```
> LQinfty0:=solve(EQ2,LQinfty0);
  LQ01:=solve(EQ3,LQ01);
```

$$LQinfty0 := -2\,P01\,Q01\,\nu_0 + 2\,Pinfty_0\,\nu_1 + \nu_0 \qquad (1.7)$$
$$LQ01 := -2\,Q01\left(P01\,Qinfty_0\,\nu_0 + P01\,\nu_1 + Pinfty_0\,\nu_0\right)$$

```
> simplify(EQ5);
```

```
simplify(EQ6);
simplify(EQ7);
```

$$0 \tag{1.8}$$

$$\frac{1}{Q01}\left(-2\,\nu_0\,tinfty_3^2\,Q01^3+\left(\left(3\,Qinfty_0^2\nu_0-6\,Qinfty_0\,\nu_1\right)tinfty_3^2+\left(4\,\nu_1\,tinfty_2-4\left(Qinfty_0\,tinfty_2-\frac{1}{2}\,tinfty_1\right)\nu_0\right)tinfty_3+\nu_0\left(P01^2+tinfty_2^2\right)\right)Q01^2+\left(4\,\nu_1\,Qinfty_0^3\,tinfty_3^2-6\,\nu_1\left(Qinfty_0^2\,tinfty_2-\frac{2}{3}\,Qinfty_0\,tinfty_1+\frac{1}{3}\,tinfty_0-\frac{1}{6}\right)tinfty_3+\left(2\,Qinfty_0\,tinfty_2^2-2\,tinfty_1\,tinfty_2\right)\nu_1-LPinfty0\right)Q01-t00^2\,\nu_0\right)$$

$$\frac{1}{Q01}\left(\left(\left(4\,Qinfty_0\,tinfty_3^2-4\,tinfty_2\,tinfty_3\right)\nu_0-2\,tinfty_3^2\,\nu_1\right)Q01^3+\left(\left(-Qinfty_0^3\,tinfty_3^2+\left(2\,Qinfty_0^2\,tinfty_2-2\,Qinfty_0\,tinfty_1+2\,tinfty_0-1\right)tinfty_3+\left(-P01^2-tinfty_2^2\right)Qinfty_0-2\,P01\,Pinfty_0+2\,tinfty_1\,tinfty_2\right)\nu_0+3\,tinfty_3^2\,\nu_1\,Qinfty_0^2+2\,\nu_1\left(-2\,Qinfty_0\,tinfty_2+tinfty_1\right)tinfty_3+\left(-P01^2+tinfty_2^2\right)\nu_1+LP01\right)Q01^2-t00^2\left(Qinfty_0\,\nu_0+\nu_1\right)\right)$$

```
> LPinfty0:=solve(EQ6,LPinfty0);
  LP01:=solve(EQ7,LP01);
```

$$LPinfty0 := \frac{1}{Q01}\left(3\,Q01^2\,Qinfty_0^2\,\nu_0\,tinfty_3^2+4\,Q01\,Qinfty_0^3\,\nu_1\,tinfty_3^2-2\,Q01^3\,\nu_0\,tinfty_3^2-4\,Q01^2\,Qinfty_0\,\nu_0\,tinfty_2\,tinfty_3-6\,Q01^2\,Qinfty_0\,\nu_1\,tinfty_3^2-6\,Q01\,Qinfty_0^2\,\nu_1\,tinfty_2\,tinfty_3+P01^2\,Q01^2\,\nu_0+2\,Q01^2\,\nu_0\,tinfty_1\,tinfty_3+Q01^2\,\nu_0\,tinfty_2^2+4\,Q01^2\,\nu_1\,tinfty_2\,tinfty_3+4\,Q01\,Qinfty_0\,\nu_1\,tinfty_1\,tinfty_3+2\,Q01\,Qinfty_0\,\nu_1\,tinfty_2^2-2\,Q01\,\nu_1\,tinfty_0\,tinfty_3-2\,Q01\,\nu_1\,tinfty_1\,tinfty_2+Q01\,\nu_1\,tinfty_3-t00^2\,\nu_0\right) \tag{1.9}$$

$$LP01 := \frac{1}{Q01^2}\left(Q01^2\,Qinfty_0^3\,\nu_0\,tinfty_3^2-4\,Q01^3\,Qinfty_0\,\nu_0\,tinfty_3^2-2\,Q01^2\,Qinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3-3\,Q01^2\,Qinfty_0^2\,\nu_1\,tinfty_3^2+P01^2\,Q01^2\,Qinfty_0\,\nu_0+4\,Q01^3\,\nu_0\,tinfty_2\,tinfty_3+2\,Q01^3\,\nu_1\,tinfty_3^2+2\,Q01^2\,Qinfty_0\,\nu_0\,tinfty_1\,tinfty_3+Q01^2\,Qinfty_0\,\nu_0\,tinfty_2^2+4\,Q01^2\,Qinfty_0\,\nu_1\,tinfty_2\,tinfty_3+P01^2\,Q01^2\,\nu_1+2\,P01\,Q01^2\,Pinfty_0\,\nu_0-2\,Q01^2\,\nu_0\,tinfty_0\,tinfty_3-2\,Q01^2\,\nu_0\,tinfty_1\,tinfty_2-2\,Q01^2\,\nu_1\,tinfty_1\,tinfty_3-Q01^2\,\nu_1\,tinfty_2^2+Q01^2\,\nu_0\,tinfty_3+t00^2\,Qinfty_0\,\nu_0+t00^2\,\nu_1\right)$$

```
> Ham:=-2*P01*Q01*Pinfty[0]*nu[0]+Pinfty[0]^2*nu[1]+Pinfty[0]*nu
  [0]-P01^2*Q01*(Qinfty[0]*nu[0]+nu[1])
  -((-2*nu[0]*tinfty[3]^2*Q01^3+((2*nu[0]*tinfty[1]+4*nu[1]*
  tinfty[2])*tinfty[3]+nu[0]*tinfty[2]^2)*Q01^2-2*nu[1]*((tinfty
  [0]-1/2)*tinfty[3]+tinfty[1]*tinfty[2])*Q01-t00^2*nu[0])*Qinfty
  [0]/Q01+((1/2)*(-6*Q01*tinfty[3]^2+4*tinfty[1]*tinfty[3]+2*
  tinfty[2]^2)*nu[1]-2*Q01*nu[0]*tinfty[2]*tinfty[3])*Qinfty[0]
  ^2+tinfty[3]*(Q01*nu[0]*tinfty[3]-2*nu[1]*tinfty[2])*Qinfty[0]
```

```
^3+tinfty[3]^2*nu[1]*Qinfty[0]^4)
-(2*Q01^2*nu[0]*tinfty[2]*tinfty[3]+Q01^2*nu[1]*tinfty[3]^2-2*
Q01*nu[0]*tinfty[0]*tinfty[3]-2*Q01*nu[0]*tinfty[1]*tinfty[2]
-2*Q01*nu[1]*tinfty[1]*tinfty[3]-Q01*nu[1]*tinfty[2]^2+nu[0]*
tinfty[3]*Q01-t00^2*nu[1]/Q01);

simplify(diff(Ham,Pinfty[0])- LQinfty0);
simplify(diff(Ham,P01)- LQ01);
simplify(diff(Ham,Qinfty[0])+LPinfty0);
simplify(diff(Ham,Q01)+LP01);
```

$$Ham := -2\,P01\,Q01\,Pinfty_0\,\nu_0 + Pinfty_0^2\,\nu_1 + \nu_0\,Pinfty_0 - P01^2\,Q01\left(Qinfty_0\,\nu_0 + \nu_1\right) \tag{1.10}$$
$$-\frac{1}{Q01}\left(\left(-2\,\nu_0\,tinfty_3^2\,Q01^3 + \left(\left(2\,\nu_0\,tinfty_1 + 4\,\nu_1\,tinfty_2\right)tinfty_3 + \nu_0\,tinfty_2^2\right)Q01^2\right.\right.$$
$$\left.-2\,\nu_1\left(\left(tinfty_0 - \frac{1}{2}\right)tinfty_3 + tinfty_1\,tinfty_2\right)Q01 - t00^2\,\nu_0\right)Qinfty_0\Bigg) - \left(\frac{1}{2}\left(\right.\right.$$
$$\left.-6\,Q01\,tinfty_3^2 + 4\,tinfty_1\,tinfty_3 + 2\,tinfty_2^2\right)\nu_1 - 2\,Q01\,\nu_0\,tinfty_2\,tinfty_3\Bigg)Qinfty_0^2$$
$$-tinfty_3\left(Q01\,\nu_0\,tinfty_3 - 2\,\nu_1\,tinfty_2\right)Qinfty_0^3 - tinfty_3^2\,\nu_1\,Qinfty_0^4$$
$$-2\,Q01^2\,\nu_0\,tinfty_2\,tinfty_3 - Q01^2\,\nu_1\,tinfty_3^2 + 2\,Q01\,\nu_0\,tinfty_0\,tinfty_3$$
$$+2\,Q01\,\nu_0\,tinfty_1\,tinfty_2 + 2\,Q01\,\nu_1\,tinfty_1\,tinfty_3 + Q01\,\nu_1\,tinfty_2^2 - \nu_0\,tinfty_3\,Q01$$
$$+\frac{t00^2\,\nu_1}{Q01}$$

$$\begin{matrix}0\\0\\0\\0\end{matrix}$$

```
> Hambis:=-(Qinfty[0]*nu[0]+nu[1])*Q01*P01^2-2*nu[0]*Q01*P01*
Pinfty[0]+nu[0]*Pinfty[0]+nu[1]*(Pinfty[0])^2+t00^2*(Qinfty[0]*
nu[0]+nu[1])/Q01
+2*nu[0]*tinfty[3]^2*Qinfty[0]*Q01^2 -(Qinfty[0]^3*tinfty[3]^2+
tinfty[2]^2*Qinfty[0])*nu[0]*Q01 -(2*nu[0]*tinfty[2]*tinfty[3]+
nu[1]*tinfty[3]^2)*Q01^2+ 2*tinfty[2]*tinfty[3]*nu[0]*Q01*
Qinfty[0]^2+3*tinfty[3]^2*nu[1]*Qinfty[0]^2+(-2*tinfty[3]*(nu
[0]*tinfty[1]+2*nu[1]*tinfty[2]))*Q01*Qinfty[0]+3*Qinfty[0]^2*
nu[1]*tinfty[3]^2*Q01
-(3*(tinfty[3]^2)+2*tinfty[3]*tinfty[1]+tinfty[2]^2)*nu[1]*
Qinfty[0]^2+nu[1]*(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-
tinfty[3])*Qinfty[0]+((2*tinfty[1]*nu[1]+nu[0]*(2*tinfty[0]-1))
*tinfty[3]+2*nu[0]*tinfty[1]*tinfty[2]+nu[1]*tinfty[2]^2)*Q01
+2*tinfty[2]*tinfty[3]*nu[1]*(Qinfty[0])^3-tinfty[3]^2*nu[1]*
(Qinfty[0])^4:
simplify(series(simplify(Ham-Hambis),Q01)) ;
```

(1.11)

$$0 \tag{1.11}$$

```
> M:=Matrix(2*d,2*d,0):
  for i from 1 to 2*d do for j from 1 to i do M[i,j]:=tinfty[2*
  d+1-(i-j)]: od: od:
  M;
  alphaVector:=Matrix(2*d,1,0):
  for i from 1 to 2*d do alphaVector[i]:=alpha[2*d+1-i]/(2*d+1-i)
  : od:
  alphaVector;
  nuVector:=Multiply(M^(-1),alphaVector);
  for i from 1 to 2*d do nu[i-1]:=nuVector[i,1]: od:
  nu[0]:=nu[0];
  nu[1]:=nu[1];
```

$$\begin{bmatrix} tinfty_3 & 0 \\ tinfty_2 & tinfty_3 \end{bmatrix} \tag{1.12}$$

$$\begin{bmatrix} \frac{1}{2}\,\alpha_2 \\ \alpha_1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\,\frac{\alpha_2}{tinfty_3} \\ -\frac{1}{2}\,\frac{tinfty_2\,\alpha_2}{tinfty_3^2} + \frac{\alpha_1}{tinfty_3} \end{bmatrix}$$

$$\nu_0 := \frac{1}{2}\,\frac{\alpha_2}{tinfty_3}$$

$$\nu_1 := -\frac{1}{2}\,\frac{tinfty_2\,\alpha_2}{tinfty_3^2} + \frac{\alpha_1}{tinfty_3}$$

```
> Hamfunction:=unapply(simplify(Ham),alpha[1],alpha[2]):
  Hamtinfty1:=simplify(Hamfunction(1,0));
  Hamtinfty2:=simplify(Hamfunction(0,1));
```

