In this Maple file, we compute the Lax matrices and the Hamiltonian system for d=2.

```
> restart:
  with(LinearAlgebra):
  d:=2:
```

Formulas for the member of the Painlevé IV hierarchy and symmetry reduction in the (Q_\infty, Q_{0,1},P_\infty, P_{0,1}) variables

Definition of \check{L} in the (Q_\infty, Q_{0,1},P_\infty, P_{0,1}) variables. We omit the check for compactness.

```
> L12:=Q01/lambda+lambda^(2*d-1):
  for k from 0 to 2*d-2 do L12:=L12+ Qinfty[k]*lambda^k: od:
  L12:=L12*omega;
  L11:=P01*Q01/lambda:
  for k from 0 to 2*d-2 do L11:=L11-Pinfty[2*d-2-k]*lambda^k: od:
  for k from 0 to 2*d-3 do for m from 0 to 2*d-3-k do L11:=L11-
  Pinfty[m]*Qinfty[k+1+m]*lambda^k: od: od:
  L11:=L11- (tinfty[2*d+1]*lambda+tinfty[2*d]-tinfty[2*d+1]*
  Qinfty[2*d-2])*(L12/omega):
  L11:=simplify(L11):
  series(L11,lambda=infinity,20);
```

$$L12 := \left(\frac{Q01}{\lambda} + \lambda^3 + Qinfty_0 + Qinfty_1\,\lambda + Qinfty_2\,\lambda^2 \right) \omega \tag{1.1}$$

$$\begin{aligned} &-tinfty_5\,\lambda^4 - tinfty_4\,\lambda^3 + \left(\left(Qinfty_2^2 - Qinfty_1 \right) tinfty_5 - tinfty_4\,Qinfty_2 - Pinfty_0 \right) \lambda^2 \\ &\quad + \left(\left(Qinfty_1\,Qinfty_2 - Qinfty_0 \right) tinfty_5 - tinfty_4\,Qinfty_1 - Pinfty_0\,Qinfty_2 - Pinfty_1 \right) \lambda \\ &\quad + \left(Qinfty_0\,Qinfty_2 - Q01 \right) tinfty_5 - tinfty_4\,Qinfty_0 - Pinfty_0\,Qinfty_1 - Pinfty_1\,Qinfty_2 \\ &\quad - Pinfty_2 + \frac{Q01\left(Qinfty_2\,tinfty_5 + P01 - tinfty_4 \right)}{\lambda} \end{aligned}$$

```
> series(simplify(series((t00^2/lambda^2- L11^2)/(L12/omega),
  lambda=0,3)),lambda=0,1);
  factor(residue((t00^2/lambda^2- L11^2)/(L12/omega),lambda=0));
  ResidueBis:=1/omega*(t00^2/Q01-Q01*(P01-tinfty[2*d]+tinfty[2*
  d+1]*Qinfty[2*d-2])^2);
  simplify( factor(residue((t00^2/lambda^2- L11^2)/L12,lambda=0))
  - ResidueBis);
```

$$\begin{aligned} &-\frac{1}{\lambda}\,\frac{1}{Q01}\left(\left(Q01\left(Qinfty_2\,tinfty_5 + P01 - tinfty_4 \right) + t00 \right) \left(Q01\left(Qinfty_2\,tinfty_5 + P01 \right. \right. \right. \\ &\quad \left. \left. \left. - tinfty_4 \right) - t00 \right) \right) + \frac{1}{Q01^2}\left(2\,tinfty_5\left(Qinfty_2\,tinfty_5 + P01 - tinfty_4 \right) Q01^3 \right. \\ &\quad \left. + \left(Qinfty_2\,tinfty_5 + P01 - tinfty_4 \right) \left(\left(-Qinfty_0\,tinfty_5 + 2\,Pinfty_1 \right) Qinfty_2 + P01\,Qinfty_0 \right. \right. \end{aligned} \tag{1.2}$$

$$+ tinfty_4\, Qinfty_0 + 2\, Pinfty_0\, Qinfty_1 + 2\, Pinfty_2)\, Q01^2 - t00^2\, Qinfty_0) + O(\lambda)$$

$$-\frac{1}{Q01}\left((Q01\, Qinfty_2\, tinfty_5 + P01\, Q01 - Q01\, tinfty_4 + t00)\,(Q01\, Qinfty_2\, tinfty_5 + P01\, Q01 - Q01\, tinfty_4 - t00)\right)$$

$$ResidueBis := \frac{\frac{t00^2}{Q01} - Q01\,(Qinfty_2\, tinfty_5 + P01 - tinfty_4)^2}{\omega}$$

$$0$$

```
> L21infty:=0:
  for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do L21infty:=
  L21infty+ tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j: od: od:
  Numm:=simplify((L21infty-L11^2)/(L12/omega)):
  L21:=0:
  for j from 1 to 60 do L21:=L21- 1/omega*simplify(residue(Numm*
  lambda^(-j),lambda=infinity))*lambda^(j-1): od:
  L21:=simplify(L21+ (t00^2/Q01-Q01*(Qinfty[2*d-2]*tinfty[2*d+1]+
  P01-tinfty[2*d])^2)/omega/lambda):
```

Definition of the auxiliary matrix in the (Q,P) variables.

```
> A12:=0:
  for i from 0 to 2*d-1 do A12:=A12+nu[i]*lambda^(2*d-1-i): od:
  for k from 0 to 2*d-2 do for i from 0 to k do A12:=A12+Qinfty
  [k]*nu[i]*lambda^(k-i) od: od:
  A12:=omega*A12;
  cinfty0:= 1/2/omega*Lomega+ tinfty[2*d+1]*Q01*nu[0]:
  for j from 1 to 2*d-1 do cinfty0:=cinfty0 +tinfty[2*d+1]*nu[j]*
  Qinfty[j-1]: od:
  A11:=cinfty0:
  PolA12:=0:
  for i from 0 to 2*d-1 do PolA12:=PolA12+nu[i]*lambda^(-i): od:
  PolA12:=PolA12*L11:
  for k from 0 to 60 do A11:=A11-residue(PolA12/lambda^(k+1),
  lambda=infinity)*lambda^k od:
  A11:=A11;
  A22:=-A11:

  A21:=(-tinfty[2*d+1]*Lomega/omega^2) *lambda- (tinfty[2*d]-
  tinfty[2*d+1]*Qinfty[2*d-2])*Lomega/omega^2+ (alpha[2*d]-tinfty
  [2*d+1]*LQinfty2)/omega -tinfty[2*d+1]*nu[0]/omega:
  L21oper:=0:
  for j from 2*d-1 to 4*d do for m from 0 to 4*d-j do
  L21oper:=L21oper+tinfty[2*d+1-m]*tinfty[j+m-2*d+1]*lambda^j od:
```

```
od:
L21oper:=L21oper;
A12oper:=0:
for i from 0 to 2*d+1 do A12oper:=A12oper+nu[i]* lambda^(-i):
od:
A12oper:=A12oper;
for k from 0 to 60 do A21:=A21-residue( (L21oper*A12oper/(L12))
/lambda^(k+1),lambda=infinity)*lambda^k od:
for k from 0 to 60 do A21:=A21-residue( ((L11/(L12))*(L11*
A12/L12 -2*A11))/lambda^(k+1),lambda=infinity)*lambda^k od:
nu[2*d]:=0:
for j from 1 to 2*d-1 do nu[2*d]:=nu[2*d]-nu[j]*Qinfty[j-1]:
od:
nu[2*d]:=nu[2*d]-nu[0]*Q01;
nu[2*d+1]:=0:
for j from 2 to 2*d do nu[2*d+1]:=nu[2*d+1] - nu[j]*Qinfty[j-2]
od:
nu[2*d+1]:=nu[2*d+1]-nu[1]*Q01;
A21:=series(simplify(series(simplify(A21),lambda=infinity)),
lambda=infinity);
```

$$A12 := \omega \left(\lambda^3 \nu_0 + \lambda^2 Qinfty_2 \nu_0 + \lambda^2 \nu_1 + \lambda Qinfty_1 \nu_0 + \lambda Qinfty_2 \nu_1 + \lambda \nu_2 + Qinfty_0 \nu_0 + Qinfty_1 \nu_1 + Qinfty_2 \nu_2 + \nu_3\right) \tag{1.3}$$

$$A11 := Qinfty_2 \nu_3 tinfty_5 + Qinfty_1 \nu_2 tinfty_5 + Qinfty_0 \nu_1 tinfty_5 + \frac{1}{2} \frac{Lomega}{\omega} + tinfty_5 Q01 \nu_0 + \nu_0 \left(\left(Qinfty_0 Qinfty_2 - Q01\right) tinfty_5 - tinfty_4 Qinfty_0 - Pinfty_0 Qinfty_1 - Pinfty_1 Qinfty_2 - Pinfty_2\right) + \nu_1 \left(\left(Qinfty_1 Qinfty_2 - Qinfty_0\right) tinfty_5 - tinfty_4 Qinfty_1 - Pinfty_0 Qinfty_2 - Pinfty_1\right) + \nu_2 \left(\left(Qinfty_2^2 - Qinfty_1\right) tinfty_5 - tinfty_4 Qinfty_2 - Pinfty_0\right) - \nu_3 tinfty_4 - \left(-\nu_0 \left(\left(Qinfty_1 Qinfty_2 - Qinfty_0\right) tinfty_5 - tinfty_4 Qinfty_1 - Pinfty_0 Qinfty_2 - Pinfty_1\right) - \nu_1 \left(\left(Qinfty_2^2 - Qinfty_1\right) tinfty_5 - tinfty_4 Qinfty_2 - Pinfty_0\right) + \nu_2 tinfty_4 + \nu_3 tinfty_5\right) \lambda - \left(-\nu_0 \left(\left(Qinfty_2^2 - Qinfty_1\right) tinfty_5 - tinfty_4 Qinfty_2 - Pinfty_0\right) + \nu_1 tinfty_4 + \nu_2 tinfty_5\right) \lambda^2 - \left(\nu_0 tinfty_4 + \nu_1 tinfty_5\right) \lambda^3 - \nu_0 tinfty_5 \lambda^4$$

$$L21oper := \lambda^8 tinfty_5^2 + 2\lambda^7 tinfty_4 tinfty_5 + 2\lambda^6 tinfty_3 tinfty_5 + \lambda^6 tinfty_4^2 + 2\lambda^5 tinfty_2 tinfty_5 + 2\lambda^5 tinfty_3 tinfty_4 + 2\lambda^4 tinfty_1 tinfty_5 + 2\lambda^4 tinfty_2 tinfty_4 + \lambda^4 tinfty_3^2 + 2\lambda^3 tinfty_0 tinfty_5 + 2\lambda^3 tinfty_1 tinfty_4 + 2\lambda^3 tinfty_2 tinfty_3$$

$$A12oper := \nu_0 + \frac{\nu_1}{\lambda} + \frac{\nu_2}{\lambda^2} + \frac{\nu_3}{\lambda^3} + \frac{\nu_4}{\lambda^4} + \frac{\nu_5}{\lambda^5}$$

$$\nu_4 := -Q01\,\nu_0 - Qinfty_0\,\nu_1 - Qinfty_1\,\nu_2 - Qinfty_2\,\nu_3$$

$$\nu_5 := -\nu_1\,Q01 - Qinfty_2\left(-Q01\,\nu_0 - Qinfty_0\,\nu_1 - Qinfty_1\,\nu_2 - Qinfty_2\,\nu_3\right) - Qinfty_0\,\nu_2 - Qinfty_1\,\nu_3$$

$$A21 := \frac{1}{\omega}\left(\left(\left(2\,Qinfty_2^2 - 2\,Qinfty_1\right)\nu_0\,tinfty_5^2 + \left(-2\,Qinfty_2\,tinfty_4 - 2\,Pinfty_0 + 2\,tinfty_3\right)\nu_0\,tinfty_5\right)\lambda^3\right) + \frac{1}{\omega}\left(\left(\left(\left(-2\,Qinfty_2^3 + 4\,Qinfty_1\,Qinfty_2 - 2\,Qinfty_0\right)\nu_0 + 2\,\nu_1\,Qinfty_2^2 - 2\,Qinfty_1\,\nu_1\right)tinfty_5^2 + \left(\left(4\,Qinfty_2^2\,tinfty_4 - 4\,Qinfty_1\,tinfty_4 - 2\,Qinfty_2\,tinfty_3 - 2\,Pinfty_1 + 2\,tinfty_2\right)\nu_0 - 2\,\nu_1\,tinfty_4\,Qinfty_2 + 2\,\nu_1\left(tinfty_3 - Pinfty_0\right)\right)tinfty_5 + \left(-2\,tinfty_4^2\,Qinfty_2 + 2\,tinfty_4\left(tinfty_3 - Pinfty_0\right)\right)\nu_0\right)\lambda^2\right) + \frac{1}{\omega}\left(\left(\left(\left(Qinfty_2^4 - 4\,Qinfty_1\,Qinfty_2^2 + 4\,Qinfty_0\,Qinfty_2 + Qinfty_1^2 - 2\,Q01\right)\nu_0 - 2\,\nu_1\,Qinfty_2^3 + 2\,\nu_2\,Qinfty_2^2 + 4\,Qinfty_1\,\nu_1\,Qinfty_2 - 2\,Qinfty_0\,\nu_1 - 2\,Qinfty_1\,\nu_2\right)tinfty_5^2 + \left(\left(-2\,Qinfty_2^3\,tinfty_4 + \left(2\,tinfty_3 + 2\,Pinfty_0\right)Qinfty_2^2 + \left(6\,Qinfty_1\,tinfty_4 - 2\,tinfty_2\right)Qinfty_2 - 4\,tinfty_4\,Qinfty_0 + \left(-2\,tinfty_3 - 2\,Pinfty_0\right)Qinfty_1 - 2\,Pinfty_2 + 2\,tinfty_1\right)\nu_0 + 4\,\nu_1\,tinfty_4\,Qinfty_2^2 + \left(-2\,\nu_1\,tinfty_3 - 2\,\nu_2\,tinfty_4\right)Qinfty_2 - 4\,Qinfty_1\,tinfty_4\,\nu_1 + \left(2\,tinfty_2 - 2\,Pinfty_1\right)\nu_1 + 2\,\nu_2\left(tinfty_3 - Pinfty_0\right)\right)tinfty_5 + \left(Qinfty_2^2\,tinfty_4^2 - 2\,tinfty_4\left(tinfty_3 + Pinfty_0\right)Qinfty_2 - 2\,tinfty_4^2\,Qinfty_1 + \left(2\,tinfty_2 - 2\,Pinfty_1\right)tinfty_4 + tinfty_3^2 - Pinfty_0^2\right)\nu_0 - 2\,Qinfty_2\,\nu_1\,tinfty_4^2 + 2\,tinfty_4\,\nu_1\left(tinfty_3 - Pinfty_0\right)\right)\lambda\right) + \frac{1}{\omega}\left(\left(\left(-Qinfty_2^5 + 4\,Qinfty_2^3\,Qinfty_1 - 4\,Qinfty_0\,Qinfty_2^2 + \left(-3\,Qinfty_1^2 + 4\,Q01\right)Qinfty_2 + 2\,Qinfty_0\,Qinfty_1\right)\nu_0 + Qinfty_2^4\,\nu_1 - 2\,\nu_2\,Qinfty_2^3 + \left(-4\,Qinfty_1\,\nu_1 + 2\,\nu_3\right)Qinfty_2^2 + \left(4\,Qinfty_0\,\nu_1 + 4\,Qinfty_1\,\nu_2\right)Qinfty_2 + \left(Qinfty_1^2 - 2\,Q01\right)\nu_1 - 2\,Qinfty_0\,\nu_2 - 2\,Qinfty_1\,\nu_3\right)tinfty_5^2 + \left(\left(2\,Qinfty_2^4\,tinfty_4 - 2\,Qinfty_2^3\,tinfty_3 + \left(-6\,Qinfty_1\,tinfty_4 + 2\,Pinfty_1 + 2\,tinfty_2\right)Qinfty_2^2 + \left(6\,tinfty_4\,Qinfty_0 + \left(4\,tinfty_3 + 2\,Pinfty_0\right)Qinfty_1 + 2\,Pinfty_2 - 2\,tinfty_1\right)Qinfty_2 + \left(2\,Qinfty_1^2 - 4\,Q01\right)tinfty_4 - 2\,Qinfty_0\,tinfty_3 - 2\,tinfty_2\,Qinfty_1 + 2\,tinfty_0 - 1\right)\nu_0 - 2\,Qinfty_2^3\,tinfty_4\,\nu_1 + \left(4\,\nu_2\,tinfty_4 + 2\,\nu_1\left(tinfty_3 + Pinfty_0\right)\right)Qinfty_2^2 + \left(\left(6\,Qinfty_1\,\nu_1 - 2\,\nu_3\right)tinfty_4 + \left(2\,Pinfty_1 - 2\,tinfty_2\right)\nu_1 - 2\,\nu_2\left(tinfty_3 - Pinfty_0\right)\right)Qinfty_2 + \left(-4\,Qinfty_0\,\nu_1 - 4\,Qinfty_1\,\nu_2\right)tinfty_4 + \left(-2\,Qinfty_1\,tinfty_3 + 2\,tinfty_1\right)\nu_1 + 2\,tinfty_2\,\nu_2 + 2\,tinfty_3\,\nu_3 - LQinfty2\right)tinfty_5 + \left(-Qinfty_2^3\,tinfty_4^2 + 2\,Qinfty_2^2\,tinfty_3\,tinfty_4 + \left(2\,tinfty_4^2\,Qinfty_1 + \left(-2\,Pinfty_1 - 2\,tinfty_2\right)tinfty_4 - tinfty_3^2 - Pinfty_0^2\right)Qinfty_2 - 2\,tinfty_4^2\,Qinfty_0 + \left(\left(-2\,tinfty_3 - 2\,Pinfty_0\right)Qinfty_1 - 2\,Pinfty_2 + 2\,tinfty_1\right)tinfty_4 - 2\,Pinfty_1\,Pinfty_0 + 2\,tinfty_2\,tinfty_3\right)\nu_0 + Qinfty_2^2\,\nu_1\,tinfty_4^2 - 2\,tinfty_4\left(\nu_2\,tinfty_4\right.$$

$$+\nu_1\left(tinfty_3+Pinfty_0\right)\Big)\,Qinfty_2-2\,Qinfty_1\,\nu_1\,tinfty_4^2+\Big(\left(2\,tinfty_2-2\,Pinfty_1\right)\nu_1$$
$$+2\,\nu_2\left(tinfty_3-Pinfty_0\right)\Big)\,tinfty_4+\left(-Pinfty_0^2+tinfty_3^2\right)\nu_1+\alpha_4\Big)$$