$$Hamtinfty1 := \frac{1}{Q01\,tinfty_3}\left(-Q01^3\,tinfty_3^2 + \left(3\,Qinfty_0^2\,tinfty_3^2 + \left(-4\,Qinfty_0\,tinfty_2 + 2\,tinfty_1\right)tinfty_3 - P01^2 + tinfty_2^2\right)Q01^2 + \left(-Qinfty_0^4\,tinfty_3^2 + \left(2\,Qinfty_0^3\,tinfty_2 - 2\,tinfty_1\,Qinfty_0^2 + \left(2\,tinfty_0 - 1\right)Qinfty_0\right)tinfty_3 - Qinfty_0^2\,tinfty_2^2 + 2\,Qinfty_0\,tinfty_1\,tinfty_2 + Pinfty_0^2\right)Q01 + t00^2\right) \tag{1.13}$$

$$Hamtinfty2 := \frac{1}{2}\,\frac{1}{Q01\,tinfty_3^2}\left(\left(2\,Qinfty_0\,tinfty_3^3 - tinfty_2\,tinfty_3^2\right)Q01^3 + \left(-Qinfty_0^3\,tinfty_3^3\right.\right.$$

$$+\left(-Qinfty_0^2\,tinfty_2-2\,Qinfty_0\,tinfty_1+2\,tinfty_0-1\right)tinfty_3^2+\left(-P01^2\,Qinfty_0+3\,Qinfty_0\,tinfty_2^2-2\,P01\,Pinfty_0\right)tinfty_3+P01^2\,tinfty_2-tinfty_2^3\Big)\,Q01^2+\Big(Qinfty_0^4\,tinfty_2\,tinfty_3^2+\left(-2\,tinfty_2^2\,Qinfty_0^3+\left(2\,tinfty_1\,Qinfty_0^2+\left(-2\,tinfty_0+1\right)Qinfty_0\right)tinfty_2+Pinfty_0\right)tinfty_3+Qinfty_0^2\,tinfty_2^3-2\,Qinfty_0\,tinfty_1\,tinfty_2^2-Pinfty_0^2\,tinfty_2\Big)\,Q01+t00^2\left(Qinfty_0\,tinfty_3-tinfty_2\right)\Big)$$

```
> Hamtinfty1bis:=-Q01*P01^2/tinfty[3]+1/(tinfty[3])*(Pinfty[0])^2
  -tinfty[3]*Q01^2 +t00^2/(Q01*tinfty[3]) -Qinfty[0]^4*tinfty[3]
  +2*Qinfty[0]^3*tinfty[2] +3*Qinfty[0]^2*tinfty[3]*Q01-4*tinfty
  [2]*Qinfty[0]*Q01+Q01*(2*tinfty[1]*tinfty[3]+tinfty[2]^2)
  /tinfty[3]-(2*tinfty[1]*tinfty[3]+tinfty[2]^2)*Qinfty[0]
  ^2/tinfty[3]-Qinfty[0] +2*(tinfty[0]*tinfty[3]+tinfty[1]*tinfty
  [2])*Qinfty[0]/tinfty[3];
  simplify(series(Hamtinfty1-Hamtinfty1bis, Qinfty[0]));
```

$$Hamtinfty1bis := -\frac{Q01\,P01^2}{tinfty_3}+\frac{Pinfty_0^2}{tinfty_3}-tinfty_3\,Q01^2+\frac{t00^2}{Q01\,tinfty_3}-Qinfty_0^4\,tinfty_3+2\,Qinfty_0^3\,tinfty_2+3\,Qinfty_0^2\,tinfty_3\,Q01-4\,tinfty_2\,Qinfty_0\,Q01+\frac{Q01\left(2\,tinfty_1\,tinfty_3+tinfty_2^2\right)}{tinfty_3}-\frac{\left(2\,tinfty_1\,tinfty_3+tinfty_2^2\right)Qinfty_0^2}{tinfty_3}-Qinfty_0+\frac{2\left(tinfty_0\,tinfty_3+tinfty_1\,tinfty_2\right)Qinfty_0}{tinfty_3} \tag{1.14}$$

$$0$$

```
> Hamtinfty2bis:=-(1/2)*(Qinfty[0]*tinfty[3]-tinfty[2])*Q01*
  P01^2/tinfty[3]^2-Q01*P01*Pinfty[0]/tinfty[3]+(1/2)*Pinfty[0]
  /tinfty[3]-(1/2)*Pinfty[0]^2*tinfty[2]/tinfty[3]^2+t00^2*
  (Qinfty[0]*tinfty[3]-tinfty[2])/((2*tinfty[3]^2)*Q01)+(Qinfty
  [0]*tinfty[3]-(1/2)*tinfty[2])*Q01^2+(1/2)*Qinfty[0]^4*tinfty
  [2]
  -(1/2)*(Q01*tinfty[3]^2+2*tinfty[2]^2)*Qinfty[0]^3/tinfty[3]+
  Q01*(2*tinfty[0]*tinfty[3]^2-tinfty[2]^3-tinfty[3]^2)/(2*tinfty
  [3]^2)-(1/2)*tinfty[2]*(Q01*tinfty[3]^2-2*tinfty[1]*tinfty[3]-
  tinfty[2]^2)*Qinfty[0]^2/tinfty[3]^2
  +3/2*tinfty[2]^2*Qinfty[0]*Q01/tinfty[3] -Q01*Qinfty[0]*tinfty
  [1]+((-2*tinfty[0]+1)*tinfty[2]*tinfty[3]-2*tinfty[1]*tinfty[2]
  ^2)*Qinfty[0]/(2*tinfty[3]^2);
  simplify(series(Hamtinfty2-Hamtinfty2bis, Qinfty[0]));
```

$$Hamtinfty2bis := -\frac{1}{2}\,\frac{\left(Qinfty_0\,tinfty_3-tinfty_2\right)Q01\,P01^2}{tinfty_3^2}-\frac{Q01\,P01\,Pinfty_0}{tinfty_3} \tag{1.15}$$

$$+\frac{1}{2}\,\frac{Pinfty_0}{tinfty_3}-\frac{1}{2}\,\frac{Pinfty_0^2\,tinfty_2}{tinfty_3^2}+\frac{1}{2}\,\frac{t00^2\left(Qinfty_0\,tinfty_3-tinfty_2\right)}{Q01\,tinfty_3^2}$$

$$+\left(Qinfty_0\,tinfty_3-\frac{1}{2}\,tinfty_2\right)Q01^2+\frac{1}{2}\,Qinfty_0^4\,tinfty_2$$

$$-\frac{1}{2}\,\frac{\left(Q01\,tinfty_3^2+2\,tinfty_2^2\right)Qinfty_0^3}{tinfty_3}+\frac{1}{2}\,\frac{Q01\left(2\,tinfty_0\,tinfty_3^2-tinfty_2^3-tinfty_2^2\right)}{tinfty_3^2}$$

$$-\frac{1}{2}\,\frac{tinfty_2\left(Q01\,tinfty_3^2-2\,tinfty_1\,tinfty_3-tinfty_2^2\right)Qinfty_0^2}{tinfty_3^2}+\frac{3}{2}\,\frac{tinfty_2^2\,Qinfty_0\,Q01}{tinfty_3}$$

$$-Q01\,Qinfty_0\,tinfty_1+\frac{1}{2}\,\frac{\left(\left(-2\,tinfty_0+1\right)tinfty_2\,tinfty_3-2\,tinfty_1\,tinfty_2^2\right)Qinfty_0}{tinfty_3^2}$$

$$0$$

We impose the symmetry \check{L}(-\lambda)=-\sigma_1 \check{L}(\lambda) \sigma_1

```
> R01:=0:
  Rinfty[1]:=0:
  tinfty[2]:=0:
  tinfty[0]:=0:
  alpha[2]:=0:
  alpha[1]:=1:
```

```
> L12function:=unapply(L12,lambda):
  L21function:=unapply(L21,lambda):
  L11function:=unapply(L11,lambda);
  Qinfty[0]:=-P01/tinfty[3];
  Pinfty[0]:=-Q01*tinfty[3]+P01^2/tinfty[3]+tinfty[1]-t00^2/(2*
  tinfty[3]*Q01^2);
  omega:=-t00/Q01;
  Lomega:=t00*LQ01/Q01^2:
  simplify(L11function(-lambda)-L11function(lambda));
  series(simplify(L12function(-lambda)+L21function(lambda)),
  lambda=0);
  series(simplify(L21function(-lambda)+L12function(lambda)),
  lambda=0);
```

$$L11function := \lambda \rightarrow \frac{-\lambda^3\,tinfty_3+\left(\left(Qinfty_0^2-Q01\right)tinfty_3-Pinfty_0\right)\lambda+Q01\left(Qinfty_0\,tinfty_3+P01\right)}{\lambda} \tag{1.16}$$

$$Qinfty_0 := -\frac{P01}{tinfty_3}$$

$$Pinfty_0 := -Q01\,tinfty_3+\frac{P01^2}{tinfty_3}+tinfty_1-\frac{1}{2}\,\frac{t00^2}{tinfty_3\,Q01^2}$$

$$\omega := \begin{matrix} -\frac{t00}{Q01} \\ 0 \\ 0 \\ 0 \end{matrix}$$

```
> L:=simplify(L):
  L11:=expand(series(simplify(series(L11,lambda=infinity)),
  lambda=infinity));
  L12:=expand(series(simplify(series(L12,lambda=infinity)),
  lambda=infinity));
  L21:=expand(series(simplify(series(L21,lambda=infinity)),
  lambda=infinity));
  A:=simplify(A);
```

$$L11 := -tinfty_3\,\lambda^2 - tinfty_1 + \frac{1}{2}\,\frac{t00^2}{tinfty_3\,Q01^2} \tag{1.17}$$

$$L12 := -\frac{t00\,\lambda}{Q01} + \frac{t00\,P01}{Q01\,tinfty_3} - \frac{t00}{\lambda}$$

$$L21 := -\frac{t00\,\lambda}{Q01} - \frac{t00\,P01}{Q01\,tinfty_3} - \frac{t00}{\lambda}$$

$$\begin{bmatrix} -\lambda & -\frac{t00}{Q01\,tinfty_3} \\ -\frac{t00}{Q01\,tinfty_3} & \lambda \end{bmatrix}$$

```
> LQinfty0:=expand(LQinfty0);
  LQ01:=LQ01;
  LP01:=expand(simplify(LP01));
```

$$LQinfty0 := -2\,Q01 + \frac{2\,P01^2}{tinfty_3^2} + \frac{2\,tinfty_1}{tinfty_3} - \frac{t00^2}{tinfty_3^2\,Q01^2} \tag{1.18}$$

$$LQ01 := -\frac{2\,Q01\,P01}{tinfty_3}$$

$$LP01 := 2\,Q01\,tinfty_3 - \frac{2\,P01^2}{tinfty_3} - 2\,tinfty_1 + \frac{t00^2}{tinfty_3\,Q01^2}$$

```
> simplify(LP01-tinfty[3]*(-LQinfty0));
  LRinfty0Reduced:=unapply(expand(simplify(-LPinfty0+2*tinfty[3]*
  Qinfty[0]*LQinfty0 -tinfty[3]* LQ01)) ,Q01,P01);
  LSinfty0Reduced:=unapply(expand(simplify(-LQinfty0)) ,Q01,P01);
  LS01Reduced:=unapply(expand(simplify(LQ01)) ,Q01,P01);
  LS01Reduced(S01,Sinfty[0]*tinfty[3]);
  LRinfty0Reduced(S01,Sinfty[0]*tinfty[3]);
  LSinfty0Reduced(S01,Sinfty[0]*tinfty[3]);
```

$$0 \tag{1.19}$$

$$LRinfty0Reduced := (Q01, P01) \to -1 + \frac{2\,P01\,t00^2}{Q01^2\,tinfty_3^2}$$

$$LSinfty0Reduced := (Q01, P01) \to 2\,Q01 - \frac{2\,P01^2}{tinfty_3^2} - \frac{2\,tinfty_1}{tinfty_3} + \frac{t00^2}{tinfty_3^2\,Q01^2}$$

$$LS01Reduced := (Q01, P01) \to -\frac{2\,Q01\,P01}{tinfty_3}$$

$$-2\,S01\,Sinfty_0$$

$$-1 + \frac{2\,Sinfty_0\,t00^2}{S01^2\,tinfty_3}$$

$$2\,S01 - 2\,Sinfty_0^2 - \frac{2\,tinfty_1}{tinfty_3} + \frac{t00^2}{tinfty_3^2\,S01^2}$$

We now obtain the standard FN Hamiltonian system. We take q=-t00/tinfty[3]/Q01 and p= -P01* t00/2/tinfty[3]/Q01

```
> Lq:=unapply(t00/tinfty[3]/(Q01)^2*LQ01,Q01,P01,tinfty[3]);
  Lp:=unapply(-t00/2/tinfty[3]/Q01*LP01+P01*t00/2/tinfty[3]
  /Q01^2*LQ01,Q01,P01,tinfty[3]);
  Lqnew:=unapply(expand(simplify(Lq(-t00/tinfty[3]/q,-p/t00*2*
  tinfty[3]*(-t00/tinfty[3]/q),tinfty[3]))),tinfty[3]);
  Lpnew:=unapply(expand(simplify(Lp(-t00/tinfty[3]/q,-p/t00*2*
  tinfty[3]*(-t00/tinfty[3]/q),tinfty[3]))),tinfty[3]);
  Hamqp:=unapply(2*p^2/tinfty[3]-(1/8)*q^4*tinfty[3]+(1/2)*q^2*
  tinfty[1]+t00*q,tinfty[3]);
  simplify(diff(Hamqp(tinfty[3]),p)- Lqnew(tinfty[3]));
  series(simplify(diff(Hamqp(tinfty[3]),q)+Lpnew(tinfty[3])),p);
```

$$Lq := (Q01, P01, tinfty\_3) \to -\frac{2\,t00\,P01}{tinfty\_3^2\,Q01} \tag{1.20}$$

$$Lp := (Q01, P01, tinfty\_3) \to -\frac{1}{2}\,\frac{t00\left(2\,Q01\,tinfty\_3 - \frac{2\,P01^2}{tinfty\_3} - 2\,tinfty_1 + \frac{t00^2}{tinfty\_3\,Q01^2}\right)}{tinfty\_3\,Q01} - \frac{P01^2\,t00}{tinfty\_3^2\,Q01}$$

$$Lqnew := tinfty\_3 \to \frac{4\,p}{tinfty\_3}$$

$$Lpnew := tinfty\_3 \to \frac{1}{2}\,tinfty\_3\,q^3 - tinfty_1\,q - t00$$

$$Hamqp := tinfty\_3 \to \frac{2\,p^2}{tinfty\_3} - \frac{1}{8}\,q^4\,tinfty\_3 + \frac{1}{2}\,q^2\,tinfty_1 + t00\,q$$

$$0$$

$$0$$

We now take Q1= t00/Q01 and P1=P01/(4*tinfty[3])*t00/Q01 +(t00/Q01)^2/(8*tinfty[3])- t_{\infty, 1}/4*Q01/t00 following the proposal of Mazzocco et al.

```
> Rinfty[0]:=t00^2/2/tinfty[3]/Q01^2-tinfty[1];
```

```
Qinfty[0]:=(4*P1-Rinfty[0])/omega;
P01theo:=unapply( -tinfty[3]*Qinfty[0],Q01);
P01theo2:=expand(P01theo(t00/Q1));
Sol:=unapply( expand(solve(P01theo2=P01bis,P1)),Q1);
PP1:=P01/(4*tinfty[3])*t00/Q01 +(t00/Q01)^2/(8*tinfty[3])-
tinfty[1]/4;
```

$$Rinfty_0 := -tinfty_1 + \frac{1}{2}\,\frac{t00^2}{tinfty_3\,Q01^2} \tag{1.21}$$

$$Qinfty_0 := -\frac{\left(4\,P1 + tinfty_1 - \frac{1}{2}\,\frac{t00^2}{tinfty_3\,Q01^2}\right) Q01}{t00}$$

$$P01theo := Q01 \rightarrow \frac{\left(4\,P1 + tinfty_1 - \frac{1}{2}\,\frac{t00^2}{tinfty_3\,Q01^2}\right) Q01\,tinfty_3}{t00}$$

$$P01theo2 := \frac{4\,tinfty_3\,P1}{Q1} + \frac{tinfty_3\,tinfty_1}{Q1} - \frac{1}{2}\,Q1$$

$$Sol := Q1 \rightarrow \frac{1}{4}\,\frac{P01bis\,Q1}{tinfty_3} + \frac{1}{8}\,\frac{Q1^2}{tinfty_3} - \frac{1}{4}\,tinfty_1$$

$$PP1 := \frac{1}{4}\,\frac{t00\,P01}{Q01\,tinfty_3} + \frac{1}{8}\,\frac{t00^2}{tinfty_3\,Q01^2} - \frac{1}{4}\,tinfty_1$$

```
> LQ1:=unapply(-t00/Q01^2*LQ01,Q01,P01,tinfty[3]);
LP1:=unapply(diff(PP1,P01)*LP01+diff(PP1,Q01)*LQ01+diff(PP1,
tinfty[1]),Q01,P01,tinfty[3]):
LQ1new:=unapply(expand(simplify(LQ1(t00/Q1,4*tinfty[3]*P1/Q1
-1/2*Q1+tinfty[3]*tinfty[1]/Q1,tinfty[3]))),tinfty[3]);
LP1new:=unapply(expand(simplify(LP1(t00/Q1,4*tinfty[3]*P1/Q1
-1/2*Q1+tinfty[3]*tinfty[1]/Q1,tinfty[3]))),tinfty[3]);
int(LQ1new(tinfty[3]),P1);
```

$$LQ1 := (Q01, P01, tinfty\_3) \rightarrow \frac{2\,t00\,P01}{Q01\,tinfty\_3} \tag{1.22}$$

$$LQ1new := tinfty\_3 \rightarrow 8\,P1 - \frac{Q1^2}{tinfty\_3} + 2\,tinfty_1$$

$$LP1new := tinfty\_3 \rightarrow \frac{2\,P1\,Q1}{tinfty\_3} + \frac{1}{2}\,t00 - \frac{1}{4}$$

$$4\,P1^2 - \frac{Q1^2\,P1}{tinfty_3} + 2\,tinfty_1\,P1$$

```
> Hamnew:=unapply(4*P1^2-Q1^2*P1/tinfty[3]+2*tinfty[1]*P1- ((1/2)
*t00-1/4)*Q1,tinfty[3]  );
simplify(diff(Hamnew(tinfty[3]),P1)- LQ1new(tinfty[3]));
series(simplify(diff(Hamnew(tinfty[3]),Q1)+LP1new(tinfty[3])),
```

```
P1);
```

$$Hamnew := tinfty\_3 \to 4\,P1^2 - \frac{Q1^2\,P1}{tinfty\_3} + 2\,tinfty_1\,P1 - \left(\frac{1}{2}\,t00 - \frac{1}{4}\right)Q1 \qquad \textbf{(1.23)}$$

$$0$$

$$0$$

```
> int(-t00-qFN*tinfty[1]+(1/2)*tinfty[3]*qFN^3,qFN);
```

$$-t00\,qFN - \frac{1}{2}\,qFN^2\,tinfty_1 + \frac{1}{8}\,tinfty_3\,qFN^4 \qquad \textbf{(1.24)}$$

We define qFN=-t00/tinfty[3]/Q01 and pFN=-t00/2/tinfty[3]*P01/Q01

```
> LqFN:=unapply(t00/tinfty[3]*LQ01/Q01^2,Q01,P01);
  LpFN:=unapply(expand(simplify(-t00/2/tinfty[3]*
  LP01/Q01+t00/2/tinfty[3]*P01/Q01^2*LQ01)) ,Q01,P01);
  LqFNnew:=LqFN(-t00/tinfty[3]/qFN,-pFN*2*tinfty[3]/t00*(-
  t00/tinfty[3]/qFN));
  LpFNnew:=LpFN(-t00/tinfty[3]/qFN,-pFN*2*tinfty[3]/t00*(-
  t00/tinfty[3]/qFN));
  HamFNnew:=2*pFN^2/tinfty[3]+t00*qFN+(1/2)*qFN^2*tinfty[1]-(1/8)
  *tinfty[3]*qFN^4;
  LqFNnew-diff(HamFNnew,pFN);
  LpFNnew+diff(HamFNnew,qFN);
  ddotqFN=expand(4/tinfty[3]*LpFNnew);
```

$$LqFN := (Q01, P01) \to -\frac{2\,t00\,P01}{tinfty_3^2\,Q01} \qquad \textbf{(1.25)}$$

$$LpFN := (Q01, P01) \to -t00 + \frac{t00\,tinfty_1}{Q01\,tinfty_3} - \frac{1}{2}\,\frac{t00^3}{Q01^3\,tinfty_3^2}$$

$$LqFNnew := \frac{4\,pFN}{tinfty_3}$$

$$LpFNnew := -t00 - qFN\,tinfty_1 + \frac{1}{2}\,tinfty_3\,qFN^3$$

$$HamFNnew := \frac{2\,pFN^2}{tinfty_3} + t00\,qFN + \frac{1}{2}\,qFN^2\,tinfty_1 - \frac{1}{8}\,tinfty_3\,qFN^4$$

$$0$$

$$0$$

$$ddotqFN = -\frac{4\,t00}{tinfty_3} - \frac{4\,qFN\,tinfty_1}{tinfty_3} + 2\,qFN^3$$

We set x=mu*tinfty[1] and q=nu*qFN. This gives d^2/dtinfty[1]^2= mu^2*d^2/dx^2 and thus d^2/dtinfty[1]^2 qFN= mu^2/nu*d^2/dx^2 q

```
> test:=unapply(expand(nu/mu^2*( -4*t00/tinfty[3]-4*1/nu*q*(x/mu)
  /tinfty[3]+2*(1/nu*q)^3)),mu,nu);
```

```
test((-4/tinfty[3])^(1/3), (-4/tinfty[3])^(-1/3));
```

$$test := (\mu, \nu) \to -\frac{4\,\nu\,t00}{\mu^2\,tinfty_3} - \frac{4\,q\,x}{\mu^3\,tinfty_3} + \frac{2\,q^3}{\nu^2\,\mu^2} \qquad (1.26)$$

$$2\,q^3 + q\,x + t00$$

Test of FN Lax pair

```
> LFN:=Matrix(2,2,0):
  LFN[1,1]:= -4*lambda^2-x+2*qFN^2:
  LFN[1,2]:=4*qFN*lambda-2*pFN+alphaFN/lambda:
  LFN[2,1]:=4*qFN*lambda+2*pFN+alphaFN/lambda:
  LFN[2,2]:=-LFN[1,1]:
  AFN:=Matrix(2,2,0):
  AFN[1,1]:= -lambda:
  AFN[1,2]:=qFN:
  AFN[2,1]:=qFN:
  AFN[2,2]:=-AFN[1,1]:
  LFN;
  AFN;
```

$$\begin{bmatrix} -4\,\lambda^2 + 2\,qFN^2 - x & 4\,qFN\,\lambda - 2\,pFN + \frac{alphaFN}{\lambda} \\ 4\,qFN\,\lambda + 2\,pFN + \frac{alphaFN}{\lambda} & 4\,\lambda^2 - 2\,qFN^2 + x \end{bmatrix} \qquad (2.1)$$

$$\begin{bmatrix} -\lambda & qFN \\ qFN & \lambda \end{bmatrix}$$

```
> dAFNdlambda:=Matrix(2,2,0):
  for i from 1 to 2 do for j from 1 to 2 do dAFNdlambda[i,j]:=
  diff(AFN[i,j],lambda): od: od:
  dAFNdlambda:
> BigLLFN:=Matrix(2,2,0):
  BigLLFN[1,1]:= diff(LFN[1,1],qFN)*LqFN+ diff(LFN[1,1],pFN)*
  LpFN+diff(LFN[1,1],x):
  BigLLFN[1,2]:= diff(LFN[1,2],qFN)*LqFN+ diff(LFN[1,2],pFN)*
  LpFN+diff(LFN[1,2],x):
  BigLLFN[2,1]:= diff(LFN[2,1],qFN)*LqFN+ diff(LFN[2,1],pFN)*
  LpFN+diff(LFN[2,1],x):
  BigLLFN[2,2]:=-BigLLFN[1,1]:
  BigLLFN;
```

(2.2)

$$\begin{bmatrix} 4\,LqFN\,qFN-1 & 4\,LqFN\,\lambda-2\,LpFN \\ 4\,LqFN\,\lambda+2\,LpFN & -4\,LqFN\,qFN+1 \end{bmatrix} \tag{2.2}$$

```
> CommutatorFN:=Multiply(AFN,LFN)-Multiply(LFN,AFN):
  CompatibilityEquationFN:= simplify(dAFNdlambda-BigLLFN+
  CommutatorFN):
```

```
> LqFN:=pFN;
  LpFN:=2*qFN^3-x*qFN+alphaFN;
```

$$LqFN := pFN \tag{2.3}$$
$$LpFN := 2\,qFN^3 - qFN\,x + alphaFN$$

```
> series(CompatibilityEquationFN[1,2], lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquationFN[1,2]/lambda^3,
  lambda=infinity));
  EQ1:=simplify(residue(CompatibilityEquationFN[1,2]/lambda^2,
  lambda=infinity));
  EQ2:=simplify(residue(CompatibilityEquationFN[1,2]/lambda,
  lambda=infinity));
  EQ3:=simplify(residue(CompatibilityEquationFN[1,2],lambda=
  infinity));

  series(simplify(series(CompatibilityEquationFN[1,1], lambda=
  infinity)),lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquationFN[1,1]/lambda^4,
  lambda=infinity));
  EQ4:=simplify(residue(CompatibilityEquationFN[1,1]/lambda^3,
  lambda=infinity));
  EQ5:=simplify(residue(CompatibilityEquationFN[1,1]/lambda^2,
  lambda=infinity));
  EQ6:=simplify(residue(CompatibilityEquationFN[1,1]/lambda,
  lambda=infinity));
  EQ7:=simplify(residue(CompatibilityEquationFN[1,1],lambda=
  infinity));
```

$$EQ0 := 0 \tag{2.4}$$
$$EQ1 := 0$$
$$EQ2 := 0$$
$$EQ3 := 0$$
$$EQ0 := 0$$
$$EQ4 := 0$$
$$EQ5 := 0$$
$$EQ6 := 0$$
$$EQ7 := 0$$