Compatibility equation and evolutions

```
> L:=Matrix(2,2,0):
  L[1,1]:=L11:
  L[1,2]:=L12:
  L[2,1]:=L21:
  L[2,2]:=-L[1,1]:
  A:=Matrix(2,2,0):
  A[1,1]:=A11:
  A[1,2]:=A12:
  A[2,1]:=A21:
  A[2,2]:=-A[1,1]:
> dAdlambda:=Matrix(2,2,0):
  for i from 1 to 2 do for j from 1 to 2 do dAdlambda[i,j]:=diff
  (A[i,j],lambda): od: od:
  dAdlambda:
> BigLL:=Matrix(2,2,0):
  for i from 1 to 2 do for j from 1 to 2 do
  BigLL[i,j]:= diff(L[i,j],Qinfty[2])*LQinfty2+diff(L[i,j],Qinfty
  [1])*LQinfty1+diff(L[i,j],Qinfty[0])*LQinfty0+ diff(L[i,j],Q01)
  *LQ01
  +diff(L[i,j],Pinfty[2])*LPinfty2+diff(L[i,j],Pinfty[1])*
  LPinfty1+diff(L[i,j],Pinfty[0])*LPinfty0+ diff(L[i,j],P01)*LP01
  +diff(L[i,j],tinfty[4])*alpha[4]+diff(L[i,j],tinfty[3])*alpha
  [3]+diff(L[i,j],tinfty[2])*alpha[2] +diff(L[i,j],tinfty[1])*
  alpha[1]+ diff(L[i,j],omega)*Lomega:
  od: od:
> Commutator:=Multiply(A,L)-Multiply(L,A):
  CompatibilityEquation:= simplify(dAdlambda-BigLL+Commutator):
> series(CompatibilityEquation[1,2], lambda=infinity):
  EQ120:=simplify(residue(CompatibilityEquation[1,2]/lambda^4,
  lambda=infinity));
  EQ121:=simplify(residue(CompatibilityEquation[1,2]/lambda^3,
  lambda=infinity));
  EQ122:=simplify(residue(CompatibilityEquation[1,2]/lambda^2,
  lambda=infinity));
  EQ123:=simplify(residue(CompatibilityEquation[1,2]/lambda^1,
  lambda=infinity));
  EQ124:=simplify(residue(CompatibilityEquation[1,2]/lambda^0,
  lambda=infinity));
```

```
series(simplify(series(CompatibilityEquation[1,1], lambda=
infinity)),lambda=infinity):

EQ110:=simplify(residue(CompatibilityEquation[1,1]/lambda^5,
lambda=infinity));
EQ111:=simplify(residue(CompatibilityEquation[1,1]/lambda^4,
lambda=infinity));
EQ112:=simplify(residue(CompatibilityEquation[1,1]/lambda^3,
lambda=infinity));
EQ113:=simplify(residue(CompatibilityEquation[1,1]/lambda^2,
lambda=infinity)):
EQ114:=simplify(residue(CompatibilityEquation[1,1]/lambda^1,
lambda=infinity)):
EQ115:=simplify(residue(CompatibilityEquation[1,1]/lambda^0,
lambda=infinity)):
```

$$EQ120 := 0 \tag{1.4}$$

$$EQ121 := 2\,\omega \left(\left(-Pinfty_0\,Qinfty_1 - Pinfty_1\,Qinfty_2 - Pinfty_2 \right) \nu_1 + \left(-Qinfty_2\,\nu_2 - \nu_3 \right) Pinfty_0 + P01\,Q01\,\nu_0 - \nu_2\,Pinfty_1 - \frac{3}{2}\,\nu_0 + \frac{1}{2}\,LQinfty2 \right)$$

$$EQ122 := 2\,\omega \left(\left(-Pinfty_0\,\nu_2 - Pinfty_1\,\nu_1 \right) Qinfty_2^2 + \left(\left(-Pinfty_0\,Qinfty_1 - Pinfty_2 \right) \nu_1 + P01\,Q01\,\nu_0 - 2\,\nu_2\,Pinfty_1 - \nu_3\,Pinfty_0 - \nu_0 \right) Qinfty_2 + \left(P01\,Q01 + Pinfty_0\,Qinfty_0 - 1 \right) \nu_1 + Pinfty_0\,Q01\,\nu_0 - \nu_2\,Pinfty_2 - \nu_3\,Pinfty_1 + \frac{1}{2}\,LQinfty1 \right)$$

$$EQ123 := 2\,\omega \left(-\nu_2\,Pinfty_1\,Qinfty_2^2 + \left(\left(-\frac{1}{2} + P01\,Q01 + Pinfty_0\,Qinfty_0 - Pinfty_1\,Qinfty_1 \right) \nu_1 - Qinfty_1\,\nu_2\,Pinfty_0 + Pinfty_0\,Q01\,\nu_0 - \nu_2\,Pinfty_2 - \nu_3\,Pinfty_1 \right) Qinfty_2 + \left(-Pinfty_0\,Qinfty_1^2 + Q01\,Pinfty_0 + Pinfty_1\,Qinfty_0 - Pinfty_2\,Qinfty_1 \right) \nu_1 + \left(P01\,Q01\,\nu_0 - \nu_3\,Pinfty_0 - \frac{1}{2}\,\nu_0 \right) Qinfty_1 + Pinfty_0\,Qinfty_0\,\nu_2 + \left(P01\,Q01 - \frac{1}{2} \right) \nu_2 + Q01\,Pinfty_1\,\nu_0 - \nu_3\,Pinfty_2 + \frac{1}{2}\,LQinfty0 \right)$$

$$EQ124 := 2 \left(\left(\left(P01\,Qinfty_0 + Pinfty_0\,Qinfty_1 + Pinfty_1\,Qinfty_2 + Pinfty_2 \right) \nu_0 + \left(Qinfty_1\,\nu_1 + Qinfty_2\,\nu_2 + \nu_3 \right) P01 + Qinfty_2\,\nu_1\,Pinfty_0 + \nu_1\,Pinfty_1 + \nu_2\,Pinfty_0 \right) Q01 + \frac{1}{2}\,LQ01 \right) \omega$$

$$EQ110 := 0$$

$$EQ111 := -2 \left(\left(-Qinfty_1\,\nu_1 - Qinfty_2\,\nu_2 - \nu_3 \right) Pinfty_0 + \left(-Pinfty_1\,Qinfty_2 - Pinfty_2 \right) \nu_1 + P01\,Q01\,\nu_0 - \nu_2\,Pinfty_1 - \frac{3}{2}\,\nu_0 + \frac{1}{2}\,LQinfty2 \right) tinfty_5$$

$$EQ112 := \frac{1}{Q01}\left(\left(\left(3\,Qinfty_2^2\,\nu_0 - 2\,Qinfty_1\,\nu_0 - 4\,Qinfty_2\,\nu_1 + 2\,\nu_2\right)tinfty_5^2 + \left(-4\,tinfty_4\,\nu_0\,Qinfty_2 + \left(-2\,P01 + 4\,tinfty_4\right)\nu_1 - 2\,\nu_0\left(Pinfty_0 - tinfty_3\right)\right)tinfty_5 + \nu_0\left(P01 - tinfty_4\right)^2\right)Q01^2 + \left(\left(\nu_1\,Qinfty_2^5 - \nu_2\,Qinfty_2^4 + \left(-4\,Qinfty_1\,\nu_1 + 4\,\nu_3\right)Qinfty_2^3 + \left(6\,Qinfty_0\,\nu_1 + 6\,Qinfty_1\,\nu_2\right)Qinfty_2^2 + \left(3\,Qinfty_1^2\,\nu_1 - 4\,Qinfty_0\,\nu_2 - 6\,Qinfty_1\,\nu_3\right)Qinfty_2 - 4\,\nu_1\,Qinfty_0\,Qinfty_1 - 3\,\nu_2\,Qinfty_1^2 + 2\,\nu_3\,Qinfty_0\right)tinfty_5^2 + \left(-2\,\nu_1\,Qinfty_2^4\,tinfty_4 + \left(2\,\nu_1\,tinfty_3 + 2\,\nu_2\,tinfty_4\right)Qinfty_2^3 + \left(\left(6\,Qinfty_1\,tinfty_4 - 2\,tinfty_2\right)\nu_1 - 6\,\nu_3\,tinfty_4 - 2\,\nu_2\,tinfty_3\right)Qinfty_2^2 + \left(\left(-8\,Qinfty_0\,tinfty_4 - 4\,Qinfty_1\,tinfty_3 + 2\,tinfty_1\right)\nu_1 - 8\,Qinfty_1\,\nu_2\,tinfty_4 + \left(2\,Pinfty_1 + 2\,tinfty_2\right)\nu_2 + 4\,tinfty_3\,\nu_3 - \nu_0 + LQinfty2\right)Qinfty_2 + \left(-2\,Qinfty_1^2\,tinfty_4 - 2\,Pinfty_0\,Qinfty_0 + 4\,Qinfty_0\,tinfty_3 + 2\,Qinfty_1\,tinfty_2 - 2\,tinfty_0 + 3\right)\nu_1 + \left(4\,Qinfty_0\,\nu_2 + 4\,Qinfty_1\,\nu_3\right)tinfty_4 + \left(4\,Qinfty_1\,tinfty_3 + 2\,Pinfty_2 - 2\,tinfty_1\right)\nu_2 + \left(2\,Pinfty_1 - 2\,tinfty_2\right)\nu_3 - LQinfty1\right)tinfty_5 + tinfty_4^2\,\nu_1\,Qinfty_2^3 + \left(-2\,\nu_1\,tinfty_3\,tinfty_4 - \nu_2\,tinfty_4^2\right)Qinfty_2^2 + \left(\left(-2\,tinfty_4^2\,Qinfty_1 + \left(2\,Pinfty_1 + 2\,tinfty_2\right)tinfty_4 + Pinfty_0^2 + tinfty_3^2\right)\nu_1 + 2\,tinfty_4\left(\nu_3\,tinfty_4 + \nu_2\left(tinfty_3 + Pinfty_0\right)\right)\right)Qinfty_2 + \left(2\,tinfty_4^2\,Qinfty_0 + \left(\left(2\,tinfty_3 + 2\,Pinfty_0\right)Qinfty_1 + 2\,Pinfty_2 - 2\,tinfty_1\right)tinfty_4 + 2\,Pinfty_1\,Pinfty_0 - 2\,tinfty_2\,tinfty_3\right)\nu_1 + 2\,tinfty_4^2\,\nu_2\,Qinfty_1 + \left(\left(2\,Pinfty_1 - 2\,tinfty_2\right)\nu_2 + 2\,\nu_3\,Pinfty_0 - 2\,tinfty_3\,\nu_3 + 3\,\nu_0 - LQinfty2\right)tinfty_4 + \left(Pinfty_0^2 - tinfty_3^2\right)\nu_2 - LPinfty0\right)Q01 - \nu_0\,t00^2\right)$$

```
> LQinfty2:=simplify(solve(EQ121,LQinfty2));
  LQinfty1:=simplify(solve(EQ122,LQinfty1));
  LQinfty0:=simplify(solve(EQ123,LQinfty0));
  LQ01:=simplify(solve(EQ124,LQ01));
```

$$LQinfty2 := \left(2\,Qinfty_1\,\nu_1 + 2\,Qinfty_2\,\nu_2 + 2\,\nu_3\right)Pinfty_0 + \left(2\,Pinfty_1\,Qinfty_2 + 2\,Pinfty_2\right)\nu_1 - 2\,P01\,Q01\,\nu_0 + 2\,\nu_2\,Pinfty_1 + 3\,\nu_0 \tag{1.5}$$

$$LQinfty1 := \left(2\,Pinfty_0\,\nu_2 + 2\,Pinfty_1\,\nu_1\right)Qinfty_2^2 + \left(\left(2\,Pinfty_0\,Qinfty_1 + 2\,Pinfty_2\right)\nu_1 - 2\,P01\,Q01\,\nu_0 + 4\,\nu_2\,Pinfty_1 + 2\,\nu_3\,Pinfty_0 + 2\,\nu_0\right)Qinfty_2 + \left(-2\,P01\,Q01 - 2\,Pinfty_0\,Qinfty_0 + 2\right)\nu_1 - 2\,Pinfty_0\,Q01\,\nu_0 + 2\,\nu_2\,Pinfty_2 + 2\,\nu_3\,Pinfty_1$$

$$LQinfty0 := 2\,\nu_2\,Pinfty_1\,Qinfty_2^2 + \left(\left(-2\,P01\,Q01 - 2\,Pinfty_0\,Qinfty_0 + 2\,Pinfty_1\,Qinfty_1 + 1\right)\nu_1 + 2\,Qinfty_1\,\nu_2\,Pinfty_0 - 2\,Pinfty_0\,Q01\,\nu_0 + 2\,\nu_2\,Pinfty_2 + 2\,\nu_3\,Pinfty_1\right)Qinfty_2 + \left(2\,Pinfty_0\,Qinfty_1^2 - 2\,Q01\,Pinfty_0 - 2\,Pinfty_1\,Qinfty_0 + 2\,Pinfty_2\,Qinfty_1\right)\nu_1 + \left(-2\,P01\,Q01\,\nu_0 + 2\,Pinfty_0\,\nu_3 + \nu_0\right)Qinfty_1 - 2\,Pinfty_0\,Qinfty_0\,\nu_2 + \left(-2\,P01\,Q01 + 1\right)\nu_2 - 2\,Q01\,Pinfty_1\,\nu_0 + 2\,\nu_3\,Pinfty_2$$

$$LQ01 := -2\left(\left(P01\,Qinfty_0 + Pinfty_0\,Qinfty_1 + Pinfty_1\,Qinfty_2 + Pinfty_2\right)\nu_0 + \left(Qinfty_1\,\nu_1\right.\right.$$

$$+ Qinfty_2\,\nu_2 + \nu_3\Big)\,P01 + Qinfty_2\,\nu_1\,Pinfty_0 + \nu_1\,Pinfty_1 + \nu_2\,Pinfty_0\Big)\,Q01$$

```
> EQ111:=simplify(EQ111);
  EQ112:=simplify(EQ112):
  EQ113:=simplify(EQ113):
  EQ114:=simplify(EQ114):
  EQ115:=simplify(EQ115):
```

$$EQ111 := 0 \tag{1.6}$$

```
> LPinfty0:=simplify(solve(EQ112,LPinfty0)):
  LPinfty1:=simplify(solve(EQ113,LPinfty1)):
  LPinfty2:=simplify(solve(EQ114,LPinfty2)):
  LP01:=simplify(solve(EQ115,LP01)):

  EQ121:=simplify(EQ121);
  EQ122:=simplify(EQ122);
  EQ123:=simplify(EQ123):
  EQ124:=simplify(EQ124):
  EQ125:=simplify(EQ125):
  EQ111:=simplify(EQ111);
  EQ112:=simplify(EQ112);
  EQ113:=simplify(EQ113);
  EQ114:=simplify(EQ114);
  EQ115:=simplify(EQ115);
```

$$EQ121 := 0 \tag{1.7}$$
$$EQ122 := 0$$
$$EQ111 := 0$$
$$EQ112 := 0$$
$$EQ113 := 0$$
$$EQ114 := 0$$
$$EQ115 := 0$$

```
> Ham:=2*nu[2]*Pinfty[1]*Qinfty[2]^2*Pinfty[0]-Qinfty[2]*(nu[1]*
  (2*P01*Q01*Pinfty[0]+Pinfty[0]^2*Qinfty[0]-2*Pinfty[0]*Pinfty
  [1]*Qinfty[1]-Pinfty[0])-Qinfty[1]*nu[2]*Pinfty[0]^2+Pinfty[0]
  ^2*Q01*nu[0]-2*nu[2]*Pinfty[2]*Pinfty[0]-2*nu[3]*Pinfty[1]*
  Pinfty[0])-nu[1]*(-Pinfty[0]^2*Qinfty[1]^2+Q01*Pinfty[0]^2+2*
  Pinfty[0]*Pinfty[1]*Qinfty[0]-2*Pinfty[0]*Pinfty[2]*Qinfty[1])-
  Qinfty[1]*(2*P01*Q01*Pinfty[0]*nu[0]-Pinfty[0]^2*nu[3]-Pinfty
  [0]*nu[0])-Pinfty[0]^2*Qinfty[0]*nu[2]-(2*P01*Q01-1)*nu[2]*
  Pinfty[0]-2*Q01*Pinfty[1]*nu[0]*Pinfty[0]+2*Pinfty[0]*Pinfty[2]
  *nu[3]
  +Pinfty[1]^2*Qinfty[2]^2*nu[1]-Qinfty[2]*(2*P01*Q01*Pinfty[1]*
  nu[0]-2*Pinfty[1]^2*nu[2]-2*Pinfty[1]*Pinfty[2]*nu[1]-2*Pinfty
  [1]*nu[0])-(2*P01*Q01-2)*nu[1]*Pinfty[1]+2*nu[2]*Pinfty[2]*
  Pinfty[1]+nu[3]*Pinfty[1]^2
```

```
-2*P01*Q01*Pinfty[2]*nu[0]+Pinfty[2]^2*nu[1]+3*Pinfty[2]*nu[0]
-P01^2*Q01*(Qinfty[0]*nu[0]+Qinfty[1]*nu[1]+Qinfty[2]*nu[2]+nu
[3])

+((Qinfty[0]*nu[0]+Qinfty[1]*nu[1]+Qinfty[2]*nu[2]+nu[3])*
t00^2/Q01+(-(tinfty[4]^2*Qinfty[0]+(2*Qinfty[1]*tinfty[3]-2*
tinfty[1])*tinfty[4]-2*tinfty[2]*tinfty[3])*nu[0]-((-2*Qinfty
[1]*tinfty[4]^2+2*tinfty[2]*tinfty[4]+tinfty[3]^2)*nu[0]+2*nu
[1]*tinfty[3]*tinfty[4]+tinfty[4]^2*nu[2])*Qinfty[2]-(Qinfty[2]
^5*nu[0]-Qinfty[2]^4*nu[1]+(-4*Qinfty[1]*nu[0]+nu[2])*Qinfty[2]
^3+(3*Qinfty[0]*nu[0]+3*Qinfty[1]*nu[1]-3*nu[3])*Qinfty[2]^2+
(3*Qinfty[1]^2*nu[0]-4*Qinfty[0]*nu[1]-4*Qinfty[1]*nu[2])*
Qinfty[2]-nu[1]*Qinfty[1]^2-2*Qinfty[0]*Qinfty[1]*nu[0]+2*
Qinfty[1]*nu[3]+2*Qinfty[0]*nu[2])*tinfty[5]^2-(-2*Qinfty[2]^4*
tinfty[4]*nu[0]+(2*nu[0]*tinfty[3]+2*nu[1]*tinfty[4])*Qinfty[2]
^3+((6*Qinfty[1]*tinfty[4]-2*tinfty[2])*nu[0]-2*nu[1]*tinfty[3]
-2*nu[2]*tinfty[4])*Qinfty[2]^2+((-4*Qinfty[0]*tinfty[4]-4*
Qinfty[1]*tinfty[3]+2*tinfty[1])*nu[0]+(-4*Qinfty[1]*nu[1]+4*nu
[3])*tinfty[4]+2*nu[1]*tinfty[2]+2*nu[2]*tinfty[3])*Qinfty[2]+
(-2*Qinfty[1]^2*tinfty[4]+2*Qinfty[0]*tinfty[3]+2*Qinfty[1]*
tinfty[2]-2*tinfty[0]+1)*nu[0]+(4*Qinfty[0]*nu[1]+4*Qinfty[1]*
nu[2])*tinfty[4]+2*nu[1]*Qinfty[1]*tinfty[3]-2*nu[1]*tinfty[1]
-2*tinfty[2]*nu[2]-2*tinfty[3]*nu[3])*tinfty[5]-(Qinfty[1]*nu
[1]-nu[3])*tinfty[4]^2-(-2*nu[0]*tinfty[3]*tinfty[4]-nu[1]*
tinfty[4]^2)*Qinfty[2]^2-(-2*nu[1]*tinfty[2]-2*nu[2]*tinfty[3])
*tinfty[4]-Qinfty[2]^3*nu[0]*tinfty[4]^2+tinfty[3]^2*nu[1])*
Q01+(-(1/2)*(-4*Qinfty[2]*nu[0]+2*nu[1])*tinfty[5]^2-2*nu[0]*
tinfty[4]*tinfty[5])*Q01^2)
+(-tinfty[5]^2*(Qinfty[2]^5*nu[2]*Qinfty[1]-5*Qinfty[2]^4*nu[3]
*Qinfty[1]+Qinfty[2]^3*(-4*Qinfty[0]*Qinfty[1]*nu[1]-4*Qinfty
[1]^2*nu[2])+Qinfty[2]^2*(6*Qinfty[0]*Qinfty[1]*nu[2]+6*Qinfty
[1]^2*nu[3])+Qinfty[2]*(3*Qinfty[1]^3*nu[2]+6*Qinfty[0]*((1/2)*
nu[1]*Qinfty[1]^2-Qinfty[1]*nu[3]))-nu[3]*Qinfty[1]^3-3*nu[2]*
Qinfty[1]^2*Qinfty[0]-2*nu[1]*Qinfty[0]^2*Qinfty[1])-tinfty[5]*
(-2*tinfty[4]*nu[2]*Qinfty[2]^4*Qinfty[1]+(2*nu[2]*tinfty[3]+8*
nu[3]*tinfty[4])*Qinfty[2]^3*Qinfty[1]+Qinfty[2]^2*(nu[2]*(6*
Qinfty[1]^2*tinfty[4]-2*Qinfty[1]*tinfty[2])+6*nu[1]*Qinfty[0]*
tinfty[4]*Qinfty[1]-6*Qinfty[1]*tinfty[3]*nu[3])+Qinfty[2]*(nu
[2]*(-8*Qinfty[0]*Qinfty[1]*tinfty[4]-4*Qinfty[1]^2*tinfty[3]
+2*Qinfty[1]*tinfty[1])-4*nu[1]*Qinfty[0]*tinfty[3]*Qinfty[1]
-6*tinfty[4]*nu[3]*Qinfty[1]^2+4*tinfty[2]*nu[3]*Qinfty[1])+nu
[2]*(-2*Qinfty[1]^3*tinfty[4]+4*Qinfty[0]*Qinfty[1]*tinfty[3]
```

```
+2*Qinfty[1]^2*tinfty[2]-2*Qinfty[1]*tinfty[0]+Qinfty[1])-4*
Qinfty[0]*tinfty[4]*((1/2)*nu[1]*Qinfty[1]^2-Qinfty[1]*nu[3])
+2*Qinfty[0]*nu[1]*tinfty[2]*Qinfty[1]+2*Qinfty[1]^2*tinfty[3]*
nu[3]-2*nu[3]*tinfty[1]*Qinfty[1])-Qinfty[2]^3*tinfty[4]^2*nu
[2]*Qinfty[1]-(-2*nu[2]*tinfty[3]*tinfty[4]-3*nu[3]*tinfty[4]
^2)*Qinfty[2]^2*Qinfty[1]-Qinfty[2]*(nu[2]*(-2*Qinfty[1]^2*
tinfty[4]^2+2*Qinfty[1]*tinfty[2]*tinfty[4]+Qinfty[1]*tinfty[3]
^2)-2*Qinfty[0]*nu[1]*tinfty[4]^2*Qinfty[1]+4*tinfty[3]*tinfty
[4]*nu[3]*Qinfty[1])-nu[2]*(2*tinfty[4]^2*Qinfty[0]*Qinfty[1]+
tinfty[4]*(2*Qinfty[1]^2*tinfty[3]-2*Qinfty[1]*tinfty[1])-2*
tinfty[2]*tinfty[3]*Qinfty[1])-Qinfty[1]^2*nu[3]*tinfty[4]^2-
(2*Qinfty[0]*nu[1]*tinfty[3]-2*nu[3]*tinfty[2])*tinfty[4]*
Qinfty[1]+tinfty[3]^2*nu[3]*Qinfty[1])

+(-Qinfty[2]^6*nu[3]*tinfty[5]^2-(1/5)*(5*Qinfty[0]*nu[1]*
tinfty[5]^2-10*nu[3]*tinfty[4]*tinfty[5])*Qinfty[2]^5-(1/4)*
(-4*tinfty[5]^2*nu[2]*Qinfty[0]+(-8*Qinfty[0]*nu[1]*tinfty[4]
+8*nu[3]*tinfty[3])*tinfty[5]+4*nu[3]*tinfty[4]^2)*Qinfty[2]^4-
(1/3)*(12*Qinfty[0]*nu[3]*tinfty[5]^2+((6*nu[1]*tinfty[3]+6*nu
[2]*tinfty[4])*Qinfty[0]-6*nu[3]*tinfty[2])*tinfty[5]+3*Qinfty
[0]*nu[1]*tinfty[4]^2-6*tinfty[3]*tinfty[4]*nu[3])*Qinfty[2]^3-
(1/2)*(6*Qinfty[0]^2*nu[1]*tinfty[5]^2+((-4*nu[1]*tinfty[2]-4*
nu[2]*tinfty[3]-12*nu[3]*tinfty[4])*Qinfty[0]+4*nu[3]*tinfty[1]
)*tinfty[5]+(-4*nu[1]*tinfty[3]*tinfty[4]-2*nu[2]*tinfty[4]^2)*
Qinfty[0]+(4*tinfty[2]*tinfty[4]+2*tinfty[3]^2)*nu[3])*Qinfty
[2]^2+2*Qinfty[0]^2*nu[2]*tinfty[5]^2*Qinfty[2]-(-4*nu[1]*
tinfty[4]*Qinfty[0]^2+(2*nu[1]*tinfty[1]+2*nu[2]*tinfty[2]+4*nu
[3]*tinfty[3])*Qinfty[0]+(-2*tinfty[0]+1)*nu[3])*tinfty[5]*
Qinfty[2]-(2*nu[3]*tinfty[4]^2+(2*nu[1]*tinfty[2]+2*nu[2]*
tinfty[3])*tinfty[4]+tinfty[3]^2*nu[1])*Qinfty[0]*Qinfty[2]+2*
nu[3]*(tinfty[1]*tinfty[4]+tinfty[2]*tinfty[3])*Qinfty[2])

+(-Qinfty[0]^2*nu[3]*tinfty[5]^2-tinfty[5]*(nu[1]*(2*Qinfty[0]
^2*tinfty[3]-2*Qinfty[0]*tinfty[0]+Qinfty[0])+2*tinfty[4]*nu[2]
*Qinfty[0]^2-2*nu[2]*tinfty[1]*Qinfty[0]-2*nu[3]*tinfty[2]*
Qinfty[0])-nu[1]*(Qinfty[0]^2*tinfty[4]^2-2*Qinfty[0]*tinfty[1]
*tinfty[4]-2*Qinfty[0]*tinfty[2]*tinfty[3])-(-2*nu[2]*tinfty[2]
-2*nu[3]*tinfty[3])*tinfty[4]*Qinfty[0]+tinfty[3]^2*nu[2]*
Qinfty[0])
:
```

```
simplify(series(Ham,P01)):
simplify(diff(Ham,Pinfty[0])- LQinfty0);
simplify(diff(Ham,Pinfty[1])- LQinfty1);
simplify(diff(Ham,Pinfty[2])- LQinfty2);
simplify(diff(Ham,P01)- LQ01);

series(simplify(diff(Ham,Q01)+ LP01),Q01);
simplify(diff(Ham,Qinfty[1])+ LPinfty1);
simplify(diff(Ham,Qinfty[2])+ LPinfty2);
simplify(diff(Ham,Qinfty[0])+ LPinfty0);
```

$$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \qquad (1.8)$$

```
> Ham:=(Qinfty[0]*nu[0]+Qinfty[1]*nu[1]+Qinfty[2]*nu[2]+nu[3])*
  t00^2/Q01-Qinfty[2]^6*nu[3]*tinfty[5]^2+((-Qinfty[0]*nu[1]-
  Qinfty[1]*nu[2])*tinfty[5]^2+2*tinfty[5]*tinfty[4]*nu[3])*
  Qinfty[2]^5+((Qinfty[0]*nu[2]+5*Qinfty[1]*nu[3])*tinfty[5]^2+
  (2*Qinfty[0]*nu[1]*tinfty[4]+2*Qinfty[1]*nu[2]*tinfty[4]-2*nu
  [3]*tinfty[3])*tinfty[5]-nu[3]*tinfty[4]^2)*Qinfty[2]^4+((4*
  Qinfty[0]*Qinfty[1]*nu[1]+4*Qinfty[1]^2*nu[2]-4*Qinfty[0]*nu[3]
  )*tinfty[5]^2+((-2*nu[2]*tinfty[3]-8*nu[3]*tinfty[4])*Qinfty[1]
  -2*nu[1]*Qinfty[0]*tinfty[3]-2*tinfty[4]*nu[2]*Qinfty[0]+2*nu
  [3]*tinfty[2])*tinfty[5]-tinfty[4]^2*nu[2]*Qinfty[1]-Qinfty[0]*
  nu[1]*tinfty[4]^2+2*tinfty[3]*tinfty[4]*nu[3])*Qinfty[2]^3+(
  (-3*Qinfty[0]^2*nu[1]-6*Qinfty[0]*Qinfty[1]*nu[2]-6*Qinfty[1]
  ^2*nu[3])*tinfty[5]^2+(-6*Qinfty[1]^2*tinfty[4]*nu[2]+(-6*
  Qinfty[0]*nu[1]*tinfty[4]+2*nu[2]*tinfty[2]+6*nu[3]*tinfty[3])*
  Qinfty[1]+6*nu[3]*Qinfty[0]*tinfty[4]+(2*nu[1]*tinfty[2]+2*nu
  [2]*tinfty[3])*Qinfty[0]-2*nu[3]*tinfty[1])*tinfty[5]+(2*nu[2]*
  tinfty[3]*tinfty[4]+3*nu[3]*tinfty[4]^2)*Qinfty[1]+Qinfty[0]*
  tinfty[4]^2*nu[2]+(2*Qinfty[0]*nu[1]*tinfty[3]-2*nu[3]*tinfty
  [2])*tinfty[4]+Pinfty[1]^2*nu[1]+2*Pinfty[1]*Pinfty[0]*nu[2]-
  tinfty[3]^2*nu[3])*Qinfty[2]^2+((-3*Qinfty[0]*Qinfty[1]^2*nu[1]
  -3*Qinfty[1]^3*nu[2]+2*Qinfty[0]^2*nu[2]+6*Qinfty[0]*Qinfty[1]*
  nu[3])*tinfty[5]^2+((4*nu[2]*tinfty[3]+6*nu[3]*tinfty[4])*
  Qinfty[1]^2+(4*Qinfty[0]*nu[1]*tinfty[3]+8*Qinfty[0]*nu[2]*
  tinfty[4]-2*nu[2]*tinfty[1]-4*nu[3]*tinfty[2])*Qinfty[1]+4*nu
  [1]*tinfty[4]*Qinfty[0]^2+(-2*nu[1]*tinfty[1]-2*nu[2]*tinfty[2]
  -4*nu[3]*tinfty[3])*Qinfty[0]+nu[3]*(2*tinfty[0]-1))*tinfty[5]
```

```
+2*Qinfty[1]^2*tinfty[4]^2*nu[2]+(2*Qinfty[0]*nu[1]*tinfty[4]
^2+(-2*nu[2]*tinfty[2]-4*nu[3]*tinfty[3])*tinfty[4]+2*Pinfty[1]
*nu[1]*Pinfty[0]+(Pinfty[0]^2-tinfty[3]^2)*nu[2])*Qinfty[1]-2*
Qinfty[0]*nu[3]*tinfty[4]^2+((-2*nu[1]*tinfty[2]-2*nu[2]*tinfty
[3])*Qinfty[0]+2*nu[3]*tinfty[1])*tinfty[4]-nu[1]*(Pinfty[0]^2+
tinfty[3]^2)*Qinfty[0]+(2*Pinfty[1]*Pinfty[2]+Pinfty[0])*nu[1]+
(2*Pinfty[0]*Pinfty[2]+2*Pinfty[1]^2)*nu[2]+(2*Pinfty[0]*Pinfty
[1]+2*tinfty[2]*tinfty[3])*nu[3]+2*nu[0]*Pinfty[1])*Qinfty[2]+
(2*Qinfty[0]^2*Qinfty[1]*nu[1]+3*Qinfty[0]*Qinfty[1]^2*nu[2]+
Qinfty[1]^3*nu[3]-Qinfty[0]^2*nu[3])*tinfty[5]^2+(2*tinfty[4]*
Qinfty[1]^3*nu[2]+(2*Qinfty[0]*nu[1]*tinfty[4]-2*nu[2]*tinfty
[2]-2*nu[3]*tinfty[3])*Qinfty[1]^2+(-4*nu[3]*Qinfty[0]*tinfty
[4]+(-2*nu[1]*tinfty[2]-4*nu[2]*tinfty[3])*Qinfty[0]+(2*tinfty
[0]-1)*nu[2]+2*nu[3]*tinfty[1])*Qinfty[1]+2*Qinfty[0]*(-tinfty
[4]*nu[2]*Qinfty[0]-nu[1]*Qinfty[0]*tinfty[3]+(tinfty[0]-1/2)*
nu[1]+nu[2]*tinfty[1]+nu[3]*tinfty[2]))*tinfty[5]+(Pinfty[0]^2*
nu[1]-2*nu[2]*tinfty[3]*tinfty[4]-nu[3]*tinfty[4]^2)*Qinfty[1]
^2+(-2*Qinfty[0]*tinfty[4]^2*nu[2]+(-2*Qinfty[0]*nu[1]*tinfty
[3]+2*nu[2]*tinfty[1]+2*nu[3]*tinfty[2])*tinfty[4]+2*Pinfty[2]*
nu[1]*Pinfty[0]+2*tinfty[2]*tinfty[3]*nu[2]+(Pinfty[0]^2+tinfty
[3]^2)*nu[3]+nu[0]*Pinfty[0])*Qinfty[1]-nu[1]*Qinfty[0]^2*
tinfty[4]^2+2*Qinfty[0]*(nu[1]*tinfty[1]+nu[2]*tinfty[2]+nu[3]*
tinfty[3])*tinfty[4]+((-2*Pinfty[0]*Pinfty[1]+2*tinfty[2]*
tinfty[3])*nu[1]+(-Pinfty[0]^2+tinfty[3]^2)*nu[2])*Qinfty[0]+
(Pinfty[2]^2+2*Pinfty[1])*nu[1]+(2*Pinfty[1]*Pinfty[2]+Pinfty
[0])*nu[2]+(2*Pinfty[0]*Pinfty[2]+Pinfty[1]^2)*nu[3]+3*Pinfty
[2]*nu[0]+(-Qinfty[2]^5*tinfty[5]^2*nu[0]+(2*nu[0]*tinfty[4]*
tinfty[5]+nu[1]*tinfty[5]^2)*Qinfty[2]^4+((4*Qinfty[1]*nu[0]-nu
[2])*tinfty[5]^2+(-2*nu[0]*tinfty[3]-2*nu[1]*tinfty[4])*tinfty
[5]-tinfty[4]^2*nu[0])*Qinfty[2]^3+((-3*Qinfty[0]*nu[0]-3*
Qinfty[1]*nu[1]+3*nu[3])*tinfty[5]^2+(-6*Qinfty[1]*nu[0]*tinfty
[4]+2*nu[0]*tinfty[2]+2*nu[1]*tinfty[3]+2*nu[2]*tinfty[4])*
tinfty[5]+2*tinfty[3]*nu[0]*tinfty[4]+nu[1]*tinfty[4]^2)*Qinfty
[2]^2+((-3*Qinfty[1]^2*nu[0]+4*Qinfty[0]*nu[1]+4*Qinfty[1]*nu
[2])*tinfty[5]^2+((4*nu[0]*tinfty[3]+4*nu[1]*tinfty[4])*Qinfty
[1]+(4*Qinfty[0]*nu[0]-4*nu[3])*tinfty[4]-2*tinfty[1]*nu[0]-2*
nu[1]*tinfty[2]-2*nu[2]*tinfty[3])*tinfty[5]+2*Qinfty[1]*tinfty
[4]^2*nu[0]-tinfty[4]^2*nu[2]+(-2*nu[0]*tinfty[2]-2*nu[1]*
tinfty[3])*tinfty[4]-2*P01*nu[1]*Pinfty[0]-nu[2]*P01^2-2*nu[0]*
(P01*Pinfty[1]+(1/2)*Pinfty[0]^2+(1/2)*tinfty[3]^2))*Qinfty[2]+
(nu[1]*Qinfty[1]^2+(2*Qinfty[0]*nu[0]-2*nu[3])*Qinfty[1]-2*
Qinfty[0]*nu[2])*tinfty[5]^2+(2*Qinfty[1]^2*tinfty[4]*nu[0]+
```

```
(-2*nu[0]*tinfty[2]-2*nu[1]*tinfty[3]-4*nu[2]*tinfty[4])*Qinfty
[1]-4*nu[1]*Qinfty[0]*tinfty[4]-2*tinfty[3]*nu[0]*Qinfty[0]+2*
nu[1]*tinfty[1]+2*tinfty[2]*nu[2]+2*tinfty[3]*nu[3]+nu[0]*(2*
tinfty[0]-1))*tinfty[5]+(-P01^2*nu[1]-2*P01*Pinfty[0]*nu[0]-2*
nu[0]*tinfty[3]*tinfty[4]-nu[1]*tinfty[4]^2)*Qinfty[1]+(-Qinfty
[0]*nu[0]+nu[3])*tinfty[4]^2+(2*nu[0]*tinfty[1]+2*nu[1]*tinfty
[2]+2*nu[2]*tinfty[3])*tinfty[4]-P01^2*nu[0]*Qinfty[0]+(-2*P01*
Pinfty[1]-Pinfty[0]^2+tinfty[3]^2)*nu[1]-2*P01*nu[2]*Pinfty[0]-
nu[3]*P01^2-2*nu[0]*(P01*Pinfty[2]+Pinfty[0]*Pinfty[1]-tinfty
[2]*tinfty[3]))*Q01+(2*Qinfty[2]*nu[0]*tinfty[5]^2-2*nu[0]*
tinfty[4]*tinfty[5]-nu[1]*tinfty[5]^2)*Q01^2;
```