Mazzocco's Lax matrix

```
> QQ:=Q1*lambda^2;
  PP:=P1*lambda^(-2);
  TT:=-z*(2*lambda)^(-2);
  series(lambda^(-2)*QQ*(1+PP),lambda);
  Beven:=series(Q1*lambda^2,lambda,infinity);
  series( 1/4*(2*lambda)^(2*d+1)*(1+PP- (1+TT)^2/(1+PP))+(2*
  lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
  Bodd:=(4*P1+z+(1/8)*Q1^2)*lambda;
  series( 1/4*(2*lambda)^(2*d+1)*(1+PP+ (1+TT)^2/(1+PP))-(2*
  lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
  A:=4*lambda^3+(-z-(1/8)*Q1^2)*lambda;

  LMazzocco11:=series( A/lambda,lambda=infinity);
  LMazzocco12:=series( (Beven+Bodd)/lambda +b0/lambda,lambda=
  infinity);
  LMazzocco21:=series( (Beven-Bodd)/lambda+b0/lambda,lambda=
  infinity);
```

$$QQ := Q1\,\lambda^2 \tag{3.1}$$

$$PP := \frac{P1}{\lambda^2}$$

$$TT := -\frac{1}{4}\,\frac{z}{\lambda^2}$$

$$\frac{P1\,Q1}{\lambda^2} + Q1$$

$$Beven := Q1\,\lambda^2$$

$$\left(4\,P1 + z + \frac{1}{8}\,Q1^2\right)\lambda + \frac{-\frac{1}{8}\,z^2 - 2\left(\frac{1}{2}\,z + P1\right)P1 + \frac{1}{8}\,Q1^2\,P1}{\lambda} + O\left(\frac{1}{\lambda^3}\right)$$

$$Bodd := \left(4\,P1 + z + \frac{1}{8}\,Q1^2\right)\lambda$$

$$4\,\lambda^3 + \left(-z - \frac{1}{8}\,Q1^2\right)\lambda + \frac{\frac{1}{8}\,z^2 + 2\left(\frac{1}{2}\,z + P1\right)P1 - \frac{1}{8}\,Q1^2\,P1}{\lambda} + O\left(\frac{1}{\lambda^3}\right)$$

$$A := 4\,\lambda^3 + \left(-z - \frac{1}{8}\,Q1^2\right)\lambda$$

$$LMazzocco11 := 4\,\lambda^2 - z - \frac{1}{8}\,Q1^2$$

$$LMazzocco12 := Q1\,\lambda + 4\,P1 + z + \frac{1}{8}\,Q1^2 + \frac{b0}{\lambda}$$

$$LMazzocco21 := Q1\,\lambda - 4\,P1 - z - \frac{1}{8}\,Q1^2 + \frac{b0}{\lambda}$$

```
> DetMazzocco:=series(simplify(-LMazzocco11^2-LMazzocco12*
  LMazzocco21),lambda=infinity);
  DetL:=series(simplify(-L11^2-L12*L21),lambda=infinity);
```

$$DetMazzocco := -16\,\lambda^4 + 8\,z\,\lambda^2 + 16\,P1^2 + \left(Q1^2 + 8\,z\right)P1 - 2\,Q1\,b0 - \frac{b0^2}{\lambda^2} \tag{3.2}$$

$$DetL := -tinfty_3^2\,\lambda^4 - 2\,tinfty_1\,tinfty_3\,\lambda^2 + \frac{1}{4}\,\frac{1}{tinfty_3^2\,Q01^4}\left(-4\,Q01^4\,tinfty_1^2\,tinfty_3^2 - 8\,Q01^3\,t00^2\,tinfty_3^2 + 4\,t00^2\left(P01^2 + tinfty_1\,tinfty_3\right)Q01^2 - t00^4\right) - \frac{t00^2}{\lambda^2}$$

```
> series(simplify(L11),lambda=infinity);
  series(simplify(L12),lambda=infinity);
  series(simplify(L21),lambda=infinity);
```

$$-tinfty_3\,\lambda^2 + \frac{1}{2}\,\frac{-2\,Q01^2\,tinfty_1\,tinfty_3 + t00^2}{Q01^2\,tinfty_3} \tag{3.3}$$

$$-\frac{t00\,\lambda}{Q01} + \frac{t00\,P01}{Q01\,tinfty_3} - \frac{t00}{\lambda}$$

$$-\frac{t00\,\lambda}{Q01} - \frac{t00\,P01}{Q01\,tinfty_3} - \frac{t00}{\lambda}$$

```
> tinfty[3]:=-4;
  Q1:=-t00/Q01;
  z:=tinfty[1];
  b0:=-t00;
  P1:=-P01*t00/(16*Q01)-(1/4)*tinfty[1]-t00^2/(32*Q01^2);
  simplify(series(simplify(L11-LMazzocco11),Q01));
  series(simplify(L12-LMazzocco12),Q01);
  series(simplify(L21-LMazzocco21),Q01);
```

$$tinfty_3 := -4 \tag{3.4}$$

$$Q1 := -\frac{t00}{Q01}$$

$$z := tinfty_1$$

$$b0 := -t00$$

$$P1 := -\frac{1}{16}\,\frac{P01\,t00}{Q01} - \frac{1}{4}\,tinfty_1 - \frac{1}{32}\,\frac{t00^2}{Q01^2}$$

$$0$$

$$0$$

$$0$$

Coordinates (R,S)

```
> restart:
  with(LinearAlgebra):
  d:=1:
```

```
> L12:=Q01/lambda+lambda^(2*d-1):
  for k from 0 to 2*d-2 do L12:=L12+ Qinfty[k]*lambda^k: od:
  L12:=L12*omega;
  L11:=P01*Q01/lambda:
  for k from 0 to 2*d-2 do L11:=L11-Pinfty[2*d-2-k]*lambda^k: od:
  L11:=L11- (tinfty[2*d+1]*lambda+tinfty[2*d]-tinfty[2*d+1]*
  Qinfty[2*d-2])*(L12/omega):
  L11:=simplify(L11):
  series(L11,lambda=infinity,20);
```

$$L12 := \left(\frac{Q01}{\lambda} + \lambda + Qinfty_0 \right) \omega \tag{4.1}$$

$$-tinfty_3\,\lambda^2 - tinfty_2\,\lambda + \left(Qinfty_0^2 - Q01\right) tinfty_3 - Qinfty_0\,tinfty_2 - Pinfty_0 + \frac{Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)}{\lambda}$$

```
> series(simplify(series((t00^2/lambda^2- L11^2)/(L12/omega),
  lambda=0,3)),lambda=0,1);
  factor(residue((t00^2/lambda^2- L11^2)/(L12/omega),lambda=0));
  ResidueBis:=1/omega*(t00^2/Q01-Q01*(P01-tinfty[2*d]+tinfty[2*
  d+1]*Qinfty[2*d-2])^2);
  simplify( factor(residue((t00^2/lambda^2- L11^2)/L12,lambda=0))
  - ResidueBis);
```

$$-\frac{1}{\lambda}\,\frac{1}{Q01}\left(\left(Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right) - t00\right)\left(Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right) + t00\right)\right) + \frac{1}{Q01^2}\left(2\,tinfty_3\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right) Q01^3 + \left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)\left(-Qinfty_0^2\,tinfty_3 + \left(P01 + tinfty_2\right) Qinfty_0 + 2\,Pinfty_0\right) Q01^2 - t00^2\,Qinfty_0\right) + O(\lambda) \tag{4.2}$$

$$-\frac{1}{Q01}\left(\left(Q01\,Qinfty_0\,tinfty_3 + P01\,Q01 - Q01\,tinfty_2 - t00\right)\left(Q01\,Qinfty_0\,tinfty_3 + P01\,Q01 - Q01\,tinfty_2 + t00\right)\right)$$

$$ResidueBis := \frac{\frac{t00^2}{Q01} - Q01\left(Qinfty_0\,tinfty_3 + P01 - tinfty_2\right)^2}{\omega}$$

$$0$$

```
> L21infty:=0:
  for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21infty:=
  L21infty+ tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j: od: od:
```

```
Numm:=simplify((L21infty-L11^2)/(L12/omega)):
L21:=0:
for j from 1 to 60 do L21:=L21- 1/omega*simplify(residue(Numm*
lambda^(-j),lambda=infinity))*lambda^(j-1): od:
L21:=simplify(L21+ (t00^2/Q01-Q01*(Qinfty[0]*tinfty[3]+P01-
tinfty[2])^2)/omega/lambda):
```

```
> series(simplify(series(L21,lambda=infinity,2)),lambda=infinity,
2):
CoefflambdaL21:=-factor(residue(L21/lambda^2,lambda=infinity));
Coefflambda0L21:=-factor(residue(L21/lambda,lambda=infinity));
Coefflambda0L21bis:=1/omega*( 2*tinfty[3]*Q01*P01+2*(tinfty[3]*
Qinfty[0]-tinfty[2])*Pinfty[0]+4*(tinfty[3]^2*Qinfty[0]-tinfty
[2]*tinfty[3])*Q01-2*tinfty[3]^2*Qinfty[0]^3+4*tinfty[3]*tinfty
[2]*Qinfty[0]^2-2*(tinfty[2]^2+tinfty[3]*tinfty[1])*Qinfty[0]
+2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2] );
simplify(series(Coefflambda0L21-Coefflambda0L21bis,P01));
CoefflambdaMinus1L21:=-factor(residue(L21,lambda=infinity)):
```

$$CoefflambdaL21 := -\frac{2\,tinfty_3\left(-Qinfty_0^2\,tinfty_3+Q01\,tinfty_3+Qinfty_0\,tinfty_2+Pinfty_0-tinfty_1\right)}{\omega} \tag{4.3}$$

$$Coefflambda0L21 := \frac{1}{\omega}\left(2\left(-Qinfty_0^3\,tinfty_3^2+2\,Q01\,Qinfty_0\,tinfty_3^2+2\,Qinfty_0^2\,tinfty_2\,tinfty_3+P01\,Q01\,tinfty_3-2\,Q01\,tinfty_2\,tinfty_3+Pinfty_0\,Qinfty_0\,tinfty_3-Qinfty_0\,tinfty_1\,tinfty_3-Qinfty_0\,tinfty_2^2-Pinfty_0\,tinfty_2+tinfty_0\,tinfty_3+tinfty_1\,tinfty_2\right)\right)$$

$$Coefflambda0L21bis := \frac{1}{\omega}\left(2\,P01\,Q01\,tinfty_3+2\left(Qinfty_0\,tinfty_3-tinfty_2\right)Pinfty_0+4\left(Qinfty_0\,tinfty_3^2-tinfty_2\,tinfty_3\right)Q01-2\,Qinfty_0^3\,tinfty_3^2+4\,Qinfty_0^2\,tinfty_2\,tinfty_3-2\left(tinfty_1\,tinfty_3+tinfty_2^2\right)Qinfty_0+2\,tinfty_0\,tinfty_3+2\,tinfty_1\,tinfty_2\right)$$

$$0$$

Definition of the auxiliary matrix in the (Q,P) variables.

```
> A12:=0:
for i from 0 to 2*d-1 do A12:=A12+nu[i]*lambda^(2*d-1-i): od:
for k from 0 to 2*d-2 do for i from 0 to k do A12:=A12+Qinfty
[k]*nu[i]*lambda^(k-i) od: od:
omega*nu[0]*lambda^omega*nu[1]*lambda^(2*d-2):
A12:=omega*A12;
cinfty0:= 1/2/omega*Lomega+ tinfty[3]*(Q01*nu[0]+nu[1]*Qinfty
[0]):
A11:=cinfty0:
```

```
PolA12:=0:
for i from 0 to 2*d-1 do PolA12:=PolA12+nu[i]*lambda^(-i): od:
PolA12:=PolA12*L11:
for k from 0 to 60 do A11:=A11-residue(PolA12/lambda^(k+1),
lambda=infinity)*lambda^k od:
A11:=A11;
A22:=-A11:

A21:=(-tinfty[2*d+1]*Lomega/omega^2) *lambda- (tinfty[2*d]-
tinfty[2*d+1]*Qinfty[2*d-2])*Lomega/omega^2+ (alpha[2*d]-tinfty
[2*d+1]*LQinfty0)/omega -tinfty[2*d+1]*nu[0]/omega:
L21oper:=0:
for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21oper:=
L21oper+tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j od: od:
L21oper:=L21oper;
A12oper:=0:
for i from 0 to 2*d+1 do A12oper:=A12oper+nu[i]* lambda^(-i):
od:
A12oper:=A12oper;
for k from 0 to 60 do A21:=A21-residue( (L21oper*A12oper/(L12))
/lambda^(k+1),lambda=infinity)*lambda^k od:
for k from 0 to 60 do A21:=A21-residue( ((L11/(L12))*(L11*
A12/L12 -2*A11))/lambda^(k+1),lambda=infinity)*lambda^k od:
nu[2]:=-(Q01*nu[0]+Qinfty[0]*nu[1]);
Testnu2:=-nu[0]*Q01:
nu[3]:=Q01*Qinfty[0]*nu[0]+Qinfty[0]^2*nu[1]-Q01*nu[1]:
Testnu3:=-nu[1]*Q01:
for j from 2 to 2*d do Testnu3:=Testnu3- nu[j]*Qinfty[j-2]: od:
simplify(nu[3]-Testnu3);
for j from 1 to 2*d-1 do Testnu2:=Testnu2- nu[j]*Qinfty[j-1]:
od:
simplify(nu[2]-Testnu2);
A21:=series(simplify(series(simplify(A21),lambda=infinity)),
lambda=infinity);
```