$$Ham := \frac{\left(Qinfty_0\,\nu_0 + Qinfty_1\,\nu_1 + Qinfty_2\,\nu_2 + \nu_3\right)\,t00^2}{Q01} - Qinfty_2^6\,\nu_3\,tinfty_5^2 + \left(\left(\right.\right. \tag{1.9}$$
$$\left.\left. -Qinfty_0\,\nu_1 - Qinfty_1\,\nu_2\right)\,tinfty_5^2 + 2\,tinfty_5\,tinfty_4\,\nu_3\right)\,Qinfty_2^5 + \left(\left(Qinfty_0\,\nu_2\right.\right.$$
$$\left.\left. + 5\,Qinfty_1\,\nu_3\right)\,tinfty_5^2 + \left(2\,Qinfty_0\,\nu_1\,tinfty_4 + 2\,Qinfty_1\,\nu_2\,tinfty_4 - 2\,\nu_3\,tinfty_3\right)\,tinfty_5\right.$$
$$\left. - \nu_3\,tinfty_4^2\right)\,Qinfty_2^4 + \left(\left(4\,Qinfty_0\,Qinfty_1\,\nu_1 + 4\,Qinfty_1^2\,\nu_2 - 4\,Qinfty_0\,\nu_3\right)\,tinfty_5^2 + \left(\right.\right.$$
$$\left.\left. -2\,\nu_2\,tinfty_3 - 8\,\nu_3\,tinfty_4\right)\,Qinfty_1 - 2\,tinfty_3\,\nu_1\,Qinfty_0 - 2\,Qinfty_0\,tinfty_4\,\nu_2\right.$$
$$\left. + 2\,\nu_3\,tinfty_2\right)\,tinfty_5 - tinfty_4^2\,\nu_2\,Qinfty_1 - Qinfty_0\,\nu_1\,tinfty_4^2 + 2\,tinfty_3\,tinfty_4\,\nu_3\Big)\,Qinfty_2^3$$
$$+ \left(\left(-3\,Qinfty_0^2\,\nu_1 - 6\,Qinfty_0\,Qinfty_1\,\nu_2 - 6\,Qinfty_1^2\,\nu_3\right)\,tinfty_5^2 + \left(-6\,Qinfty_1^2\,tinfty_4\,\nu_2\right.\right.$$
$$\left.\left. + \left(-6\,Qinfty_0\,\nu_1\,tinfty_4 + 2\,\nu_2\,tinfty_2 + 6\,\nu_3\,tinfty_3\right)\,Qinfty_1 + 6\,\nu_3\,Qinfty_0\,tinfty_4\right.\right.$$
$$\left.\left. + \left(2\,\nu_1\,tinfty_2 + 2\,\nu_2\,tinfty_3\right)\,Qinfty_0 - 2\,\nu_3\,tinfty_1\right)\,tinfty_5 + \left(2\,\nu_2\,tinfty_3\,tinfty_4 + 3\,\nu_3\right.\right.$$
$$\left.\left. tinfty_4^2\right)\,Qinfty_1 + Qinfty_0\,\nu_2\,tinfty_4^2 + \left(2\,Qinfty_0\,\nu_1\,tinfty_3 - 2\,\nu_3\,tinfty_2\right)\,tinfty_4 +\right.$$
$$\left. Pinfty_1^2\,\nu_1 + 2\,Pinfty_1\,Pinfty_0\,\nu_2 - tinfty_3^2\,\nu_3\right)\,Qinfty_2^2 + \left(\left(-3\,Qinfty_0\,Qinfty_1^2\,\nu_1 - 3\right.\right.$$
$$\left.\left. Qinfty_1^3\,\nu_2 + 2\,Qinfty_0^2\,\nu_2 + 6\,Qinfty_0\,Qinfty_1\,\nu_3\right)\,tinfty_5^2 + \left(\left(4\,\nu_2\,tinfty_3 + 6\,\nu_3\,tinfty_4\right)\right.\right.$$
$$\left.\left. Qinfty_1^2 + \left(4\,Qinfty_0\,\nu_1\,tinfty_3 + 8\,Qinfty_0\,\nu_2\,tinfty_4 - 2\,\nu_2\,tinfty_1 - 4\,\nu_3\,tinfty_2\right)\,Qinfty_1\right.\right.$$
$$\left.\left. + 4\,Qinfty_0^2\,\nu_1\,tinfty_4 + \left(-2\,\nu_1\,tinfty_1 - 2\,\nu_2\,tinfty_2 - 4\,\nu_3\,tinfty_3\right)\,Qinfty_0 + \nu_3\,\left(2\,tinfty_0\right.\right.\right.$$
$$\left.\left.\left. - 1\right)\right)\,tinfty_5 + 2\,Qinfty_1^2\,tinfty_4^2\,\nu_2 + \left(2\,Qinfty_0\,\nu_1\,tinfty_4^2 + \left(-2\,\nu_2\,tinfty_2\right.\right.\right.$$
$$\left.\left.\left. - 4\,\nu_3\,tinfty_3\right)\,tinfty_4 + 2\,Pinfty_1\,\nu_1\,Pinfty_0 + \left(Pinfty_0^2 - tinfty_3^2\right)\,\nu_2\right)\,Qinfty_1$$
$$- 2\,Qinfty_0\,\nu_3\,tinfty_4^2 + \left(\left(-2\,\nu_1\,tinfty_2 - 2\,\nu_2\,tinfty_3\right)\,Qinfty_0 + 2\,\nu_3\,tinfty_1\right)\,tinfty_4$$
$$- \nu_1\,\left(Pinfty_0^2 + tinfty_3^2\right)\,Qinfty_0 + \left(2\,Pinfty_1\,Pinfty_2 + Pinfty_0\right)\,\nu_1 + \left(2\,Pinfty_0\,Pinfty_2\right.$$
$$\left. + 2\,Pinfty_1^2\right)\,\nu_2 + \left(2\,Pinfty_0\,Pinfty_1 + 2\,tinfty_2\,tinfty_3\right)\,\nu_3 + 2\,\nu_0\,Pinfty_1\Big)\,Qinfty_2 + \Big(2$$
$$Qinfty_0^2\,Qinfty_1\,\nu_1 + 3\,Qinfty_0\,Qinfty_1^2\,\nu_2 + Qinfty_1^3\,\nu_3 - Qinfty_0^2\,\nu_3\Big)\,tinfty_5^2 + \Big(2$$
$$Qinfty_1^3\,tinfty_4\,\nu_2 + \left(2\,Qinfty_0\,\nu_1\,tinfty_4 - 2\,\nu_2\,tinfty_2 - 2\,\nu_3\,tinfty_3\right)\,Qinfty_1^2 + \Big(\Big.$$
$$\Big. -4\,\nu_3\,Qinfty_0\,tinfty_4 + \left(-2\,\nu_1\,tinfty_2 - 4\,\nu_2\,tinfty_3\right)\,Qinfty_0 + \left(2\,tinfty_0 - 1\right)\,\nu_2$$

$$+2\,\nu_3\,tinfty_1\Big)\,Qinfty_1+2\,Qinfty_0\Big(-Qinfty_0\,tinfty_4\,\nu_2-tinfty_3\,\nu_1\,Qinfty_0+\Big(tinfty_0-\frac{1}{2}\Big)\,\nu_1+tinfty_1\,\nu_2+\nu_3\,tinfty_2\Big)\Big)\,tinfty_5+\Big(Pinfty_0^2\,\nu_1-2\,\nu_2\,tinfty_3\,tinfty_4-\nu_3\,tinfty_4^2\Big)\,Qinfty_1^2+\Big(-2\,Qinfty_0\,\nu_2\,tinfty_4^2+\Big(-2\,Qinfty_0\,\nu_1\,tinfty_3+2\,\nu_2\,tinfty_1+2\,\nu_3\,tinfty_2\Big)\,tinfty_4+2\,Pinfty_2\,\nu_1\,Pinfty_0+2\,tinfty_2\,tinfty_3\,\nu_2+\Big(Pinfty_0^2+tinfty_3^2\Big)\,\nu_3+\nu_0\,Pinfty_0\Big)\,Qinfty_1-tinfty_4^2\,\nu_1\,Qinfty_0^2+2\,Qinfty_0\Big(\nu_1\,tinfty_1+\nu_2\,tinfty_2+\nu_3\,tinfty_3\Big)\,tinfty_4+\Big(\Big(-2\,Pinfty_0\,Pinfty_1+2\,tinfty_2\,tinfty_3\Big)\,\nu_1+\Big(-Pinfty_0^2+tinfty_3^2\Big)\,\nu_2\Big)\,Qinfty_0+\Big(Pinfty_2^2+2\,Pinfty_1\Big)\,\nu_1+\Big(2\,Pinfty_1\,Pinfty_2+Pinfty_0\Big)\,\nu_2+\Big(2\,Pinfty_0\,Pinfty_2+Pinfty_1^2\Big)\,\nu_3+3\,Pinfty_2\,\nu_0+\Big(-Qinfty_2^5\,tinfty_5^2\,\nu_0+\Big(2\,\nu_0\,tinfty_4\,tinfty_5+\nu_1\,tinfty_5^2\Big)\,Qinfty_2^4+\Big(\Big(4\,Qinfty_1\,\nu_0-\nu_2\Big)\,tinfty_5^2+\Big(-2\,\nu_0\,tinfty_3-2\,\nu_1\,tinfty_4\Big)\,tinfty_5-tinfty_4^2\,\nu_0\Big)\,Qinfty_2^3+\Big(\Big(-3\,Qinfty_0\,\nu_0-3\,Qinfty_1\,\nu_1+3\,\nu_3\Big)\,tinfty_5^2+\Big(-6\,Qinfty_1\,\nu_0\,tinfty_4+2\,\nu_0\,tinfty_2+2\,\nu_1\,tinfty_3+2\,\nu_2\,tinfty_4\Big)\,tinfty_5+2\,\nu_0\,tinfty_3\,tinfty_4+\nu_1\,tinfty_4^2\Big)\,Qinfty_2^2+\Big(\Big(-3\,Qinfty_1^2\,\nu_0+4\,Qinfty_0\,\nu_1+4\,Qinfty_1\,\nu_2\Big)\,tinfty_5^2+\Big(\Big(4\,\nu_0\,tinfty_3+4\,\nu_1\,tinfty_4\Big)\,Qinfty_1+\Big(4\,Qinfty_0\,\nu_0-4\,\nu_3\Big)\,tinfty_4-2\,tinfty_1\,\nu_0-2\,\nu_1\,tinfty_2-2\,\nu_2\,tinfty_3\Big)\,tinfty_5+2\,Qinfty_1\,tinfty_4^2\,\nu_0-tinfty_4^2\,\nu_2+\Big(-2\,\nu_0\,tinfty_2-2\,\nu_1\,tinfty_3\Big)\,tinfty_4-2\,P01\,\nu_1\,Pinfty_0-\nu_2\,P01^2-2\,\nu_0\Big(P01\,Pinfty_1+\frac{1}{2}\,Pinfty_0^2+\frac{1}{2}\,tinfty_3^2\Big)\Big)\,Qinfty_2+\Big(\nu_1\,Qinfty_1^2+\Big(2\,Qinfty_0\,\nu_0-2\,\nu_3\Big)\,Qinfty_1-2\,Qinfty_0\,\nu_2\Big)\,tinfty_5^2+\Big(2\,Qinfty_1^2\,tinfty_4\,\nu_0+\Big(-2\,\nu_0\,tinfty_2-2\,\nu_1\,tinfty_3-4\,\nu_2\,tinfty_4\Big)\,Qinfty_1-4\,\nu_1\,Qinfty_0\,tinfty_4-2\,tinfty_3\,\nu_0\,Qinfty_0+2\,\nu_1\,tinfty_1+2\,tinfty_2\,\nu_2+2\,tinfty_3\,\nu_3+\nu_0\,\big(2\,tinfty_0-1\big)\Big)\,tinfty_5+\Big(-P01^2\,\nu_1-2\,P01\,Pinfty_0\,\nu_0-2\,\nu_0\,tinfty_3\,tinfty_4-\nu_1\,tinfty_4^2\Big)\,Qinfty_1+\Big(-Qinfty_0\,\nu_0+\nu_3\Big)\,tinfty_4^2+\Big(2\,\nu_0\,tinfty_1+2\,\nu_1\,tinfty_2+2\,\nu_2\,tinfty_3\Big)\,tinfty_4-P01^2\,\nu_0\,Qinfty_0+\Big(-2\,P01\,Pinfty_1-Pinfty_0^2+tinfty_3^2\Big)\,\nu_1-2\,P01\,\nu_2\,Pinfty_0-\nu_3\,P01^2-2\,\nu_0\,\big(P01\,Pinfty_2+Pinfty_0\,Pinfty_1-tinfty_2\,tinfty_3\big)\Big)\,Q01+\Big(2\,Qinfty_2\,\nu_0\,tinfty_5^2-2\,\nu_0\,tinfty_4\,tinfty_5-\nu_1\,tinfty_5^2\Big)\,Q01^2$$

```
> simplify(diff(Ham,Pinfty[0])- LQinfty0);
  simplify(diff(Ham,Pinfty[1])- LQinfty1);
  simplify(diff(Ham,Pinfty[2])- LQinfty2);
  simplify(diff(Ham,P01)- LQ01);

  simplify(diff(Ham,Q01)+ LP01);
```

```
simplify(diff(Ham,Qinfty[1])+ LPinfty1);
simplify(diff(Ham,Qinfty[2])+ LPinfty2);
simplify(diff(Ham,Qinfty[0])+ LPinfty0);
```

$$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \tag{1.10}$$

```
> M:=Matrix(2*d,2*d,0):
  for i from 1 to 2*d do for j from 1 to i do M[i,j]:=tinfty[2*
  d+1-(i-j)]: od: od:
  M;
  alphaVector:=Matrix(2*d,1,0):
  for i from 1 to 2*d do alphaVector[i]:=alpha[2*d+1-i]/(2*d+1-i)
  : od:
  alphaVector;
  nuVector:=Multiply(M^(-1),alphaVector);
  for i from 1 to 2*d do nu[i-1]:=nuVector[i,1]: od:
  nu[0]:=nu[0];
  nu[1]:=nu[1];
  nu[2]:=nu[2];
  nu[3]:=nu[3];
```

$$\begin{bmatrix} tinfty_5 & 0 & 0 & 0 \\ tinfty_4 & tinfty_5 & 0 & 0 \\ tinfty_3 & tinfty_4 & tinfty_5 & 0 \\ tinfty_2 & tinfty_3 & tinfty_4 & tinfty_5 \end{bmatrix} \tag{1.11}$$

$$\begin{bmatrix} \frac{1}{4}\,\alpha_4 \\ \frac{1}{3}\,\alpha_3 \\ \frac{1}{2}\,\alpha_2 \\ \alpha_1 \end{bmatrix}$$

$$\left[\left[\frac{1}{4} \quad \frac{\alpha_4}{tinfty_5}\right],\right.$$

$$\left[-\frac{1}{4}\,\frac{tinfty_4\,\alpha_4}{tinfty_5^2} + \frac{1}{3}\,\frac{\alpha_3}{tinfty_5} \right],$$

$$\left[-\frac{1}{4}\,\frac{\left(tinfty_3\,tinfty_5 - tinfty_4^2\right)\alpha_4}{tinfty_5^3} - \frac{1}{3}\,\frac{tinfty_4\,\alpha_3}{tinfty_5^2} + \frac{1}{2}\,\frac{\alpha_2}{tinfty_5} \right],$$

$$\left[-\frac{1}{4}\,\frac{\left(tinfty_2\,tinfty_5^2 - 2\,tinfty_3\,tinfty_4\,tinfty_5 + tinfty_4^3\right)\alpha_4}{tinfty_5^4} - \frac{1}{3}\,\frac{\left(tinfty_3\,tinfty_5 - tinfty_4^2\right)\alpha_3}{tinfty_5^3} - \frac{1}{2}\,\frac{tinfty_4\,\alpha_2}{tinfty_5^2} + \frac{\alpha_1}{tinfty_5} \right]$$

$$\nu_0 := \frac{1}{4}\,\frac{\alpha_4}{tinfty_5}$$

$$\nu_1 := -\frac{1}{4}\,\frac{tinfty_4\,\alpha_4}{tinfty_5^2} + \frac{1}{3}\,\frac{\alpha_3}{tinfty_5}$$

$$\nu_2 := -\frac{1}{4}\,\frac{\left(tinfty_3\,tinfty_5 - tinfty_4^2\right)\alpha_4}{tinfty_5^3} - \frac{1}{3}\,\frac{tinfty_4\,\alpha_3}{tinfty_5^2} + \frac{1}{2}\,\frac{\alpha_2}{tinfty_5}$$

$$\nu_3 := -\frac{1}{4}\,\frac{\left(tinfty_2\,tinfty_5^2 - 2\,tinfty_3\,tinfty_4\,tinfty_5 + tinfty_4^3\right)\alpha_4}{tinfty_5^4} - \frac{1}{3}\,\frac{\left(tinfty_3\,tinfty_5 - tinfty_4^2\right)\alpha_3}{tinfty_5^3} - \frac{1}{2}\,\frac{tinfty_4\,\alpha_2}{tinfty_5^2} + \frac{\alpha_1}{tinfty_5}$$

```
> Hamfunction:=unapply(simplify(Ham),alpha[1],alpha[2],alpha[3],
  alpha[4]):
  Hamtinfty1:=simplify(Hamfunction(1,0,0,0)):
  Hamtinfty2:=simplify(Hamfunction(0,1,0,0)):
  Hamtinfty3:=simplify(Hamfunction(0,0,1,0)):
  Hamtinfty2:=simplify(Hamfunction(0,0,0,1)):
```

We impose the symmetry \check{L}(-\lambda)=-\sigma_1 \check{L}(\lambda) \sigma_1

```
> R01:=0:
  Rinfty[1]:=0:
  Rinfty[3]:=0:
  tinfty[4]:=0:
  tinfty[2]:=0:
  tinfty[0]:=0:
  alpha[2]:=0:
  alpha[4]:=0:
```

```
> omega:=-t00/Q01;
  Lomega:=t00*LQ01/Q01^2:
  Qinfty[2]:=-P01/tinfty[5];
```

```
Pinfty[1]:=-P01*Qinfty[1]-tinfty[5]*Qinfty[0]+Pinfty[0]*
P01/tinfty[5];
Qinfty[1]:=P01^2/tinfty[5]^2-Pinfty[0]/tinfty[5]+tinfty[3]
/tinfty[5]-t00^2/(2*tinfty[5]^2*Q01^2);
Pinfty[2]:=-2*P01*Qinfty[0]-tinfty[5]*Q01-P01^4/tinfty[5]^3+
P01^2*Pinfty[0]/tinfty[5]^2-P01^2*tinfty[3]/tinfty[5]^2+Pinfty
[0]^2/tinfty[5]-Pinfty[0]*tinfty[3]/tinfty[5]+tinfty[1]+3*
t00^2*Pinfty[0]/(2*Q01^2*tinfty[5]^2)-t00^2*tinfty[3]/(2*Q01^2*
tinfty[5]^2)+3*t00^4/(8*Q01^4*tinfty[5]^3);
L12function:=unapply(L12,lambda):
L21function:=unapply(L21,lambda):
L11function:=unapply(L11,lambda):
series(simplify(L11function(-lambda)-L11function(lambda)),
lambda=0);
series(simplify(L12function(-lambda)+L21function(lambda)),
lambda=0);
series(simplify(L21function(-lambda)+L12function(lambda)),
lambda=0):
```

$$\omega := -\frac{t00}{Q01} \tag{1.12}$$

$$Qinfty_2 := -\frac{P01}{tinfty_5}$$

$$Pinfty_1 := -P01\,Qinfty_1 - tinfty_5\,Qinfty_0 + \frac{Pinfty_0\,P01}{tinfty_5}$$

$$Qinfty_1 := \frac{P01^2}{tinfty_5^2} - \frac{Pinfty_0}{tinfty_5} + \frac{tinfty_3}{tinfty_5} - \frac{1}{2}\,\frac{t00^2}{tinfty_5^2\,Q01^2}$$

$$Pinfty_2 := -2\,P01\,Qinfty_0 - tinfty_5\,Q01 - \frac{P01^4}{tinfty_5^3} + \frac{P01^2\,Pinfty_0}{tinfty_5^2} - \frac{P01^2\,tinfty_3}{tinfty_5^2} + \frac{Pinfty_0^2}{tinfty_5} - \frac{Pinfty_0\,tinfty_3}{tinfty_5} + tinfty_1 + \frac{3}{2}\,\frac{t00^2\,Pinfty_0}{tinfty_5^2\,Q01^2} - \frac{1}{2}\,\frac{t00^2\,tinfty_3}{tinfty_5^2\,Q01^2} + \frac{3}{8}\,\frac{t00^4}{Q01^4\,tinfty_5^3}$$

$$0$$
$$0$$

```
> L:=simplify(L):
L11:=expand(series(simplify(series(L11,lambda=infinity)),
lambda=infinity));
L12:=expand(series(simplify(series(L12,lambda=infinity)),
lambda=infinity));
L21:=expand(series(simplify(series(L21,lambda=infinity)),
```

```
  lambda=infinity));
  A:=simplify(A);
```

$$L11 := -tinfty_5\,\lambda^4 - \lambda^2\,tinfty_3 + \frac{1}{2}\,\frac{\lambda^2\,t00^2}{Q01^2\,tinfty_5} - tinfty_1 + \frac{1}{2}\,\frac{P01^2\,t00^2}{tinfty_5^3\,Q01^2} - \frac{t00^2\,Pinfty_0}{tinfty_5^2\,Q01^2} + \frac{1}{2}\,\frac{t00^2\,tinfty_3}{tinfty_5^2\,Q01^2} - \frac{3}{8}\,\frac{t00^4}{Q01^4\,tinfty_5^3} \tag{1.13}$$

$$L12 := -\frac{t00\,\lambda^3}{Q01} + \frac{P01\,t00\,\lambda^2}{tinfty_5\,Q01} - \frac{t00\,\lambda\,P01^2}{tinfty_5^2\,Q01} + \frac{t00\,\lambda\,Pinfty_0}{tinfty_5\,Q01} - \frac{t00\,\lambda\,tinfty_3}{tinfty_5\,Q01} + \frac{1}{2}\,\frac{t00^3\,\lambda}{tinfty_5^2\,Q01^3} - \frac{Qinfty_0\,t00}{Q01} - \frac{t00}{\lambda}$$

$$L21 := -\frac{t00\,\lambda^3}{Q01} - \frac{P01\,t00\,\lambda^2}{tinfty_5\,Q01} - \frac{t00\,\lambda\,P01^2}{tinfty_5^2\,Q01} + \frac{t00\,\lambda\,Pinfty_0}{tinfty_5\,Q01} - \frac{t00\,\lambda\,tinfty_3}{tinfty_5\,Q01} + \frac{1}{2}\,\frac{t00^3\,\lambda}{tinfty_5^2\,Q01^3} + \frac{Qinfty_0\,t00}{Q01} - \frac{t00}{\lambda}$$

$$\left[\left[-\frac{1}{3}\,\frac{\lambda\left(Q01^2\left(\lambda^2\,\alpha_3 + 3\,\alpha_1\right)tinfty_5^2 - \frac{1}{2}\,t00^2\,\alpha_3\right)}{tinfty_5^2\,Q01^2},\ -\frac{1}{3}\,\frac{1}{Q01^3\,tinfty_5^3}\left(t00\left(\left(\left(\lambda^2\,\alpha_3 + 3\,\alpha_1\right)tinfty_5^2 - \alpha_3\left(P01\,\lambda + Pinfty_0\right)tinfty_5 + P01^2\,\alpha_3\right)Q01^2 - \frac{1}{2}\,t00^2\,\alpha_3\right)\right)\right],$$

$$\left[-\frac{1}{3}\,\frac{1}{Q01^3\,tinfty_5^3}\left(t00\left(\left(\left(\lambda^2\,\alpha_3 + 3\,\alpha_1\right)tinfty_5^2 + \alpha_3\left(P01\,\lambda - Pinfty_0\right)tinfty_5 + P01^2\,\alpha_3\right)Q01^2 - \frac{1}{2}\,t00^2\,\alpha_3\right)\right),\ \frac{1}{3}\,\frac{\lambda\left(Q01^2\left(\lambda^2\,\alpha_3 + 3\,\alpha_1\right)tinfty_5^2 - \frac{1}{2}\,t00^2\,\alpha_3\right)}{tinfty_5^2\,Q01^2}\right]\right]$$

```
> LQinfty0:=expand(simplify(LQinfty0));
  LPinfty0:=expand(simplify(LPinfty0));
  LQ01:=expand(simplify(LQ01));
  LP01:=expand(simplify(LP01));

  LQinfty1:=expand(simplify(LQinfty1)):
  LQinfty2:=expand(simplify(LQinfty2)):
  LPinfty1:=expand(simplify(LPinfty1)):
```

```
LPinfty2:=expand(simplify(LPinfty2)):
```

$$LQinfty0 := \frac{2}{3}\,Qinfty_0^2\,\alpha_3 - \frac{2\,P01\,Qinfty_0\,\alpha_1}{tinfty_5} + \frac{2}{3}\,\frac{P01\,Qinfty_0\,\alpha_3\,tinfty_3}{tinfty_5^2} - 2\,Q01\,\alpha_1 \qquad \textbf{(1.14)}$$

$$+\frac{2}{3}\,\frac{P01^2\,\alpha_3\,tinfty_1}{tinfty_5^3} - \frac{2}{3}\,\frac{Pinfty_0\,\alpha_3\,tinfty_1}{tinfty_5^2} + \frac{2\,\alpha_1\,tinfty_1}{tinfty_5} - \frac{1}{3}\,\frac{P01\,\alpha_3}{tinfty_5^2}$$

$$+\frac{1}{3}\,\frac{t00^2\,\alpha_3}{tinfty_5^2\,Q01} - \frac{1}{3}\,\frac{P01^4\,t00^2\,\alpha_3}{tinfty_5^6\,Q01^2} + \frac{P01^2\,t00^2\,Pinfty_0\,\alpha_3}{tinfty_5^5\,Q01^2} - \frac{P01^2\,t00^2\,\alpha_1}{tinfty_5^4\,Q01^2}$$

$$-\frac{1}{3}\,\frac{P01^2\,t00^2\,\alpha_3\,tinfty_3}{tinfty_5^5\,Q01^2} - \frac{2}{3}\,\frac{t00^2\,Pinfty_0^2\,\alpha_3}{tinfty_5^4\,Q01^2} + \frac{2\,t00^2\,Pinfty_0\,\alpha_1}{tinfty_5^3\,Q01^2}$$

$$+\frac{1}{3}\,\frac{t00^2\,Pinfty_0\,\alpha_3\,tinfty_3}{tinfty_5^4\,Q01^2} - \frac{t00^2\,\alpha_1\,tinfty_3}{tinfty_5^3\,Q01^2} - \frac{1}{3}\,\frac{t00^2\,\alpha_3\,tinfty_1}{tinfty_5^3\,Q01^2}$$

$$+\frac{5}{12}\,\frac{P01^2\,t00^4\,\alpha_3}{tinfty_5^6\,Q01^4} - \frac{7}{12}\,\frac{t00^4\,Pinfty_0\,\alpha_3}{tinfty_5^5\,Q01^4} + \frac{3}{4}\,\frac{t00^4\,\alpha_1}{tinfty_5^4\,Q01^4} + \frac{1}{6}\,\frac{t00^4\,\alpha_3\,tinfty_3}{tinfty_5^5\,Q01^4}$$

$$-\frac{1}{8}\,\frac{t00^6\,\alpha_3}{tinfty_5^6\,Q01^6}$$

$$LPinfty0 := \frac{2}{3}\,\frac{P01^2\,Qinfty_0\,\alpha_3}{tinfty_5} + \frac{4}{3}\,Q01\,P01\,\alpha_3 + \frac{2}{3}\,Pinfty_0\,Qinfty_0\,\alpha_3$$

$$+2\,tinfty_5\,Qinfty_0\,\alpha_1 - \frac{2}{3}\,\alpha_3\,Qinfty_0\,tinfty_3 + \frac{2\,P01^3\,\alpha_1}{tinfty_5^2} - \frac{2}{3}\,\frac{P01^3\,\alpha_3\,tinfty_3}{tinfty_5^3}$$

$$-\frac{6\,P01\,Pinfty_0\,\alpha_1}{tinfty_5} + \frac{2\,P01\,Pinfty_0\,\alpha_3\,tinfty_3}{tinfty_5^2} + \frac{2\,P01\,\alpha_1\,tinfty_3}{tinfty_5}$$

$$-\frac{2}{3}\,\frac{P01\,\alpha_3\,tinfty_1}{tinfty_5} - \frac{2}{3}\,\frac{P01\,\alpha_3\,tinfty_3^2}{tinfty_5^2} + \frac{1}{3}\,\alpha_3 + \frac{2}{3}\,\frac{t00^2\,Qinfty_0\,\alpha_3}{tinfty_5\,Q01^2}$$

$$+\frac{1}{3}\,\frac{P01^3\,t00^2\,\alpha_3}{tinfty_5^4\,Q01^2} - \frac{2}{3}\,\frac{P01\,t00^2\,Pinfty_0\,\alpha_3}{tinfty_5^3\,Q01^2} - \frac{3\,P01\,t00^2\,\alpha_1}{tinfty_5^2\,Q01^2}$$

$$+\frac{4}{3}\,\frac{P01\,t00^2\,\alpha_3\,tinfty_3}{tinfty_5^3\,Q01^2} - \frac{1}{4}\,\frac{P01\,t00^4\,\alpha_3}{tinfty_5^4\,Q01^4}$$

$$LQ01 := -\frac{2\,Q01\,P01\,\alpha_1}{tinfty_5} + \frac{2}{3}\,Q01\,\alpha_3\,Qinfty_0 + \frac{2}{3}\,\frac{Q01\,\alpha_3\,tinfty_3\,P01}{tinfty_5^2}$$

$$LP01 := \frac{2}{3}\,Qinfty_0\,\alpha_3\,P01 + \frac{2}{3}\,Q01\,tinfty_5\,\alpha_3 - 2\,\alpha_1\,Pinfty_0 + \frac{2}{3}\,\frac{Pinfty_0\,\alpha_3\,tinfty_3}{tinfty_5}$$

$$-\frac{2}{3}\,tinfty_1\,\alpha_3+\frac{1}{3}\,\frac{P01^2\,t00^2\,\alpha_3}{Q01^2\,tinfty_5^3}-\frac{2}{3}\,\frac{t00^2\,Pinfty_0\,\alpha_3}{Q01^2\,tinfty_5^2}+\frac{1}{3}\,\frac{t00^2\,\alpha_3\,tinfty_3}{Q01^2\,tinfty_5^2}$$
$$-\frac{1}{4}\,\frac{t00^4\,\alpha_3}{Q01^4\,tinfty_5^3}$$