$$A12 := \omega\left(\lambda\,\nu_0 + Qinfty_0\,\nu_0 + \nu_1\right) \tag{4.4}$$

$$A11 := \frac{1}{2}\,\frac{Lomega}{\omega} + tinfty_3\left(Q01\,\nu_0 + Qinfty_0\,\nu_1\right) + \nu_0\left(\left(Qinfty_0^2 - Q01\right)tinfty_3 - Qinfty_0\,tinfty_2 - Pinfty_0\right) - \nu_1\,tinfty_2 - \left(\nu_0\,tinfty_2 + \nu_1\,tinfty_3\right)\lambda - \nu_0\,tinfty_3\,\lambda^2$$

$$L21oper := \lambda^4\,tinfty_3^2 + 2\,\lambda^3\,tinfty_2\,tinfty_3 + 2\,\lambda^2\,tinfty_1\,tinfty_3 + \lambda^2\,tinfty_2^2 + 2\,\lambda\,tinfty_0\,tinfty_3 + 2\,\lambda\,tinfty_1\,tinfty_2$$

$$A12oper := \nu_0 + \frac{\nu_1}{\lambda} + \frac{\nu_2}{\lambda^2} + \frac{\nu_3}{\lambda^3}$$

$$\nu_2 := -Q01\,\nu_0 - Qinfty_0\,\nu_1$$

$$0$$

$$0$$

$$A21 := \frac{1}{\omega}\left(\left(\left(2\,Qinfty_0^2 - 2\,Q01\right)\nu_0\,tinfty_3^2 + \left(-2\,Qinfty_0\,tinfty_2 - 2\,Pinfty_0 + 2\,tinfty_1\right)\nu_0\,tinfty_3\right)\lambda\right) + \frac{1}{\omega}\left(\left(\left(-2\,Qinfty_0^3 + 4\,Q01\,Qinfty_0\right)\nu_0 - 2\,\nu_1\left(-Qinfty_0^2 + Q01\right)\right)tinfty_3^2 + \left(\left(4\,Qinfty_0^2\,tinfty_2 + \left(2\,Pinfty_0 - 2\,tinfty_1\right)Qinfty_0 - 4\,Q01\,tinfty_2 + 2\,tinfty_0 - 1\right)\nu_0 - 2\,tinfty_2\,\nu_1\,Qinfty_0 + 2\,tinfty_1\,\nu_1 - LQinfty0\right)tinfty_3 + 2\,tinfty_2\left(-Qinfty_0\,tinfty_2 - Pinfty_0 + tinfty_1\right)\nu_0 + \alpha_2\right)$$

```
> CoeffLambda1A21:=factor(-residue(A21/lambda^2,lambda=infinity))
  ;
  CoeffLambda0A21:=simplify(-residue(A21/lambda,lambda=infinity))
  :
  CoeffLambda0A21bis:=1/omega*(-tinfty[3]*LQinfty0 -2*tinfty[3]*(
  (-Qinfty[0]^2+Q01)*tinfty[3]+Qinfty[0]*tinfty[2]-tinfty[1])*nu
  [1]  +alpha[2] +4*tinfty[3]*nu[0]*(Qinfty[0]*tinfty[3]-tinfty
  [2])*Q01-2*Qinfty[0]^3*nu[0]*tinfty[3]^2+4*Qinfty[0]^2*nu[0]*
  tinfty[2]*tinfty[3]+2*nu[0]*(Qinfty[0]*tinfty[3]-tinfty[2])*
  Pinfty[0]+nu[0]*(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-
  tinfty[3])-2*nu[0]*(tinfty[1]*tinfty[3]+tinfty[2]^2)*Qinfty[0])
  ;
  simplify(series(omega*(CoeffLambda0A21-CoeffLambda0A21bis),
  Qinfty[0]));
```

$$CoeffLambda1A21 := -\frac{2\,\nu_0\,tinfty_3\left(-Qinfty_0^2\,tinfty_3 + Q01\,tinfty_3 + Qinfty_0\,tinfty_2 + Pinfty_0 - tinfty_1\right)}{\omega} \tag{4.5}$$

$$CoeffLambda0A21bis := \frac{1}{\omega}\left(-LQinfty0\,tinfty_3 - 2\,tinfty_3\left(\left(-Qinfty_0^2 + Q01\right)tinfty_3 + Qinfty_0\,tinfty_2 - tinfty_1\right)\nu_1 + \alpha_2 + 4\,tinfty_3\,\nu_0\left(Qinfty_0\,tinfty_3 - tinfty_2\right)Q01 - 2\,Qinfty_0^3\,\nu_0\,tinfty_3^2 + 4\,Qinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3 + 2\,\nu_0\left(Qinfty_0\,tinfty_3 - tinfty_2\right)Pinfty_0 + \nu_0\left(2\,tinfty_0\,tinfty_3 + 2\,tinfty_1\,tinfty_2 - tinfty_3\right) - 2\,\nu_0\left(tinfty_1\,tinfty_3 + tinfty_2^2\right)Qinfty_0\right)$$

$$0$$

Compatibility equation and evolutions

```
> L:=Matrix(2,2,0):
  L[1,1]:=L11:
  L[1,2]:=L12:
```

```
  L[2,1]:=L21:
  L[2,2]:=-L[1,1]:
  A:=Matrix(2,2,0):
  A[1,1]:=A11:
  A[1,2]:=A12:
  A[2,1]:=A21:
  A[2,2]:=-A[1,1]:
> dAdlambda:=Matrix(2,2,0):
  for i from 1 to 2 do for j from 1 to 2 do dAdlambda[i,j]:=diff
  (A[i,j],lambda): od: od:
  dAdlambda:
> BigLL:=Matrix(2,2,0):
  BigLL[1,1]:= diff(L[1,1],Qinfty[0])*LQinfty0+ diff(L[1,1],Q01)*
  LQ01+diff(L[1,1],Pinfty[0])*LPinfty0+ diff(L[1,1],P01)*LP01+
  diff(L[1,1],tinfty[2])*alpha[2] +diff(L[1,1],tinfty[1])*alpha
  [1]+ diff(L[1,1],omega)*Lomega:
  BigLL[1,2]:= diff(L[1,2],Qinfty[0])*LQinfty0+ diff(L[1,2],Q01)*
  LQ01+diff(L[1,2],Pinfty[0])*LPinfty0+ diff(L[1,2],P01)*LP01+
  diff(L[1,2],tinfty[2])*alpha[2] +diff(L[1,2],tinfty[1])*alpha
  [1]+ diff(L[1,2],omega)*Lomega:
  BigLL[2,1]:= diff(L[2,1],Qinfty[0])*LQinfty0+ diff(L[2,1],Q01)*
  LQ01+diff(L[2,1],Pinfty[0])*LPinfty0+ diff(L[2,1],P01)*LP01+
  diff(L[2,1],tinfty[2])*alpha[2] +diff(L[2,1],tinfty[1])*alpha
  [1]+ diff(L[2,1],omega)*Lomega:
  BigLL[2,2]:=-BigLL[1,1]:
> Commutator:=Multiply(A,L)-Multiply(L,A):
  CompatibilityEquation:= simplify(dAdlambda-BigLL+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
  lambda=infinity));
  EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
  lambda=infinity));
  EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
  infinity));
  EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
  infinity));

  series(simplify(series(CompatibilityEquation[1,1], lambda=
  infinity)),lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
  lambda=infinity));
  EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
```

```
lambda=infinity));
EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity));
EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda,lambda=
infinity));
EQ7:=simplify(residue(CompatibilityEquation[1,1],lambda=
infinity));
```

$$EQ0 := 0 \qquad (4.6)$$

$$EQ1 := 0$$

$$EQ2 := \left(2\, P01\, Q01\, \nu_0 - 2\, Pinfty_0\, \nu_1 + LQinfty0 - \nu_0\right) \omega$$

$$EQ3 := \left(2\, P01\, Q01\, Qinfty_0\, \nu_0 + 2\, P01\, Q01\, \nu_1 + 2\, Q01\, Pinfty_0\, \nu_0 + LQ01\right) \omega$$

$$EQ0 := 0$$

$$EQ4 := 0$$

$$EQ5 := -tinfty_3 \left(2\, P01\, Q01\, \nu_0 - 2\, Pinfty_0\, \nu_1 + LQinfty0 - \nu_0\right)$$

$$EQ6 := \frac{1}{Q01}\left(-2\,\nu_0\, tinfty_3^2\, Q01^3 + \left(\left(3\, Qinfty_0^2\,\nu_0 - 6\, Qinfty_0\,\nu_1\right) tinfty_3^2 + \left(\left(-2\, P01 + 4\, tinfty_2\right)\nu_1 - 4\,\nu_0\left(Qinfty_0\, tinfty_2 - \frac{1}{2}\, tinfty_1 + \frac{1}{2}\, Pinfty_0\right)\right) tinfty_3 + \nu_0\left(P01 - tinfty_2\right)^2\right) Q01^2 + \left(4\,\nu_1\, Qinfty_0^3\, tinfty_3^2 + \left(\left(-6\, Qinfty_0^2\, tinfty_2 + \left(-2\, Pinfty_0 + 4\, tinfty_1\right) Qinfty_0 - 2\, tinfty_0 + 1\right)\nu_1 - Qinfty_0\,\nu_0 + Qinfty_0\, LQinfty0 - LQ01\right) tinfty_3 + 2\, tinfty_2\left(Qinfty_0\, tinfty_2 + Pinfty_0 - tinfty_1\right)\nu_1 + \nu_0\, tinfty_2 - tinfty_2\, LQinfty0 - LPinfty0\right) Q01 - t00^2\,\nu_0\right)$$

$$EQ7 := \frac{1}{Q01}\left(\left(\left(4\, Qinfty_0\,\nu_0 - 2\,\nu_1\right) tinfty_3^2 - 4\, tinfty_2\,\nu_0\, tinfty_3\right) Q01^3 + \left(\left(-Qinfty_0^3\,\nu_0 + 3\, Qinfty_0^2\,\nu_1\right) tinfty_3^2 + \left(\left(\left(2\, P01 + 2\, tinfty_2\right) Qinfty_0^2 + \left(2\, Pinfty_0 - 2\, tinfty_1\right) Qinfty_0 + 2\, tinfty_0 - 1\right)\nu_0 + 2\left(\left(P01 - 2\, tinfty_2\right) Qinfty_0 + tinfty_1\right)\nu_1\right) tinfty_3 + \left(\left(P01^2 - 2\, P01\, tinfty_2 - tinfty_2^2\right) Qinfty_0 + 2\, tinfty_2\left(-Pinfty_0 + tinfty_1\right)\right)\nu_0 + P01^2\,\nu_1 - 2\, P01\,\nu_1\, tinfty_2 + \nu_1\, tinfty_2^2 + LP01\right) Q01^2 + LQ01\left(Qinfty_0\, tinfty_3 + P01 - tinfty_2\right) Q01 - t00^2\left(Qinfty_0\,\nu_0 + \nu_1\right)\right)$$

```
> LQinfty0:=solve(EQ2,LQinfty0);
  LQ01:=solve(EQ3,LQ01);
```

$$LQinfty0 := -2\, P01\, Q01\, \nu_0 + 2\, Pinfty_0\, \nu_1 + \nu_0 \qquad (4.7)$$

$$LQ01 := -2\, Q01\left(P01\, Qinfty_0\,\nu_0 + P01\,\nu_1 + Pinfty_0\,\nu_0\right)$$

```
> simplify(EQ5);
  simplify(EQ6);
  simplify(EQ7);
```

$$0 \qquad (4.8)$$

$$\frac{1}{Q01}\left(-2\,\nu_0\, tinfty_3^2\, Q01^3 + \left(\left(3\, Qinfty_0^2\,\nu_0 - 6\, Qinfty_0\,\nu_1\right) tinfty_3^2 + \left(4\,\nu_1\, tinfty_2\right.\right.$$