We have kept P01,Q01 and Qinfty[0] and Pinfty[0] as free variables from which we have computed the evolutions

Mazzocco's Lax matrix

```
> QQ:=Q1*lambda^2+Q2*lambda^4;
  PP:=P1*lambda^(-2)+P2*lambda^(-4);
  TT:=-z*(2*lambda)^(-2*d):
  for i from 1 to d-1 do TT:=TT+s[i]*(2*lambda)^(2*i-2*d): od:
  TT:=TT;
  series(lambda^(-2)*QQ*(1+PP),lambda=infinity);
  Beven:=Q2*lambda^4+(Q2*P1+Q1)*lambda^2;

  series( 1/4*(2*lambda)^(2*d+1)*(1+PP- (1+TT)^2/(1+PP))+(2*
  lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
  Bodd:=(16*P1-4*s[1]+(1/32)*Q2^2)*lambda^3+(16*P2+z-(1/2)*s[1]^2
  -(8*(-(1/2)*s[1]+P1))*P1+(1/32)*Q2^2*P1+(1/16)*Q2*Q1)*lambda;

  series( 1/4*(2*lambda)^(2*d+1)*(1+PP+ (1+TT)^2/(1+PP))-(2*
  lambda)^(-2*d-1)*QQ^2*(1+PP),lambda=infinity);
  AA:=4^d*lambda^(2*d+1)+(4*s[1]-(1/32)*Q2^2)*lambda^3+(-tinfty
  [1]+(1/2)*s[1]^2+(8*(-(1/2)*s[1]+P1))*P1-(1/32)*Q2^2*P1-(1/16)*
  Q2*Q1)*lambda;

  LMazzocco11:=series( AA/lambda,lambda=infinity);
  LMazzocco12:=series( (Beven+Bodd)/lambda +b0/lambda,lambda=
  infinity);
  LMazzocco21:=series( (Beven-Bodd)/lambda +b0/lambda,lambda=
  infinity);
```

$$QQ := Q2\,\lambda^4+Q1\,\lambda^2 \tag{2.1}$$
$$PP := \frac{P1}{\lambda^2}+\frac{P2}{\lambda^4}$$
$$TT := -\frac{1}{16}\,\frac{z}{\lambda^4}+\frac{1}{4}\,\frac{s_1}{\lambda^2}$$
$$Q2\,\lambda^2+Q2\,P1+Q1+\frac{P1\,Q1+P2\,Q2}{\lambda^2}+\frac{Q1\,P2}{\lambda^4}$$

$$Beven := Q2\,\lambda^4 + (P1\,Q2 + Q1)\,\lambda^2$$

$$\left(16\,P1 - 4\,s_1 + \frac{1}{32}\,Q2^2\right)\lambda^3 + \left(16\,P2 + z - \frac{1}{2}\,s_1^2 - 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 + \frac{1}{32}\,Q2^2\,P1 + \frac{1}{16}\,Q2\,Q1\right)\lambda + O\left(\frac{1}{\lambda}\right)$$

$$Bodd := \left(16\,P1 - 4\,s_1 + \frac{1}{32}\,Q2^2\right)\lambda^3 + \left(16\,P2 + z - \frac{1}{2}\,s_1^2 - 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 + \frac{1}{32}\,Q2^2\,P1 + \frac{1}{16}\,Q2\,Q1\right)\lambda$$

$$16\,\lambda^5 + \left(4\,s_1 - \frac{1}{32}\,Q2^2\right)\lambda^3 + \left(-z + \frac{1}{2}\,s_1^2 + 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 - \frac{1}{32}\,Q2^2\,P1 - \frac{1}{16}\,Q2\,Q1\right)\lambda + O\left(\frac{1}{\lambda}\right)$$

$$AA := 16\,\lambda^5 + \left(4\,s_1 - \frac{1}{32}\,Q2^2\right)\lambda^3 + \left(-tinfty_1 + \frac{1}{2}\,s_1^2 + 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 - \frac{1}{32}\,Q2^2\,P1 - \frac{1}{16}\,Q2\,Q1\right)\lambda$$

$$LMazzocco11 := 16\,\lambda^4 + \left(4\,s_1 - \frac{1}{32}\,Q2^2\right)\lambda^2 - tinfty_1 + \frac{1}{2}\,s_1^2 + 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 - \frac{1}{32}\,Q2^2\,P1 - \frac{1}{16}\,Q2\,Q1$$

$$LMazzocco12 := Q2\,\lambda^3 + \left(16\,P1 - 4\,s_1 + \frac{1}{32}\,Q2^2\right)\lambda^2 + (P1\,Q2 + Q1)\,\lambda + 16\,P2 + z - \frac{1}{2}\,s_1^2 - 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 + \frac{1}{32}\,Q2^2\,P1 + \frac{1}{16}\,Q2\,Q1 + \frac{b0}{\lambda}$$

$$LMazzocco21 := Q2\,\lambda^3 + \left(-16\,P1 + 4\,s_1 - \frac{1}{32}\,Q2^2\right)\lambda^2 + (P1\,Q2 + Q1)\,\lambda - 16\,P2 - z + \frac{1}{2}\,s_1^2 + 8\left(-\frac{1}{2}\,s_1 + P1\right)P1 - \frac{1}{32}\,Q2^2\,P1 - \frac{1}{16}\,Q2\,Q1 + \frac{b0}{\lambda}$$

```
> DetMazzocco:=series(simplify(-LMazzocco11^2-LMazzocco12*
  LMazzocco21),lambda=infinity):
  DetL:=series(simplify(-L11^2-L12*L21),lambda=infinity):
  series(DetL-DetMazzocco,lambda=infinity,1);
  s[1]:=-1/4*tinfty[3];
  tinfty[2*d+1]:=-4^d;
  series(DetL-DetMazzocco,lambda=infinity,1);
```

$$\left(-tinfty_5^2 + 256\right)\lambda^8 + \left(-2\,tinfty_3\,tinfty_5 + 128\,s_1\right)\lambda^6 + \left(\frac{1}{64}\,\frac{-128\,tinfty_1\,tinfty_5^7 - 64\,tinfty_3^2\,tinfty_5^6}{tinfty_5^6} + 16\,s_1^2 - 32\,tinfty_1\right)\lambda^4 + \left(-\frac{1}{tinfty_5^4\,Q01^6}\left(2\,Q01^6\,tinfty_1\,tinfty_3\,tinfty_5^4 + 2\,t00^2\,tinfty_5^4\,Q01^5 + t00^2\left(\left(2\,P01\,Qinfty_0 - tinfty_1\right)tinfty_5^3 + Pinfty_0^2\,tinfty_5^2 - 2\,P01^2\left(Pinfty_0 - \frac{1}{2}\,tinfty_3\right)tinfty_5 + P01^4\right)Q01^4\right.\right. \tag{2.2}$$

$$
\begin{aligned}
&-\frac{1}{2}\,t00^4\left(P01^2-\frac{1}{2}\,tinfty_3\,tinfty_5\right)Q01^2-\frac{1}{8}\,t00^6\Big)+256\,P1^3-128\,P1^2\,s_1\\
&-\frac{1}{16}\left(-256\,s_1^2+8192\,P2+512\,z\right)P1-\frac{1}{16}\left(-128\,z-2048\,P2+128\,tinfty_1\right)s_1\\
&-\frac{1}{16}\left(z+16\,P2-tinfty_1\right)Q2^2+2\,Q2\,b0+Q1^2\Big)\lambda^2+\frac{1}{64}\,\frac{1}{Q01^8\,tinfty_5^6}\Big(\\
&-64\,Q01^8\,tinfty_1^2\,tinfty_5^6-128\,t00^2\left(\left(tinfty_3-Pinfty_0\right)tinfty_5+P01^2\right)tinfty_5^4\,Q01^7\\
&+64\,t00^2\left(Qinfty_0^2\,tinfty_5^3-2\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)tinfty_1\,tinfty_5+P01^2\,tinfty_1\right)\\
&tinfty_5^3\,Q01^6+64\,Q01^5\,t00^4\,tinfty_5^4-16\,t00^4\left(3\,tinfty_1\,tinfty_5^3+4\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)^2\right.\\
&\left.tinfty_5^2-4\,P01^2\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)tinfty_5+P01^4\right)Q01^4+24\,t00^6\left(\left(-2\,Pinfty_0\right.\right.\\
&\left.\left.+tinfty_3\right)tinfty_5+P01^2\right)Q01^2-9\,t00^8\Big)-\frac{1}{16}\left(-256\,z-4096\,P2+256\,tinfty_1\right)P1^2\\
&-\frac{1}{16}\left(\left(128\,z+2048\,P2-128\,tinfty_1\right)s_1+Q2\left(\left(z+16\,P2-tinfty_1\right)Q2\right.\right.\\
&\left.\left.-32\,b0\right)\right)P1-\frac{1}{16}\left(-16\,z-256\,P2+16\,tinfty_1\right)s_1^2-\frac{1}{8}\,Q1\left(z+16\,P2\right.\\
&\left.-tinfty_1\right)Q2-z^2-32\,z\,P2-256\,P2^2+2\,Q1\,b0+tinfty_1^2+O\left(\frac{1}{\lambda^2}\right)
\end{aligned}
$$

$$s_1 := -\frac{1}{4}\,tinfty_3$$

$$tinfty_5 := -16$$

$$
\begin{aligned}
&\Big(-\frac{1}{65536}\,\frac{1}{Q01^6}\Big(131072\,Q01^6\,tinfty_1\,tinfty_3+131072\,t00^2\,Q01^5+t00^2\Big(\\
&-8192\,P01\,Qinfty_0+4096\,tinfty_1+256\,Pinfty_0^2+32\,P01^2\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)\\
&+P01^4\Big)Q01^4-\frac{1}{2}\,t00^4\left(P01^2+8\,tinfty_3\right)Q01^2-\frac{1}{8}\,t00^6\Big)+256\,P1^3\\
&+32\,P1^2\,tinfty_3-\frac{1}{16}\left(-16\,tinfty_3^2+8192\,P2+512\,z\right)P1+\frac{1}{64}\left(-128\,z-2048\,P2\right.\\
&\left.+128\,tinfty_1\right)tinfty_3-\frac{1}{16}\left(z+16\,P2-tinfty_1\right)Q2^2+2\,Q2\,b0+Q1^2\Big)\lambda^2\\
&+\frac{1}{1073741824}\,\frac{1}{Q01^8}\Big(-1073741824\,Q01^8\,tinfty_1^2-8388608\,t00^2\left(P01^2\right.\\
&\left.+16\,Pinfty_0-16\,tinfty_3\right)Q01^7-262144\,t00^2\left(-4096\,Qinfty_0^2+32\left(Pinfty_0\right.\right.\\
&\left.\left.-\frac{1}{2}\,tinfty_3\right)tinfty_1+P01^2\,tinfty_1\right)Q01^6+4194304\,Q01^5\,t00^4-16\,t00^4\Big(\\
&-12288\,tinfty_1+1024\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)^2+64\,P01^2\left(Pinfty_0-\frac{1}{2}\,tinfty_3\right)
\end{aligned}
$$

$$+ P01^4\Big)\, Q01^4 + 24\, t00^6\left(P01^2 + 32\, Pinfty_0 - 16\, tinfty_3\right) Q01^2 - 9\, t00^8\Big) - \frac{1}{16}\,($$

$$-256\, z - 4096\, P2 + 256\, tinfty_1\big)\, P1^2 - \frac{1}{16}\left(-\frac{1}{4}\,(128\, z + 2048\, P2\right.$$

$$\left.- 128\, tinfty_1\big)\, tinfty_3 + Q2\,\big((z + 16\, P2 - tinfty_1)\, Q2 - 32\, b0\big)\right) P1 - \frac{1}{256}\,(-16\, z$$

$$- 256\, P2 + 16\, tinfty_1\big)\, tinfty_3^2 - \frac{1}{8}\, Q1\,(z + 16\, P2 - tinfty_1)\, Q2 - z^2 - 32\, z\, P2$$

$$- 256\, P2^2 + 2\, Q1\, b0 + tinfty_1^2 + O\left(\frac{1}{\lambda^2}\right)$$

The correspondence is given by

```
Q01:=-t00/Q2;
P01:=(1/2)*Q2+256*P1/Q2+16*tinfty[3]/Q2;
Qinfty[0]:=(1/32)*Q2*P1-8*P1^2/Q2-P1*tinfty[3]/Q2+(1/16)*Q1-tinfty[3]^2/(32*Q2)+16*P2/Q2+
tinfty[1]/Q2;
Pinfty[0]:=(1/64)*Q2^2-4096*P1^2/Q2^2-512*P1*tinfty[3]/Q2^2+16*Q1/Q2-16*tinfty[3]^2/Q2^2;
```

The inverse change is

```
P1:=-P01*t00/(256*Q01)-(1/16)*tinfty[3]-t00^2/(512*Q01^2);
P2:= -P01^2*t00^2/(131072*Q01^2)-(1/16)*tinfty[1]-Qinfty0*t00/(16*Q01)-t00^2*Pinfty0/(4096*
Q01^2)+t00^2*tinfty[3]/(8192*Q01^2)+3*t00^4/(524288*Q01^4);
Q1:=-P01^2*t00/(256*Q01)-P01*t00^2/(256*Q01^2)-t00*Pinfty0/(16*Q01);
Q2:=-t00/Q01;
```

```
> Q01function:=unapply(-t00/Q2,Q1,Q2,P1,P2);
  P01function:=unapply((1/2)*Q2+256*P1/Q2+16*tinfty[3]/Q2,Q1,Q2,
  P1,P2);
  Qinfty0function:=unapply( (1/32)*Q2*P1-8*P1^2/Q2-P1*tinfty[3]
  /Q2+(1/16)*Q1-tinfty[3]^2/(32*Q2)+16*P2/Q2+tinfty[1]/Q2,Q1,Q2,
  P1,P2) ;
  Pinfty0function:=unapply( (1/64)*Q2^2-4096*P1^2/Q2^2-512*P1*
  tinfty[3]/Q2^2+16*Q1/Q2-16*tinfty[3]^2/Q2^2,Q1,Q2,P1,P2);

  P1function:=unapply(-P01bis*t00/(256*Q01bis)-(1/16)*tinfty[3]-
  t00^2/(512*Q01bis^2), Q01bis,Qinfty0bis, P01bis,Pinfty0bis);
  P2function:=unapply( -P01bis^2*t00^2/(131072*Q01bis^2)-(1/16)*
  tinfty[1]-Qinfty0bis*t00/(16*Q01bis)-t00^2*Pinfty0bis/(4096*
  Q01bis^2)+t00^2*tinfty[3]/(8192*Q01bis^2)+3*t00^4/(524288*
  Q01bis^4), Q01bis,Qinfty0bis, P01bis,Pinfty0bis);
  Q1function:=unapply(-P01bis^2*t00/(256*Q01bis)-P01bis*t00^2/
  (256*Q01bis^2)-t00*Pinfty0bis/(16*Q01bis), Q01bis,Qinfty0bis,
  P01bis,Pinfty0bis);
  Q2function:=unapply( -t00/Q01bis, Q01bis,Qinfty0bis, P01bis,
  Pinfty0bis);
```

(2.3)

$$Q01function := (Q1, Q2, P1, P2) \to -\frac{t00}{Q2} \tag{2.3}$$

$$P01function := (Q1, Q2, P1, P2) \to \frac{1}{2}\, Q2 + \frac{256\, P1}{Q2} + \frac{16\, tinfty_3}{Q2}$$

$$Qinfty0function := (Q1, Q2, P1, P2) \to \frac{1}{32}\, Q2\, P1 - \frac{8\, P1^2}{Q2} - \frac{P1\, tinfty_3}{Q2} + \frac{1}{16}\, Q1 - \frac{1}{32}\, \frac{tinfty_3^2}{Q2} + \frac{16\, P2}{Q2} + \frac{tinfty_1}{Q2}$$

$$Pinfty0function := (Q1, Q2, P1, P2) \to \frac{1}{64}\, Q2^2 - \frac{4096\, P1^2}{Q2^2} - \frac{512\, P1\, tinfty_3}{Q2^2} + \frac{16\, Q1}{Q2} - \frac{16\, tinfty_3^2}{Q2^2}$$

$$P1function := (Q01bis, Qinfty0bis, P01bis, Pinfty0bis) \to -\frac{1}{256}\, \frac{P01bis\, t00}{Q01bis} - \frac{1}{16}\, tinfty_3 - \frac{1}{512}\, \frac{t00^2}{Q01bis^2}$$

$$P2function := (Q01bis, Qinfty0bis, P01bis, Pinfty0bis) \to -\frac{1}{131072}\, \frac{P01bis^2\, t00^2}{Q01bis^2} - \frac{1}{16}\, tinfty_1 - \frac{1}{16}\, \frac{Qinfty0bis\, t00}{Q01bis} - \frac{1}{4096}\, \frac{t00^2\, Pinfty0bis}{Q01bis^2} + \frac{1}{8192}\, \frac{t00^2\, tinfty_3}{Q01bis^2} + \frac{3}{524288}\, \frac{t00^4}{Q01bis^4}$$

$$Q1function := (Q01bis, Qinfty0bis, P01bis, Pinfty0bis) \to -\frac{1}{256}\, \frac{P01bis^2\, t00}{Q01bis} - \frac{1}{256}\, \frac{P01bis\, t00^2}{Q01bis^2} - \frac{1}{16}\, \frac{t00\, Pinfty0bis}{Q01bis}$$

$$Q2function := (Q01bis, Qinfty0bis, P01bis, Pinfty0bis) \to -\frac{t00}{Q01bis}$$

```
> b0:=-t00;
  z:=tinfty[1];
  simplify(series(simplify(L11-LMazzocco11),lambda));
  series(simplify(L12-LMazzocco12),lambda);
  series(simplify(L21-LMazzocco21),lambda):
  series(simplify(L12-L21-(LMazzocco12-LMazzocco21)),lambda);
```

$$b0 := -t00 \tag{2.4}$$

$$z := tinfty_1$$

$$\frac{1}{32768}\, \frac{1}{Q01^4} \left(\left(1024\, P1\, Q2^2 - 262144\, P1^2 - 32768\, P1\, tinfty_3 + 2048\, Q1\, Q2 - 1024\, tinfty_3^2 \right) Q01^4 - 4\, t00^2 \left(P01^2 + 32\, Pinfty_0 - 16\, tinfty_3 \right) Q01^2 + 3\, t00^4 \right)$$

$$+\frac{1}{32}\,\frac{QO1^2\,Q2^2-t00^2}{QO1^2}\,\lambda^2$$

$$\frac{1}{512}\,\frac{1}{QO1^3}\left(\left(4096\,P1^2+\left(-16\,Q2^2+512\,tinfty_3\right)P1+16\,tinfty_3^2-32\,Q2\,Q1\right.\right.$$
$$\left.-512\,tinfty_1-8192\,P2\right)QO1^3-512\,Qinfty_0\,t00\,QO1^2\Big)+\frac{1}{512}\,\frac{1}{QO1^3}\Big(($$
$$-512\,P1\,Q2-512\,Q1)\,QO1^3-512\,t00\left(\frac{1}{256}\,PO1^2-\frac{1}{16}\,tinfty_3+\frac{1}{16}\,Pinfty_0\right)QO1^2$$
$$\left.+t00^3\right)\lambda+\frac{1}{512}\,\frac{\left(-16\,Q2^2-8192\,P1-512\,tinfty_3\right)QO1^3-32\,t00\,PO1\,QO1^2}{QO1^3}\,\lambda^2$$
$$+\frac{1}{512}\,\frac{-512\,QO1^3\,Q2-512\,QO1^2\,t00}{QO1^3}\,\lambda^3$$

$$\frac{1}{16}\,\frac{1}{QO1}\left(\left(256\,P1^2+\left(-Q2^2+32\,tinfty_3\right)P1+tinfty_3^2-2\,Q2\,Q1-32\,tinfty_1\right.\right.$$
$$\left.-512\,P2\right)QO1-32\,Qinfty_0\,t00\Big)$$
$$+\frac{1}{16}\,\frac{\left(-Q2^2-512\,P1-32\,tinfty_3\right)QO1-2\,t00\,PO1}{QO1}\,\lambda^2$$

```
> LQ1function:= unapply( diff(Q1function(Q01,Qinfty[0],P01,Pinfty
  [0]),Q01)*LQ01+diff(Q1function(Q01,Qinfty[0],P01,Pinfty[0]),
  Qinfty[0])*LQinfty0+ diff(Q1function(Q01,Qinfty[0],P01,Pinfty
  [0]),P01)*LP01+diff(Q1function(Q01,Qinfty[0],P01,Pinfty[0]),
  Pinfty[0])*LPinfty0 +  diff(Q1function(Q01,Qinfty[0],P01,Pinfty
  [0]),tinfty[1])*alpha[1]+diff(Q1function(Q01,Qinfty[0],P01,
  Pinfty[0]),tinfty[3])*alpha[3],Q01,P01,Qinfty[0],Pinfty[0]);

  LQ2function:= unapply(diff(Q2function(Q01,Qinfty[0],P01,Pinfty
  [0]),Q01)*LQ01+diff(Q2function(Q01,Qinfty[0],P01,Pinfty[0]),
  Qinfty[0])*LQinfty0+ diff(Q2function(Q01,Qinfty[0],P01,Pinfty
  [0]),P01)*LP01+diff(Q2function(Q01,Qinfty[0],P01,Pinfty[0]),
  Pinfty[0])*LPinfty0 +  diff(Q2function(Q01,Qinfty[0],P01,Pinfty
  [0]),tinfty[1])*alpha[1]+diff(Q2function(Q01,Qinfty[0],P01,
  Pinfty[0]),tinfty[3])*alpha[3],Q01,P01,Qinfty[0],Pinfty[0]);

  LP1function:= unapply(diff(P1function(Q01,Qinfty[0],P01,Pinfty
  [0]),Q01)*LQ01+diff(P1function(Q01,Qinfty[0],P01,Pinfty[0]),
  Qinfty[0])*LQinfty0+ diff(P1function(Q01,Qinfty[0],P01,Pinfty
  [0]),P01)*LP01+diff(P1function(Q01,Qinfty[0],P01,Pinfty[0]),
  Pinfty[0])*LPinfty0 +  diff(P1function(Q01,Qinfty[0],P01,Pinfty
  [0]),tinfty[1])*alpha[1]+diff(P1function(Q01,Qinfty[0],P01,
  Pinfty[0]),tinfty[3])*alpha[3],Q01,P01,Qinfty[0],Pinfty[0]);

  LP2function:= unapply(diff(P2function(Q01,Qinfty[0],P01,Pinfty
```

```
[0]),Q01)*LQ01+diff(P2function(Q01,Qinfty[0],P01,Pinfty[0]),
Qinfty[0])*LQinfty0+ diff(P2function(Q01,Qinfty[0],P01,Pinfty
[0]),P01)*LP01+diff(P2function(Q01,Qinfty[0],P01,Pinfty[0]),
Pinfty[0])*LPinfty0 +  diff(P2function(Q01,Qinfty[0],P01,Pinfty
[0]),tinfty[1])*alpha[1]+diff(P2function(Q01,Qinfty[0],P01,
Pinfty[0]),tinfty[3])*alpha[3],Q01,P01,Qinfty[0],Pinfty[0]);
```

$$LQ1function := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \left(\frac{1}{256} \frac{P01^2\, t00}{Q01^2} + \frac{1}{128} \frac{P01\, t00^2}{Q01^3} \right. \tag{2.5}$$

$$+ \frac{1}{16} \frac{t00\, Pinfty\_0}{Q01^2} \Bigg) \left(\frac{1}{8} Q01\, P01\, \alpha_1 + \frac{2}{3} Q01\, \alpha_3\, Qinfty\_0 \right.$$

$$\left. + \frac{1}{384} Q01\, \alpha_3\, tinfty_3\, P01 \right) + \left(- \frac{1}{128} \frac{P01\, t00}{Q01} \right.$$

$$\left. - \frac{1}{256} \frac{t00^2}{Q01^2} \right) \left(\frac{2}{3} Qinfty\_0\, \alpha_3\, P01 - \frac{32}{3} Q01\, \alpha_3 - 2\, \alpha_1\, Pinfty\_0 \right.$$

$$- \frac{1}{24} Pinfty\_0\, \alpha_3\, tinfty_3 - \frac{2}{3} tinfty_1\, \alpha_3 - \frac{1}{12288} \frac{P01^2\, t00^2\, \alpha_3}{Q01^2}$$

$$\left. - \frac{1}{384} \frac{t00^2\, Pinfty\_0\, \alpha_3}{Q01^2} + \frac{1}{768} \frac{t00^2\, \alpha_3\, tinfty_3}{Q01^2} + \frac{1}{16384} \frac{t00^4\, \alpha_3}{Q01^4} \right)$$

$$- \frac{1}{16} \frac{1}{Q01} \left(t00 \left(- \frac{1}{24} P01^2\, Qinfty\_0\, \alpha_3 + \frac{4}{3} Q01\, P01\, \alpha_3 + \frac{2}{3} Pinfty\_0\, Qinfty\_0\, \alpha_3 \right. \right.$$

$$- 32\, Qinfty\_0\, \alpha_1 - \frac{2}{3} \alpha_3\, Qinfty\_0\, tinfty_3 + \frac{1}{128} P01^3\, \alpha_1 + \frac{1}{6144} \alpha_3\, tinfty_3\, P01^3$$

$$+ \frac{3}{8} P01\, Pinfty\_0\, \alpha_1 + \frac{1}{128} P01\, Pinfty\_0\, \alpha_3\, tinfty_3 - \frac{1}{8} P01\, \alpha_1\, tinfty_3$$

$$+ \frac{1}{24} P01\, \alpha_3\, tinfty_1 - \frac{1}{384} P01\, \alpha_3\, tinfty_3^2 + \frac{1}{3} \alpha_3 - \frac{1}{24} \frac{t00^2\, Qinfty\_0\, \alpha_3}{Q01^2}$$

$$+ \frac{1}{196608} \frac{P01^3\, t00^2\, \alpha_3}{Q01^2} + \frac{1}{6144} \frac{P01\, t00^2\, Pinfty\_0\, \alpha_3}{Q01^2} - \frac{3}{256} \frac{P01\, t00^2\, \alpha_1}{Q01^2}$$

$$\left. \left. - \frac{1}{3072} \frac{P01\, t00^2\, \alpha_3\, tinfty_3}{Q01^2} - \frac{1}{262144} \frac{P01\, t00^4\, \alpha_3}{Q01^4} \right) \right)$$

$$LQ2function := (Q01, P01, Qinfty\_0, Pinfty\_0)$$

$$\rightarrow \frac{t00 \left(\frac{1}{8} Q01\, P01\, \alpha_1 + \frac{2}{3} Q01\, \alpha_3\, Qinfty\_0 + \frac{1}{384} Q01\, \alpha_3\, tinfty_3\, P01 \right)}{Q01^2}$$

$$LP1function := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \left(\frac{1}{256} \frac{P01\, t00}{Q01^2} \right.$$

$$\left. + \frac{1}{256} \frac{t00^2}{Q01^3} \right) \left(\frac{1}{8} Q01\, P01\, \alpha_1 + \frac{2}{3} Q01\, \alpha_3\, Qinfty\_0 + \frac{1}{384} Q01\, \alpha_3\, tinfty_3\, P01 \right)$$

$$-\frac{1}{256}\,\frac{1}{Q01}\left(t00\left(\frac{2}{3}\,Qinfty\_0\,\alpha_3\,P01-\frac{32}{3}\,Q01\,\alpha_3-2\,\alpha_1\,Pinfty\_0\right.\right.$$
$$-\frac{1}{24}\,Pinfty\_0\,\alpha_3\,tinfty_3-\frac{2}{3}\,tinfty_1\,\alpha_3-\frac{1}{12288}\,\frac{P01^2\,t00^2\,\alpha_3}{Q01^2}$$
$$\left.\left.-\frac{1}{384}\,\frac{t00^2\,Pinfty\_0\,\alpha_3}{Q01^2}+\frac{1}{768}\,\frac{t00^2\,\alpha_3\,tinfty_3}{Q01^2}+\frac{1}{16384}\,\frac{t00^4\,\alpha_3}{Q01^4}\right)\right)-\frac{1}{16}\,\alpha_3$$

$$LP2function:=(Q01,P01,Qinfty\_0,Pinfty\_0)\to\left(\frac{1}{65536}\,\frac{P01^2\,t00^2}{Q01^3}\right.$$
$$+\frac{1}{16}\,\frac{Qinfty\_0\,t00}{Q01^2}+\frac{1}{2048}\,\frac{t00^2\,Pinfty\_0}{Q01^3}-\frac{1}{4096}\,\frac{t00^2\,tinfty_3}{Q01^3}$$
$$\left.-\frac{3}{131072}\,\frac{t00^4}{Q01^5}\right)\left(\frac{1}{8}\,Q01\,P01\,\alpha_1+\frac{2}{3}\,Q01\,\alpha_3\,Qinfty\_0\right.$$
$$\left.+\frac{1}{384}\,Q01\,\alpha_3\,tinfty_3\,P01\right)-\frac{1}{16}\,\frac{1}{Q01}\left(t00\left(\frac{2}{3}\,Qinfty\_0^2\,\alpha_3\right.\right.$$
$$+\frac{1}{8}\,P01\,Qinfty\_0\,\alpha_1+\frac{1}{384}\,P01\,Qinfty\_0\,\alpha_3\,tinfty_3-2\,Q01\,\alpha_1$$
$$-\frac{1}{6144}\,P01^2\,tinfty_1\,\alpha_3-\frac{1}{384}\,\alpha_3\,Pinfty\_0\,tinfty_1-\frac{1}{8}\,\alpha_1\,tinfty_1-\frac{1}{768}\,P01\,\alpha_3$$
$$+\frac{1}{768}\,\frac{t00^2\,\alpha_3}{Q01}-\frac{1}{50331648}\,\frac{P01^4\,t00^2\,\alpha_3}{Q01^2}-\frac{1}{1048576}\,\frac{P01^2\,t00^2\,Pinfty\_0\,\alpha_3}{Q01^2}$$
$$-\frac{1}{65536}\,\frac{P01^2\,t00^2\,\alpha_1}{Q01^2}+\frac{1}{3145728}\,\frac{P01^2\,t00^2\,\alpha_3\,tinfty_3}{Q01^2}$$
$$-\frac{1}{98304}\,\frac{t00^2\,Pinfty\_0^2\,\alpha_3}{Q01^2}-\frac{1}{2048}\,\frac{t00^2\,Pinfty\_0\,\alpha_1}{Q01^2}$$
$$+\frac{1}{196608}\,\frac{t00^2\,Pinfty\_0\,\alpha_3\,tinfty_3}{Q01^2}+\frac{1}{4096}\,\frac{t00^2\,\alpha_1\,tinfty_3}{Q01^2}+\frac{1}{12288}\,\frac{t00^2\,\alpha_3\,tinfty_1}{Q01^2}$$
$$+\frac{5}{201326592}\,\frac{P01^2\,t00^4\,\alpha_3}{Q01^4}+\frac{7}{12582912}\,\frac{t00^4\,Pinfty\_0\,\alpha_3}{Q01^4}+\frac{3}{262144}\,\frac{t00^4\,\alpha_1}{Q01^4}$$
$$\left.\left.-\frac{1}{6291456}\,\frac{t00^4\,\alpha_3\,tinfty_3}{Q01^4}-\frac{1}{134217728}\,\frac{t00^6\,\alpha_3}{Q01^6}\right)\right)$$
$$-\frac{1}{65536}\,\frac{1}{Q01^2}\left(P01\,t00^2\left(\frac{2}{3}\,Qinfty\_0\,\alpha_3\,P01-\frac{32}{3}\,Q01\,\alpha_3-2\,\alpha_1\,Pinfty\_0\right.\right.$$
$$\left.\left.-\frac{1}{24}\,Pinfty\_0\,\alpha_3\,tinfty_3-\frac{2}{3}\,tinfty_1\,\alpha_3-\frac{1}{12288}\,\frac{P01^2\,t00^2\,\alpha_3}{Q01^2}\right.\right.$$

$$-\frac{1}{384}\,\frac{t00^2\,Pinfty\_0\,\alpha_3}{Q01^2}+\frac{1}{768}\,\frac{t00^2\,\alpha_3\,tinfty_3}{Q01^2}+\frac{1}{16384}\,\frac{t00^4\,\alpha_3}{Q01^4}\Bigg)\Bigg)$$

$$-\frac{1}{4096}\,\frac{1}{Q01^2}\Bigg(t00^2\Bigg(-\frac{1}{24}\,P01^2\,Qinfty\_0\,\alpha_3+\frac{4}{3}\,Q01\,P01\,\alpha_3$$

$$+\frac{2}{3}\,Pinfty\_0\,Qinfty\_0\,\alpha_3-32\,Qinfty\_0\,\alpha_1-\frac{2}{3}\,\alpha_3\,Qinfty\_0\,tinfty_3+\frac{1}{128}\,P01^3\,\alpha_1$$

$$+\frac{1}{6144}\,\alpha_3\,tinfty_3\,P01^3+\frac{3}{8}\,P01\,Pinfty\_0\,\alpha_1+\frac{1}{128}\,P01\,Pinfty\_0\,\alpha_3\,tinfty_3$$

$$-\frac{1}{8}\,P01\,\alpha_1\,tinfty_3+\frac{1}{24}\,P01\,\alpha_3\,tinfty_1-\frac{1}{384}\,P01\,\alpha_3\,tinfty_3^2+\frac{1}{3}\,\alpha_3$$

$$-\frac{1}{24}\,\frac{t00^2\,Qinfty\_0\,\alpha_3}{Q01^2}+\frac{1}{196608}\,\frac{P01^3\,t00^2\,\alpha_3}{Q01^2}+\frac{1}{6144}\,\frac{P01\,t00^2\,Pinfty\_0\,\alpha_3}{Q01^2}$$

$$-\frac{3}{256}\,\frac{P01\,t00^2\,\alpha_1}{Q01^2}-\frac{1}{3072}\,\frac{P01\,t00^2\,\alpha_3\,tinfty_3}{Q01^2}-\frac{1}{262144}\,\frac{P01\,t00^4\,\alpha_3}{Q01^4}\Bigg)\Bigg)$$