$$-4\left(Qinfty_0\,tinfty_2-\frac{1}{2}\,tinfty_1\right)\nu_0\Big)\,tinfty_3+\nu_0\left(P01^2+tinfty_2^2\right)\Big)\,Q01^2+\Big(4\,\nu_1$$
$$Qinfty_0^3\,tinfty_3^2-6\,\nu_1\left(Qinfty_0^2\,tinfty_2-\frac{2}{3}\,Qinfty_0\,tinfty_1+\frac{1}{3}\,tinfty_0-\frac{1}{6}\right)tinfty_3$$
$$+\left(2\,Qinfty_0\,tinfty_2^2-2\,tinfty_1\,tinfty_2\right)\nu_1-LPinfty0\Big)\,Q01-t00^2\,\nu_0\Big)$$
$$\frac{1}{Q01}\Big(\Big(\left(4\,Qinfty_0\,tinfty_3^2-4\,tinfty_2\,tinfty_3\right)\nu_0-2\,tinfty_3^2\,\nu_1\Big)\,Q01^3+\Big(\Big(-Qinfty_0^3\,tinfty_3^2$$
$$+\left(2\,Qinfty_0^2\,tinfty_2-2\,Qinfty_0\,tinfty_1+2\,tinfty_0-1\right)tinfty_3+\left(-P01^2-$$
$$tinfty_2^2\right)Qinfty_0-2\,P01\,Pinfty_0+2\,tinfty_1\,tinfty_2\Big)\nu_0+3\,tinfty_3^2\,\nu_1\,Qinfty_0^2+2\,\nu_1\,\Big($$
$$-2\,Qinfty_0\,tinfty_2+tinfty_1\Big)\,tinfty_3+\left(-P01^2+tinfty_2^2\right)\nu_1+LP01\Big)\,Q01^2$$
$$-t00^2\left(Qinfty_0\,\nu_0+\nu_1\right)\Big)$$

```
> LPinfty0:=solve(EQ6,LPinfty0);
  LP01:=solve(EQ7,LP01);
```

$$LPinfty0 := \frac{1}{Q01}\Big(3\,Q01^2\,Qinfty_0^2\,\nu_0\,tinfty_3^2+4\,Q01\,Qinfty_0^3\,\nu_1\,tinfty_3^2-2\,Q01^3\,\nu_0\,tinfty_3^2 \qquad \textbf{(4.9)}$$
$$-4\,Q01^2\,Qinfty_0\,\nu_0\,tinfty_2\,tinfty_3-6\,Q01^2\,Qinfty_0\,\nu_1\,tinfty_3^2-6\,Q01$$
$$Qinfty_0^2\,\nu_1\,tinfty_2\,tinfty_3+P01^2\,Q01^2\,\nu_0+2\,Q01^2\,\nu_0\,tinfty_1\,tinfty_3+Q01^2\,\nu_0\,tinfty_2^2$$
$$+4\,Q01^2\,\nu_1\,tinfty_2\,tinfty_3+4\,Q01\,Qinfty_0\,\nu_1\,tinfty_1\,tinfty_3+2\,Q01\,Qinfty_0\,\nu_1\,tinfty_2^2$$
$$-2\,Q01\,\nu_1\,tinfty_0\,tinfty_3-2\,Q01\,\nu_1\,tinfty_1\,tinfty_2+Q01\,\nu_1\,tinfty_3-t00^2\,\nu_0\Big)$$
$$LP01 := \frac{1}{Q01^2}\Big(Q01^2\,Qinfty_0^3\,\nu_0\,tinfty_3^2-4\,Q01^3\,Qinfty_0\,\nu_0\,tinfty_3^2-2\,Q01^2$$
$$Qinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3-3\,Q01^2\,Qinfty_0^2\,\nu_1\,tinfty_3^2+P01^2\,Q01^2\,Qinfty_0\,\nu_0$$
$$+4\,Q01^3\,\nu_0\,tinfty_2\,tinfty_3+2\,Q01^3\,\nu_1\,tinfty_3^2+2\,Q01^2\,Qinfty_0\,\nu_0\,tinfty_1\,tinfty_3$$
$$+Q01^2\,Qinfty_0\,\nu_0\,tinfty_2^2+4\,Q01^2\,Qinfty_0\,\nu_1\,tinfty_2\,tinfty_3+P01^2\,Q01^2\,\nu_1$$
$$+2\,P01\,Q01^2\,Pinfty_0\,\nu_0-2\,Q01^2\,\nu_0\,tinfty_0\,tinfty_3-2\,Q01^2\,\nu_0\,tinfty_1\,tinfty_2$$
$$-2\,Q01^2\,\nu_1\,tinfty_1\,tinfty_3-Q01^2\,\nu_1\,tinfty_2^2+Q01^2\,\nu_0\,tinfty_3+t00^2\,Qinfty_0\,\nu_0+t00^2\,\nu_1\Big)$$

```
> Ham:=-2*P01*Q01*Pinfty[0]*nu[0]+Pinfty[0]^2*nu[1]+Pinfty[0]*nu
  [0]-P01^2*Q01*(Qinfty[0]*nu[0]+nu[1])
  -((-2*nu[0]*tinfty[3]^2*Q01^3+((2*nu[0]*tinfty[1]+4*nu[1]*
  tinfty[2])*tinfty[3]+nu[0]*tinfty[2]^2)*Q01^2-2*nu[1]*((tinfty
  [0]-1/2)*tinfty[3]+tinfty[1]*tinfty[2])*Q01-t00^2*nu[0])*Qinfty
  [0]/Q01+((1/2)*(-6*Q01*tinfty[3]^2+4*tinfty[1]*tinfty[3]+2*
  tinfty[2]^2)*nu[1]-2*Q01*nu[0]*tinfty[2]*tinfty[3])*Qinfty[0]
  ^2+tinfty[3]*(Q01*nu[0]*tinfty[3]-2*nu[1]*tinfty[2])*Qinfty[0]
  ^3+tinfty[3]^2*nu[1]*Qinfty[0]^4)
  -(2*Q01^2*nu[0]*tinfty[2]*tinfty[3]+Q01^2*nu[1]*tinfty[3]^2-2*
  Q01*nu[0]*tinfty[0]*tinfty[3]-2*Q01*nu[0]*tinfty[1]*tinfty[2]
  -2*Q01*nu[1]*tinfty[1]*tinfty[3]-Q01*nu[1]*tinfty[2]^2+nu[0]*
```

```
tinfty[3]*Q01-t00^2*nu[1]/Q01);

simplify(diff(Ham,Pinfty[0])- LQinfty0);
simplify(diff(Ham,P01)- LQ01);
simplify(diff(Ham,Qinfty[0])+LPinfty0);
simplify(diff(Ham,Q01)+LP01);
```

$$Ham := -2\,P01\,Q01\,Pinfty_0\,\nu_0 + Pinfty_0^2\,\nu_1 + \nu_0\,Pinfty_0 - P01^2\,Q01\left(Qinfty_0\,\nu_0 + \nu_1\right) - \frac{1}{Q01}\left(\left(-2\,\nu_0\,tinfty_3^2\,Q01^3 + \left(\left(2\,\nu_0\,tinfty_1 + 4\,\nu_1\,tinfty_2\right)tinfty_3 + \nu_0\,tinfty_2^2\right)Q01^2 - 2\,\nu_1\left(\left(tinfty_0 - \frac{1}{2}\right)tinfty_3 + tinfty_1\,tinfty_2\right)Q01 - t00^2\,\nu_0\right)Qinfty_0\right) - \left(\frac{1}{2}\left(-6\,Q01\,tinfty_3^2 + 4\,tinfty_1\,tinfty_3 + 2\,tinfty_2^2\right)\nu_1 - 2\,Q01\,\nu_0\,tinfty_2\,tinfty_3\right)Qinfty_0^2 - tinfty_3\left(Q01\,\nu_0\,tinfty_3 - 2\,\nu_1\,tinfty_2\right)Qinfty_0^3 - tinfty_3^2\,\nu_1\,Qinfty_0^4 - 2\,Q01^2\,\nu_0\,tinfty_2\,tinfty_3 - Q01^2\,\nu_1\,tinfty_3^2 + 2\,Q01\,\nu_0\,tinfty_0\,tinfty_3 + 2\,Q01\,\nu_0\,tinfty_1\,tinfty_2 + 2\,Q01\,\nu_1\,tinfty_1\,tinfty_3 + Q01\,\nu_1\,tinfty_2^2 - \nu_0\,tinfty_3\,Q01 + \frac{t00^2\,\nu_1}{Q01} \tag{4.10}$$

$$\begin{matrix}0\\0\\0\\0\end{matrix}$$

```
> Qinfty[0]:=-Sinfty[0];
  Q01:=S01;
  P01:=R01+tinfty[3]*Sinfty[0];
  Pinfty[0]:=-Rinfty[0]+tinfty[3]*Sinfty[0]^2-tinfty[3]*S01;
```

$$Qinfty_0 := -Sinfty_0 \tag{4.11}$$

$$Q01 := S01$$

$$P01 := Sinfty_0\,tinfty_3 + R01$$

$$Pinfty_0 := Sinfty_0^2\,tinfty_3 - S01\,tinfty_3 - Rinfty_0$$

```
> simplify(series(L11,lambda=infinity));
  simplify(L11-(-tinfty[3]*lambda^2+Rinfty[0] +R01*S01/lambda-
  tinfty[2]*L12/omega));
  simplify(series(L12,lambda=infinity));
  ConstantTermL21:=simplify(-omega*residue(L21/lambda,lambda=
  infinity)):
  ConstantTermL21bis:=2*tinfty[3]*R01*S01+2*(Sinfty[0]*tinfty[3]+
  tinfty[2])*Rinfty[0]-2*S01*tinfty[2]*tinfty[3]+2*Sinfty[0]^2*
  tinfty[2]*tinfty[3] +2*(tinfty[1]*tinfty[3]+tinfty[2]^2)*Sinfty
  [0]+2*(tinfty[0]*tinfty[3]+tinfty[1]*tinfty[2]):
  ConstantTermL21bis;
  series(simplify(ConstantTermL21-ConstantTermL21bis),Sinfty[0]);
```

$$\frac{-\lambda^3\, tinfty_3 - \lambda^2\, tinfty_2 + \left(Sinfty_0\, tinfty_2 + Rinfty_0\right)\lambda + S01\,\left(R01 - tinfty_2\right)}{\lambda}$$

$$0$$

$$\frac{\omega\left(\lambda^2 - \lambda\, Sinfty_0 + S01\right)}{\lambda}$$

$$2\, R01\, S01\, tinfty_3 + 2\left(Sinfty_0\, tinfty_3 + tinfty_2\right) Rinfty_0 - 2\, S01\, tinfty_2\, tinfty_3 + 2\, Sinfty_0^2\, tinfty_2\, tinfty_3 + 2\left(tinfty_1\, tinfty_3 + tinfty_2^2\right) Sinfty_0 + 2\, tinfty_0\, tinfty_3 + 2\, tinfty_1\, tinfty_2$$

$$0$$

(4.12)

```
> series(A12,lambda=infinity);
  series(A11,lambda=infinity);
  series(Lomega/(2*omega)+tinfty[3]*(S01*nu[0]-Sinfty[0]*nu[1])+
  nu[0]*((Sinfty[0]^2-S01)*tinfty[3]+Sinfty[0]*tinfty[2]-Sinfty
  [0]^2*tinfty[3]+S01*tinfty[3]+Rinfty[0])-nu[1]*tinfty[2],nu[0])
  ;
```

$$\nu_0\,\omega\,\lambda + \omega\left(-Sinfty_0\,\nu_0 + \nu_1\right)$$

$$-\nu_0\, tinfty_3\,\lambda^2 + \left(-\nu_0\, tinfty_2 - \nu_1\, tinfty_3\right)\lambda + \frac{1}{2}\,\frac{Lomega}{\omega} + tinfty_3\left(S01\,\nu_0 - Sinfty_0\,\nu_1\right) + \nu_0\left(\left(Sinfty_0^2 - S01\right) tinfty_3 + Sinfty_0\, tinfty_2 - tinfty_3\, Sinfty_0^2 + tinfty_3\, S01 + Rinfty_0\right) - \nu_1\, tinfty_2$$

$$\frac{1}{2}\,\frac{Lomega}{\omega} - Sinfty_0\,\nu_1\, tinfty_3 - \nu_1\, tinfty_2 + \left(2\, tinfty_3\, S01 + \left(Sinfty_0^2 - S01\right) tinfty_3 + Sinfty_0\, tinfty_2 - tinfty_3\, Sinfty_0^2 + Rinfty_0\right)\nu_0$$

(4.13)

```
> series(simplify(A21),lambda=infinity);
  ConstantTermA21:=simplify(-omega*residue(A21/lambda,lambda=
  infinity)):
  ConstantTermA21bis:=alpha[2]+2*tinfty[3]*(tinfty[2]*Sinfty[0]+
  Rinfty[0]+tinfty[1])*nu[1]
  +nu[0]*(2*tinfty[3]*R01*S01+2*(tinfty[3]*Sinfty[0]+tinfty[2])*
  Rinfty[0] -2*tinfty[2]*tinfty[3]*S01+2*tinfty[2]*tinfty[3]*
  (Sinfty[0])^2+2*(tinfty[1]*tinfty[3]+(tinfty[2])^2)*Sinfty[0]
  +2*((tinfty[0]-1)*tinfty[3]+tinfty[1]*tinfty[2])):
  series(simplify(ConstantTermA21-ConstantTermA21bis),Sinfty[0]);
  ConstantTermA21bis;
```

$$\frac{\left(2\, Sinfty_0\, tinfty_2 + 2\, Rinfty_0 + 2\, tinfty_1\right)\nu_0\, tinfty_3\,\lambda}{\omega} + \frac{1}{\omega}\left(\left(\left(\left(2\, Sinfty_0^2 - 2\, S01\right) tinfty_2 + \left(2\, Rinfty_0 + 2\, tinfty_1\right) Sinfty_0 + 2\, S01\, R01 + 2\, tinfty_0 - 2\right)\nu_0 + 2\,\nu_1\left(Sinfty_0\, tinfty_2 + Rinfty_0 + tinfty_1\right)\right) tinfty_3 + 2\, tinfty_2\left(Sinfty_0\, tinfty_2 + Rinfty_0 + tinfty_1\right)\nu_0 + \alpha_2\right)$$

$$0$$

(4.14)

$$\alpha_2 + 2\,tinfty_3\left(Sinfty_0\,tinfty_2 + Rinfty_0 + tinfty_1\right)\nu_1 + \nu_0\left(2\,R01\,S01\,tinfty_3 + 2\left(Sinfty_0\,tinfty_3 + tinfty_2\right)Rinfty_0 - 2\,S01\,tinfty_2\,tinfty_3 + 2\,Sinfty_0^2\,tinfty_2\,tinfty_3 + 2\left(tinfty_1\,tinfty_3 + tinfty_2^2\right)Sinfty_0 + 2\left(tinfty_0 - 1\right)tinfty_3 + 2\,tinfty_1\,tinfty_2\right)$$

```
> BigLLRS:=Matrix(2,2,0):
  BigLLRS[1,1]:= diff(L[1,1],Sinfty[0])*LSinfty0+ diff(L[1,1],
  S01)*LS01+diff(L[1,1],Rinfty[0])*LRinfty0+ diff(L[1,1],R01)*
  LR01+diff(L[1,1],tinfty[2])*alpha[2] +diff(L[1,1],tinfty[1])*
  alpha[1]+ diff(L[1,1],omega)*Lomega:
  BigLLRS[1,2]:= diff(L[1,2],Sinfty[0])*LSinfty0+ diff(L[1,2],
  S01)*LS01+diff(L[1,2],Rinfty[0])*LRinfty0+ diff(L[1,2],R01)*
  LR01+diff(L[1,2],tinfty[2])*alpha[2] +diff(L[1,2],tinfty[1])*
  alpha[1]+ diff(L[1,2],omega)*Lomega:
  BigLLRS[2,1]:= diff(L[2,1],Sinfty[0])*LSinfty0+ diff(L[2,1],
  S01)*LS01+diff(L[2,1],Rinfty[0])*LRinfty0+ diff(L[2,1],R01)*
  LR01+diff(L[2,1],tinfty[2])*alpha[2] +diff(L[2,1],tinfty[1])*
  alpha[1]+ diff(L[2,1],omega)*Lomega:
  BigLLRS[2,2]:=-BigLL[1,1]:
> A:=simplify(A):
  L:=simplify(L):
> Commutator:=Multiply(A,L)-Multiply(L,A):
  CompatibilityEquation:= simplify(dAdlambda-BigLLRS+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
  lambda=infinity));
  EQ1:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
  lambda=infinity));
  EQ2:=simplify(residue(CompatibilityEquation[1,2]/lambda,lambda=
  infinity));
  EQ3:=simplify(residue(CompatibilityEquation[1,2],lambda=
  infinity));

  series(simplify(series(CompatibilityEquation[1,1], lambda=
  infinity)),lambda=infinity):
  EQ0:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
  lambda=infinity));
  EQ4:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
  lambda=infinity));
  EQ5:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
  lambda=infinity));
  EQ6:=simplify(residue(CompatibilityEquation[1,1]/lambda,lambda=
  infinity));
```

```
EQ7:=simplify(residue(CompatibilityEquation[1,1],lambda=
infinity));
```

$$EQ0 := 0$$
$$EQ1 := 0 \tag{4.15}$$

$$EQ2 := 2\,\omega\left(\left(\left(Sinfty_0\,tinfty_3 + R01\right)\nu_0 + \nu_1\,tinfty_3\right)S01 + \left(-Sinfty_0^2\,tinfty_3 + Rinfty_0\right)\nu_1 - \frac{1}{2}\,\nu_0 - \frac{1}{2}\,LSinfty0\right)$$

$$EQ3 := -2\left(S01^2\,\nu_0\,tinfty_3 + \left(\left(R01\,Sinfty_0 + Rinfty_0\right)\nu_0 - \left(Sinfty_0\,tinfty_3 + R01\right)\nu_1\right)S01 - \frac{1}{2}\,LS01\right)\omega$$

$$EQ0 := 0$$
$$EQ4 := 0$$
$$EQ5 := 0$$

$$EQ6 := \frac{1}{S01}\left(\left(\left(\left(2\,Sinfty_0\,\nu_0 + 2\,\nu_1\right)tinfty_2 - 2\,R01\,\nu_1 + 2\,\nu_0\left(Rinfty_0 + tinfty_1\right)\right)tinfty_3 + \nu_0\left(R01 - tinfty_2\right)^2\right)S01^2 + \left(\left(-4\,tinfty_2\,\nu_1\,Sinfty_0^2 + \left(\left(-4\,tinfty_1 - 4\,Rinfty_0\right)Sinfty_0 - 2\,tinfty_0 + 1\right)\nu_1 + 2\,Sinfty_0\,\nu_0\right)tinfty_3 - 2\,\nu_1\,Sinfty_0\,tinfty_2^2 + \left(\left(-2\,Rinfty_0 - 2\,tinfty_1\right)\nu_1 + \nu_0 + LSinfty0\right)tinfty_2 + LRinfty0\right)S01 - t00^2\,\nu_0\right)$$

$$EQ7 := \frac{1}{S01}\left(2\,tinfty_3\,\nu_0\left(R01 - tinfty_2\right)S01^3 + \left(\left(Sinfty_0\,tinfty_2^2 + \left(2\,Sinfty_0^2\,tinfty_3 + 2\,R01\,Sinfty_0 + 2\,Rinfty_0 + 2\,tinfty_1\right)tinfty_2 + \left(\left(2\,Rinfty_0 + 2\,tinfty_1\right)Sinfty_0 + 2\,tinfty_0 - 2\right)tinfty_3 - R01^2\,Sinfty_0\right)\nu_0 + \nu_1\,tinfty_2^2 - 2\,\nu_1\left(-Sinfty_0\,tinfty_3 + R01\right)tinfty_2 + 2\,\nu_1\left(Rinfty_0 + tinfty_1\right)tinfty_3 + R01^2\,\nu_1 + LR01\right)S01^2 + \left(R01 - tinfty_2\right)LS01\,S01 + t00^2\left(Sinfty_0\,\nu_0 - \nu_1\right)\right)$$

```
> LSinfty0:=solve(EQ2,LSinfty0);
  LS01:=solve(EQ3,LS01);
```

$$LSinfty0 := 2\,S01\,Sinfty_0\,\nu_0\,tinfty_3 - 2\,Sinfty_0^2\,\nu_1\,tinfty_3 + 2\,R01\,S01\,\nu_0 + 2\,S01\,\nu_1\,tinfty_3 + 2\,Rinfty_0\,\nu_1 - \nu_0 \tag{4.16}$$

$$LS01 := 2\,S01\left(R01\,Sinfty_0\,\nu_0 + S01\,\nu_0\,tinfty_3 - Sinfty_0\,\nu_1\,tinfty_3 - R01\,\nu_1 + Rinfty_0\,\nu_0\right)$$

```
> simplify(EQ5);
  simplify(EQ6);
  simplify(EQ7);
```

$$0 \tag{4.17}$$

$$\frac{1}{S01}\left(\left(\left(\left(-2\,R01 + 4\,tinfty_2\right)\nu_1 + 2\,\nu_0\left(2\,Sinfty_0\,tinfty_2 + Rinfty_0 + tinfty_1\right)\right)tinfty_3 + \nu_0\left(R01^2 + tinfty_2^2\right)\right)S01^2 + \left(\left(\left(-6\,Sinfty_0^2\,tinfty_2 + \left(-4\,tinfty_1 - 4\,Rinfty_0\right)Sinfty_0 - 2\,tinfty_0 + 1\right)\nu_1 + 2\,Sinfty_0\,\nu_0\right)tinfty_3 + \left(-2\,Sinfty_0\,tinfty_2^2 - 2\,tinfty_1\,tinfty_2\right)\nu_1 + LRinfty0\right)S01 - t00^2\,\nu_0\right)$$

$$\frac{1}{S01}\left(4\,tinfty_3\,\nu_0\left(R01-tinfty_2\right)S01^3+\left(\left(\left(2\,Sinfty_0^2\,tinfty_2+\left(2\,Rinfty_0+2\,tinfty_1\right)Sinfty_0+2\,tinfty_0-2\right)tinfty_3+\left(R01^2+tinfty_2^2\right)Sinfty_0+2\,R01\,Rinfty_0+2\,tinfty_1\,tinfty_2\right)\nu_0-2\,\nu_1\left(\left(R01-2\,tinfty_2\right)Sinfty_0-tinfty_1-Rinfty_0\right)tinfty_3+\left(-R01^2+tinfty_2^2\right)\nu_1+LR01\right)S01^2+t00^2\left(Sinfty_0\,\nu_0-\nu_1\right)\right)$$

```
> LRinfty0:=solve(EQ6,LRinfty0);
  LR01:=solve(EQ7,LR01);
```

$$LRinfty0 := -\frac{1}{S01}\left(4\,S01^2\,Sinfty_0\,\nu_0\,tinfty_2\,tinfty_3-6\,S01\,Sinfty_0^2\,\nu_1\,tinfty_2\,tinfty_3+R01^2\,S01^2\,\nu_0-2\,R01\,S01^2\,\nu_1\,tinfty_3+2\,S01^2\,Rinfty_0\,\nu_0\,tinfty_3+2\,S01^2\,\nu_0\,tinfty_1\,tinfty_3+S01^2\,\nu_0\,tinfty_2^2+4\,S01^2\,\nu_1\,tinfty_2\,tinfty_3-4\,S01\,Rinfty_0\,Sinfty_0\,\nu_1\,tinfty_3-4\,S01\,Sinfty_0\,\nu_1\,tinfty_1\,tinfty_3-2\,S01\,Sinfty_0\,\nu_1\,tinfty_2^2+2\,S01\,Sinfty_0\,\nu_0\,tinfty_3-2\,S01\,\nu_1\,tinfty_0\,tinfty_3-2\,S01\,\nu_1\,tinfty_1\,tinfty_2+S01\,\nu_1\,tinfty_3-t00^2\,\nu_0\right) \tag{4.18}$$

$$LR01 := -\frac{1}{S01^2}\left(2\,S01^2\,Sinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3+R01^2\,S01^2\,Sinfty_0\,\nu_0+4\,R01\,S01^3\,\nu_0\,tinfty_3-2\,R01\,S01^2\,Sinfty_0\,\nu_1\,tinfty_3-4\,S01^3\,\nu_0\,tinfty_2\,tinfty_3+2\,S01^2\,Rinfty_0\,Sinfty_0\,\nu_0\,tinfty_3+2\,S01^2\,Sinfty_0\,\nu_0\,tinfty_1\,tinfty_3+S01^2\,Sinfty_0\,\nu_0\,tinfty_2^2+4\,S01^2\,Sinfty_0\,\nu_1\,tinfty_2\,tinfty_3-R01^2\,S01^2\,\nu_1+2\,R01\,S01^2\,Rinfty_0\,\nu_0+2\,S01^2\,Rinfty_0\,\nu_1\,tinfty_3+2\,S01^2\,\nu_0\,tinfty_0\,tinfty_3+2\,S01^2\,\nu_0\,tinfty_1\,tinfty_2+2\,S01^2\,\nu_1\,tinfty_1\,tinfty_3+S01^2\,\nu_1\,tinfty_2^2-2\,S01^2\,\nu_0\,tinfty_3+t00^2\,Sinfty_0\,\nu_0-t00^2\,\nu_1\right)$$

```
> HamRS:=-(((2*Sinfty[0]^2-2*S01)*nu[1]-2*S01*nu[0]*Sinfty[0])*
  tinfty[3]*Rinfty[0]-2*R01*S01*nu[0]*Rinfty[0]-Rinfty[0]^2*nu[1]
  +Rinfty[0]*nu[0]) -(-2*S01*(tinfty[3]*(S01*nu[0]-Sinfty[0]*nu
  [1])*R01+(1/2)*R01^2*(Sinfty[0]*nu[0]-nu[1]))) -(-2*S01*Sinfty
  [0]^2*nu[0]*tinfty[2]*tinfty[3]+2*Sinfty[0]^3*nu[1]*tinfty[2]*
  tinfty[3]-2*S01*Sinfty[0]*nu[0]*tinfty[1]*tinfty[3]-S01*Sinfty
  [0]*nu[0]*tinfty[2]^2-4*S01*Sinfty[0]*nu[1]*tinfty[2]*tinfty[3]
  +2*Sinfty[0]^2*nu[1]*tinfty[1]*tinfty[3]+nu[1]*Sinfty[0]^2*
  tinfty[2]^2-Sinfty[0]^2*nu[0]*tinfty[3]+2*Sinfty[0]*nu[1]*
  tinfty[0]*tinfty[3]+2*nu[1]*tinfty[2]*Sinfty[0]*tinfty[1]-
  Sinfty[0]*tinfty[3]*nu[1]+t00^2*Sinfty[0]*nu[0]/S01) -(2*S01^2*
  nu[0]*tinfty[2]*tinfty[3]-2*S01*nu[0]*tinfty[0]*tinfty[3]-2*
  S01*nu[0]*tinfty[1]*tinfty[2]-2*S01*nu[1]*tinfty[1]*tinfty[3]-
  S01*nu[1]*tinfty[2]^2+2*S01*nu[0]*tinfty[3]-t00^2*nu[1]/S01);
  simplify(diff(HamRS,Rinfty[0])- LSinfty0);
  simplify(diff(HamRS,R01)- LS01);
  simplify(diff(HamRS,Sinfty[0])+LRinfty0);
  simplify(diff(HamRS,S01)+LR01);
```

$$HamRS := -\left(\left(2\,Sinfty_0^2 - 2\,S01\right)\nu_1 - 2\,S01\,\nu_0\,Sinfty_0\right)tinfty_3\,Rinfty_0 + 2\,R01\,S01\,\nu_0\,Rinfty_0 + Rinfty_0^2\,\nu_1 - Rinfty_0\,\nu_0 + 2\,S01\left(tinfty_3\left(S01\,\nu_0 - Sinfty_0\,\nu_1\right)R01 + \frac{1}{2}\,R01^2\left(Sinfty_0\,\nu_0 - \nu_1\right)\right) + 2\,S01\,Sinfty_0^2\,\nu_0\,tinfty_2\,tinfty_3 - 2\,Sinfty_0^3\,\nu_1\,tinfty_2\,tinfty_3 + 2\,S01\,Sinfty_0\,\nu_0\,tinfty_1\,tinfty_3 + S01\,Sinfty_0\,\nu_0\,tinfty_2^2 + 4\,S01\,Sinfty_0\,\nu_1\,tinfty_2\,tinfty_3 - 2\,Sinfty_0^2\,\nu_1\,tinfty_1\,tinfty_3 - \nu_1\,Sinfty_0^2\,tinfty_2^2 + Sinfty_0^2\,\nu_0\,tinfty_3 - 2\,Sinfty_0\,\nu_1\,tinfty_0\,tinfty_3 - 2\,\nu_1\,tinfty_2\,Sinfty_0\,tinfty_1 + Sinfty_0\,\nu_1\,tinfty_3 - \frac{t00^2\,Sinfty_0\,\nu_0}{S01} - 2\,S01^2\,\nu_0\,tinfty_2\,tinfty_3 + 2\,S01\,\nu_0\,tinfty_0\,tinfty_3 + 2\,S01\,\nu_0\,tinfty_1\,tinfty_2 + 2\,S01\,\nu_1\,tinfty_1\,tinfty_3 + S01\,\nu_1\,tinfty_2^2 - 2\,S01\,\nu_0\,tinfty_3 + \frac{t00^2\,\nu_1}{S01} \tag{4.19}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> HamRS:=simplify(HamRS):
  M:=Matrix(2*d,2*d,0):
  for i from 1 to 2*d do for j from 1 to i do M[i,j]:=tinfty[2*
  d+1-(i-j)]: od: od:
  M;
  alphaVector:=Matrix(2*d,1,0):
  for i from 1 to 2*d do alphaVector[i]:=alpha[2*d+1-i]/(2*d+1-i)
  : od:
  alphaVector;
  nuVector:=Multiply(M^(-1),alphaVector);
  for i from 1 to 2*d do nu[i-1]:=nuVector[i,1]: od:
  nu[0]:=nu[0];
  nu[1]:=nu[1];
```

$$\begin{bmatrix} tinfty_3 & 0 \\ tinfty_2 & tinfty_3 \end{bmatrix} \tag{4.20}$$

$$\begin{bmatrix} \frac{1}{2}\,\alpha_2 \\ \alpha_1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\ \frac{\alpha_2}{tinfty_3} \\ -\frac{1}{2}\ \frac{tinfty_2\,\alpha_2}{tinfty_3^2} + \frac{\alpha_1}{tinfty_3} \end{bmatrix}$$

$$\nu_0 := \frac{1}{2}\ \frac{\alpha_2}{tinfty_3}$$

$$\nu_1 := -\frac{1}{2}\ \frac{tinfty_2\,\alpha_2}{tinfty_3^2} + \frac{\alpha_1}{tinfty_3}$$

```
> HamRSfunction:=unapply(simplify(HamRS),alpha[1],alpha[2]):
  HamRStinfty1:=simplify(HamRSfunction(1,0));
  HamRStinfty2:=simplify(HamRSfunction(0,1));
```

$$HamRStinfty1 := \frac{1}{tinfty_3\,S01}\left(\left(\left(\left(-2\,R01 + 4\,tinfty_2\right)Sinfty_0 + 2\,Rinfty_0 + 2\,tinfty_1\right)tinfty_3\right. \right. \tag{4.21}$$
$$\left. - R01^2 + tinfty_2^2\right)S01^2 + \left(-2\left(Sinfty_0^2\,tinfty_2 + \left(Rinfty_0 + tinfty_1\right)Sinfty_0 + tinfty_0\right.\right.$$
$$\left.\left. - \frac{1}{2}\right)Sinfty_0\,tinfty_3 - Sinfty_0^2\,tinfty_2^2 - 2\,Sinfty_0\,tinfty_1\,tinfty_2 + Rinfty_0^2\right)S01 + t00^2\Bigg)$$

$$HamRStinfty2 := \frac{1}{2}\ \frac{1}{tinfty_3^2\,S01}\left(2\,tinfty_3^2\left(R01 - tinfty_2\right)S01^3 + \left(\left(2\,Sinfty_0^2\,tinfty_2\right.\right.\right.$$
$$\left. + \left(2\,Rinfty_0 + 2\,tinfty_1\right)Sinfty_0 + 2\,tinfty_0 - 2\right)tinfty_3^2 + \left(R01 - tinfty_2\right)\left(R01\,Sinfty_0\right.$$
$$\left.\left. + 3\,Sinfty_0\,tinfty_2 + 2\,Rinfty_0\right)tinfty_3 + R01^2\,tinfty_2 - tinfty_2^3\right)S01^2 + \left(Sinfty_0^2\,tinfty_3^2\right.$$
$$+ \left(2\,tinfty_2^2\,Sinfty_0^3 + 2\,Sinfty_0\left(\left(Rinfty_0 + tinfty_1\right)Sinfty_0 + tinfty_0 - \frac{1}{2}\right)tinfty_2\right.$$
$$\left.\left. - Rinfty_0\right)tinfty_3 + Sinfty_0^2\,tinfty_2^3 + 2\,Sinfty_0\,tinfty_1\,tinfty_2^2 - Rinfty_0^2\,tinfty_2\right)S01$$
$$\left. - t00^2\left(Sinfty_0\,tinfty_3 + tinfty_2\right)\right)$$

```
> HamRStinfty1bis:=t00^2/(S01*tinfty[3]) -S01*R01^2/tinfty[3]-2*
  S01*R01*Sinfty[0]+2*(S01-Sinfty[0]^2)*Rinfty[0]+Rinfty[0]
  ^2/tinfty[3]+4*tinfty[2]*Sinfty[0]*S01+(2*tinfty[1]*tinfty[3]+
  tinfty[2]^2)*S01/tinfty[3]-2*Sinfty[0]^3*tinfty[2]-(2*tinfty[1]
  *tinfty[3]+tinfty[2]^2)*Sinfty[0]^2/tinfty[3]
  -(2*tinfty[0]*tinfty[3]+2*tinfty[1]*tinfty[2]-tinfty[3])*Sinfty
  [0]/tinfty[3];
  simplify(series(HamRStinfty1-HamRStinfty1bis, Sinfty[0]));
```

$$HamRStinfty1bis := \frac{t00^2}{tinfty_3\,S01} - \frac{S01\,R01^2}{tinfty_3} - 2\,S01\,R01\,Sinfty_0 + 2\left(-Sinfty_0^2\right. \tag{4.22}$$

$$+ S01\Big)\, Rinfty_0 + \frac{Rinfty_0^2}{tinfty_3} + 4\, tinfty_2\, Sinfty_0\, S01 + \frac{\left(2\, tinfty_1\, tinfty_3 + tinfty_2^2\right) S01}{tinfty_3} - 2$$

$$Sinfty_0^3\, tinfty_2 - \frac{\left(2\, tinfty_1\, tinfty_3 + tinfty_2^2\right) Sinfty_0^2}{tinfty_3}$$

$$- \frac{\left(2\, tinfty_0\, tinfty_3 + 2\, tinfty_1\, tinfty_2 - tinfty_3\right) Sinfty_0}{tinfty_3}$$

$$0$$

```
> HamRStinfty2bis:=-(t00)^2*(Sinfty[0]*tinfty[3]+tinfty[2])/((2*
  tinfty[3]^2)*S01)+(1/2)*(tinfty[3]*Sinfty[0]+tinfty[2])*S01*
  R01^2/tinfty[3]^2+S01*(tinfty[3]*S01+Sinfty[0]*tinfty[2]+Rinfty
  [0])*R01/tinfty[3] -(1/2)*Rinfty[0]^2*tinfty[2]/tinfty[3]^2+
  (1/2)*((2*Sinfty[0]^2-2*S01)*tinfty[2]+2*S01*Sinfty[0]*tinfty
  [3]-1)*Rinfty[0]/tinfty[3] -S01^2*tinfty[2]+(tinfty[2]^2)/
  (tinfty[3])*(Sinfty[0])^3+(1/2)*((2*Sinfty[0]^2*tinfty[2]+2*
  Sinfty[0]*tinfty[1]+2*tinfty[0]-2)*tinfty[3]^2-3*Sinfty[0]*
  tinfty[2]^2*tinfty[3]-tinfty[2]^3)*S01/tinfty[3]^2+(tinfty[2]*
  (2*tinfty[0]-1)*tinfty[3]+2*tinfty[1]*tinfty[2]^2)*Sinfty[0]/
  (2*tinfty[3]^2)+(1/2)*(2*tinfty[1]*tinfty[2]*tinfty[3]+tinfty
  [2]^3+tinfty[3]^2)*Sinfty[0]^2/tinfty[3]^2;
  simplify(series(HamRStinfty2-HamRStinfty2bis, Sinfty[0]));
```

$$HamRStinfty2bis := -\frac{1}{2}\, \frac{t00^2\left(Sinfty_0\, tinfty_3 + tinfty_2\right)}{tinfty_3^2\, S01} \tag{4.23}$$

$$+ \frac{1}{2}\, \frac{\left(Sinfty_0\, tinfty_3 + tinfty_2\right) S01\, R01^2}{tinfty_3^2}$$

$$+ \frac{S01\left(S01\, tinfty_3 + Sinfty_0\, tinfty_2 + Rinfty_0\right) R01}{tinfty_3} - \frac{1}{2}\, \frac{Rinfty_0^2\, tinfty_2}{tinfty_3^2}$$

$$+ \frac{1}{2}\, \frac{\left(\left(2\, Sinfty_0^2 - 2\, S01\right) tinfty_2 + 2\, S01\, Sinfty_0\, tinfty_3 - 1\right) Rinfty_0}{tinfty_3} - S01^2\, tinfty_2$$

$$+ \frac{tinfty_2^2\, Sinfty_0^3}{tinfty_3} + \frac{1}{2}\, \frac{1}{tinfty_3^2}\Big(\Big(\left(2\, Sinfty_0^2\, tinfty_2 + 2\, Sinfty_0\, tinfty_1 + 2\, tinfty_0 - 2\right) tinfty_3^2 - 3\, Sinfty_0\, tinfty_2^2\, tinfty_3 - tinfty_2^3\Big)\, S01\Big)$$

$$+ \frac{1}{2}\, \frac{\left(tinfty_2\left(2\, tinfty_0 - 1\right) tinfty_3 + 2\, tinfty_1\, tinfty_2^2\right) Sinfty_0}{tinfty_3^2}$$

$$+ \frac{1}{2}\, \frac{\left(2\, tinfty_1\, tinfty_2\, tinfty_3 + tinfty_2^3 + tinfty_3^2\right) Sinfty_0^2}{tinfty_3^2}$$

$$0$$