$$-\frac{1}{16}\,\alpha_1+\frac{1}{8192}\,\frac{t00^2\,\alpha_3}{Q01^2}$$

```
> LQ1:=simplify(LQ1function(Q01function(Q1,Q2,P1,P2),P01function
  (Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,
  Q2,P1,P2)));
  LQ2:=simplify(LQ2function(Q01function(Q1,Q2,P1,P2),P01function
  (Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,
  Q2,P1,P2)));
  LP1:=simplify(LP1function(Q01function(Q1,Q2,P1,P2),P01function
  (Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,
  Q2,P1,P2)));
  LP2:=simplify(LP2function(Q01function(Q1,Q2,P1,P2),P01function
  (Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,
  Q2,P1,P2)));
```

$$LQ1 := \frac{1}{768}\,\Big(tinfty_3^3+64\,P1\,tinfty_3^2+\Big(768\,P1^2-32\,tinfty_1\Big)\,tinfty_3+8192\,P1\,P2 \qquad \textbf{(2.6)}$$

$$-16\,Q2\,(P2\,Q2+2\,t00-1)\Big)\,\alpha_3+48\,\alpha_1\,\Big(P1^2+\frac{1}{12}\,P1\,tinfty_3+\frac{1}{768}\,tinfty_3^2$$

$$-\frac{2}{3}\,P2-\frac{1}{24}\,tinfty_1\Big)$$

$$LQ2 := \frac{1}{768}\,\Big(\big(-16\,P1-tinfty_3\big)\,Q2^2-32\,Q2\,Q1+4096\,P1^2-16\,tinfty_3^2-8192\,P2$$

$$-512\,tinfty_1\Big)\,\alpha_3-32\,\alpha_1\,\Big(\frac{1}{512}\,Q2^2+P1+\frac{1}{16}\,tinfty_3\Big)$$

$$LP1 := \frac{1}{384}\left(16\,P2\,Q2 - Q1\,tinfty_3 + 16\,t00 - 24\right)\alpha_3 - \frac{1}{8}\,Q1\,\alpha_1$$

$$LP2 := \frac{1}{768}\left(\left(32\,P1\,Q2 + 2\,Q2\,tinfty_3 + 32\,Q1\right)P2 + 32\left(t00 - \frac{1}{2}\right)\left(P1 + \frac{1}{16}\,tinfty_3\right)\right)\alpha_3 + \frac{1}{8}\left(Q2\,P2 + t00 - \frac{1}{2}\right)\alpha_1$$

```
> HamQP:=unapply( -(1/384)*alpha[3]*(16*Q2*P2*Q1-(1/2)*Q1^2*
  tinfty[3]+16*t00*Q1-24*Q1)+(1/16)*Q1^2*alpha[1]
  -(1/24)*((P1+(1/16)*tinfty[3])*alpha[3]+3*alpha[1])*((1/2)*
  Q2^2*P2+t00*Q2-(1/2)*Q2)
  +(-(1/48)*alpha[3]*(-256*P1^2*P2+P2*tinfty[3]^2+256*P2^2+32*P2*
  tinfty[1])-32*alpha[1]*(P1+(1/16)*tinfty[3])*P2)+(1/768)*(alpha
  [3]*tinfty[3]+48*alpha[1])*(256*P1^3+32*P1^2*tinfty[3]+P1*
  tinfty[3]^2-32*P1*tinfty[1]),
  alpha[1],alpha[3]);
  simplify(diff(HamQP(alpha[1],alpha[3]),P1)- LQ1);
  simplify(diff(HamQP(alpha[1],alpha[3]),P2)- LQ2);

  simplify(diff(HamQP(alpha[1],alpha[3]),Q1)+ LP1);
  simplify(diff(HamQP(alpha[1],alpha[3]),Q2)+ LP2);
```

$$HamQP := (alpha\_1, alpha\_3) \to -\frac{1}{384}\,alpha\_3\left(16\,Q2\,Q1\,P2 - \frac{1}{2}\,Q1^2\,tinfty_3 + 16\,t00\,Q1 - 24\,Q1\right) + \frac{1}{16}\,Q1^2\,alpha\_1 - \frac{1}{24}\left(\left(P1 + \frac{1}{16}\,tinfty_3\right)alpha\_3 + 3\,alpha\_1\right)\left(\frac{1}{2}\,Q2^2\,P2 + Q2\,t00 - \frac{1}{2}\,Q2\right) - \frac{1}{48}\,alpha\_3\left(-256\,P1^2\,P2 + P2\,tinfty_3^2 + 256\,P2^2 + 32\,P2\,tinfty_1\right) - 32\,alpha\_1\left(P1 + \frac{1}{16}\,tinfty_3\right)P2 + \frac{1}{768}\left(alpha\_3\,tinfty_3 + 48\,alpha\_1\right)\left(256\,P1^3 + 32\,P1^2\,tinfty_3 + P1\,tinfty_3^2 - 32\,P1\,tinfty_1\right) \tag{2.7}$$

$$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

```
> simplify(HamQP(1,0)):
  HamQPtinfty1:=-(1/16)*Q2^2*P2-32*P1*P2-2*tinfty[3]*P2+2*tinfty
  [3]*P1^2+ 16*P1^3 +(tinfty[3]^2/16-2*tinfty[1])*P1+(1/16)*Q1^2+
  (1/16)*(1-2*t00)*Q2;
  simplify(HamQP(1,0)-HamQPtinfty1);
```

$$HamQPtinfty1 := -\frac{1}{16}\,Q2^2\,P2 - 32\,P1\,P2 - 2\,tinfty_3\,P2 + 2\,P1^2\,tinfty_3 + 16\,P1^3 + \left(\frac{1}{16}\,tinfty_3^2 - 2\,tinfty_1\right)P1 + \frac{1}{16}\,Q1^2 + \frac{1}{16}\,(-2\,t00 + 1)\,Q2 \tag{2.8}$$

$$0$$

```
> simplify(HamQP(0,1)):
  HamQPtinfty3:=-(1/48)*P1*P2*Q2^2-(1/24)*Q1*Q2*P2-16/3*(P2^2)-
  (1/768)*tinfty[3]*Q2^2*P2+(1/3)*P1^3*tinfty[3]+ 16/3*P1^2*P2+
  (1/24)*tinfty[3]^2*P1^2-((tinfty[3]^2)/48+(2*tinfty[1])/3)*P2+
  (1/768)*(16-32*t00)*Q2*P1+1/24*tinfty[3]*(1/32*tinfty[3]^2-
  tinfty[1])*P1+(1/768)*Q1^2*tinfty[3]+1/48*(3-2*t00)*Q1+(1/768)*
  tinfty[3]*(1-2*t00)*Q2;
  simplify(HamQP(0,1)-HamQPtinfty3);
```

$$HamQPtinfty3 := -\frac{1}{48} P1\,P2\,Q2^2 - \frac{1}{24} Q2\,Q1\,P2 - \frac{16}{3} P2^2 - \frac{1}{768} tinfty_3\,Q2^2\,P2 + \frac{1}{3} P1^3\,tinfty_3 + \frac{16}{3} P1^2\,P2 + \frac{1}{24} P1^2\,tinfty_3^2 - \left(\frac{1}{48} tinfty_3^2 + \frac{2}{3} tinfty_1\right) P2 + \frac{1}{768} (16 - 32\,t00)\,Q2\,P1 + \frac{1}{24} tinfty_3 \left(\frac{1}{32} tinfty_3^2 - tinfty_1\right) P1 + \frac{1}{768} Q1^2\,tinfty_3 + \frac{1}{48} (3 - 2\,t00)\,Q1 + \frac{1}{768} tinfty_3\,(-2\,t00 + 1)\,Q2 \qquad (2.9)$$

$$0$$

```
> s[1]:=-1/4*tinfty[3];
  tinfty[2*d+1]:=-4^d;
  b0:=-t00;
  z:=tinfty[1];
```

$$s_1 := -\frac{1}{4} tinfty_3 \qquad (2.10)$$

$$tinfty_5 := -16$$

$$b0 := -t00$$

$$z := tinfty_1$$

```
> A11function:=unapply(simplify(A11),alpha[1],alpha[3]):
  A12function:=unapply(simplify(A12),alpha[1],alpha[3]):
  A21function:=unapply(simplify(A21),alpha[1],alpha[3]):
  A11function10:=unapply(A11function(1,0) ,Q01,P01,Qinfty[0],
  Pinfty[0]);
  A12function10:=unapply(A12function(1,0) ,Q01,P01,Qinfty[0],
  Pinfty[0]);
  A21function10:=unapply(A21function(1,0) ,Q01,P01,Qinfty[0],
  Pinfty[0]);

  A11function01:=unapply(expand(A11function(0,1)),Q01,P01,Qinfty
  [0],Pinfty[0]);
  A12function01:=unapply(expand(A12function(0,1)),Q01,P01,Qinfty
  [0],Pinfty[0]);
  A21function01:=unapply(expand(A21function(0,1)),Q01,P01,Qinfty
  [0],Pinfty[0]);
```

$$A11function10 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow -\lambda \qquad (2.11)$$

$$A12function10 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \frac{1}{16}\ \frac{t00}{Q01}$$

$$A21function10 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \frac{1}{16}\ \frac{t00}{Q01}$$

$$A11function01 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow -\frac{1}{3}\ \lambda^3 + \frac{1}{1536}\ \frac{\lambda\, t00^2}{Q01^2}$$

$$A12function01 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \frac{1}{12288}\ \frac{P01^2\, t00}{Q01} + \frac{1}{768}\ \frac{t00\, P01\, \lambda}{Q01} + \frac{1}{48}\ \frac{t00\, \lambda^2}{Q01} + \frac{1}{768}\ \frac{t00\, Pinfty\_0}{Q01} - \frac{1}{24576}\ \frac{t00^3}{Q01^3}$$

$$A21function01 := (Q01, P01, Qinfty\_0, Pinfty\_0) \rightarrow \frac{1}{12288}\ \frac{P01^2\, t00}{Q01} - \frac{1}{768}\ \frac{t00\, P01\, \lambda}{Q01} + \frac{1}{48}\ \frac{t00\, \lambda^2}{Q01} + \frac{1}{768}\ \frac{t00\, Pinfty\_0}{Q01} - \frac{1}{24576}\ \frac{t00^3}{Q01^3}$$

```
> A11function10(Q01function(Q1,Q2,P1,P2),P01function(Q1,Q2,P1,
  P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,P1,P2))
  ;
  A12function10(Q01function(Q1,Q2,P1,P2),P01function(Q1,Q2,P1,
  P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,P1,P2))
  ;
  A21function10(Q01function(Q1,Q2,P1,P2),P01function(Q1,Q2,P1,
  P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,P1,P2))
  ;
```

$$\begin{matrix} -\lambda \\ -\frac{1}{16}\ Q2 \\ -\frac{1}{16}\ Q2 \end{matrix} \qquad \textbf{(2.12)}$$

```
> simplify(A11function01(Q01function(Q1,Q2,P1,P2),P01function(Q1,
  Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,
  P1,P2)));
  simplify(A12function01(Q01function(Q1,Q2,P1,P2),P01function(Q1,
  Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,
  P1,P2)));
  simplify(A21function01(Q01function(Q1,Q2,P1,P2),P01function(Q1,
  Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2), Pinfty0function(Q1,Q2,
  P1,P2)));
```

$$-\frac{1}{3}\ \lambda^3 + \frac{1}{1536}\ \lambda\, Q2^2 \qquad \textbf{(2.13)}$$

$$-\frac{1}{48}\ Q2\, \lambda^2 + \frac{1}{1536}\ \left(-Q2^2 - 512\, P1 - 32\, tinfty_3\right) \lambda + \frac{1}{1536}\ \left(-32\, P1 - 2\, tinfty_3\right) Q2 - \frac{1}{48}\ Q1$$

$$-\frac{1}{48}\,Q2\,\lambda^2+\frac{1}{1536}\left(Q2^2+512\,P1+32\,tinfty_3\right)\lambda+\frac{1}{1536}\left(-32\,P1-2\,tinfty_3\right)Q2-\frac{1}{48}\,Q1$$

```
> L11function:=unapply(L11,Q01,P01,Qinfty[0],Pinfty[0]):
  L12function:=unapply(L12,Q01,P01,Qinfty[0],Pinfty[0]):
  L21function:=unapply(L21,Q01,P01,Qinfty[0],Pinfty[0]):

  L11new:=series( simplify(L11function(Q01function(Q1,Q2,P1,P2),
  P01function(Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2),
  Pinfty0function(Q1,Q2,P1,P2))) ,lambda=infinity);
  L12new:=series(simplify(L12function(Q01function(Q1,Q2,P1,P2),
  P01function(Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2),
  Pinfty0function(Q1,Q2,P1,P2))),lambda=infinity);
  L21new:=series(simplify(L21function(Q01function(Q1,Q2,P1,P2),
  P01function(Q1,Q2,P1,P2),Qinfty0function(Q1,Q2,P1,P2),
  Pinfty0function(Q1,Q2,P1,P2))),lambda=infinity);
```

$$L11new := 16\,\lambda^4+\left(-tinfty_3-\frac{1}{32}\,Q2^2\right)\lambda^2-\frac{1}{32}\,Q2^2\,P1-\frac{1}{16}\,Q2\,Q1+8\,P1^2+P1\,tinfty_3+\frac{1}{32}\,tinfty_3^2-tinfty_1 \tag{2.14}$$

$$L12new := Q2\,\lambda^3+\left(16\,P1+tinfty_3+\frac{1}{32}\,Q2^2\right)\lambda^2+(P1\,Q2+Q1)\,\lambda-8\,P1^2+\frac{1}{32}\left(Q2^2-32\,tinfty_3\right)P1+\frac{1}{16}\,Q2\,Q1-\frac{1}{32}\,tinfty_3^2+16\,P2+tinfty_1-\frac{t00}{\lambda}$$

$$L21new := Q2\,\lambda^3+\left(-16\,P1-tinfty_3-\frac{1}{32}\,Q2^2\right)\lambda^2+(P1\,Q2+Q1)\,\lambda+8\,P1^2+\frac{1}{32}\left(-Q2^2+32\,tinfty_3\right)P1+\frac{1}{32}\,tinfty_3^2-\frac{1}{16}\,Q2\,Q1-tinfty_1-16\,P2-\frac{t00}{\lambda}$$

Hamiltonian from Mazzocco paper

```
> h12:=-32*z*P2+256*P1^2*P2-256*P2^2-2*P2*Q1*Q2-P1*P2*Q2^2+2*
  alpha2*(P1*Q2+Q1)- s1*P2*(128*P1-16*s1);
  h22:=-32*z*P1+256*P1^3-512*P1*P2-P2*Q2^2+Q1^2+2*alpha2*Q2-s1*
  (128*P1^2-128*P2-16*s1*P1);
  deltah12:=-P1*Q2+Q1;
  deltah22:=-Q2;

  Ham1:=1/16*(-h22+Q2);
  Ham2:=expand(simplify(1/3*(1/4*(h12+deltah12)-1/16*s1*(h22+
  deltah22))));
```

$$h12 := -32\,P2\,tinfty_1+256\,P1^2\,P2-256\,P2^2-2\,Q2\,Q1\,P2-P1\,P2\,Q2^2+2\,\alpha2\,(P1\,Q2+Q1)-s1\,P2\,(128\,P1-16\,s1) \tag{2.15}$$

$$h22 := -32\,P1\,tinfty_1 + 256\,P1^3 - 512\,P1\,P2 - Q2^2\,P2 + Q1^2 + 2\,\alpha 2\,Q2 - s1\,(128\,P1^2 - 16\,P1\,s1 - 128\,P2)$$

$$deltah12 := -P1\,Q2 + Q1$$

$$deltah22 := -Q2$$

$$Ham1 := 2\,P1\,tinfty_1 - 16\,P1^3 + 32\,P1\,P2 + \frac{1}{16}\,Q2^2\,P2 - \frac{1}{16}\,Q1^2 - \frac{1}{8}\,\alpha 2\,Q2 + \frac{1}{16}\,s1\,(128\,P1^2 - 16\,P1\,s1 - 128\,P2) + \frac{1}{16}\,Q2$$

$$Ham2 := -\frac{8}{3}\,P2\,tinfty_1 + \frac{64}{3}\,P1^2\,P2 - \frac{64}{3}\,P2^2 - \frac{1}{6}\,Q2\,Q1\,P2 - \frac{1}{12}\,P1\,P2\,Q2^2 + \frac{1}{6}\,P1\,Q2\,\alpha 2 + \frac{1}{6}\,Q1\,\alpha 2 - \frac{4}{3}\,s1^2\,P2 - \frac{1}{12}\,Q2\,P1 + \frac{1}{12}\,Q1 - \frac{16}{3}\,P1^3\,s1 + \frac{8}{3}\,P1^2\,s1^2 - \frac{1}{3}\,P1\,s1^3 + \frac{1}{48}\,P2\,Q2^2\,s1 + \frac{2}{3}\,P1\,s1\,tinfty_1 - \frac{1}{48}\,Q1^2\,s1 - \frac{1}{24}\,Q2\,\alpha 2\,s1 + \frac{1}{48}\,Q2\,s1$$

```
> s1:=-tinfty[3]/4;

  simplify(Ham1+HamQPtinfty1);
  simplify(series(Ham2-4*HamQPtinfty3,Q2));
  alpha2:=1-t00;

  simplify(Ham1+HamQPtinfty1);
  simplify(series(Ham2-4*HamQPtinfty3,Q2));
  simplify(Ham1+HamQPtinfty1);
```

$$s1 := -\frac{1}{4}\,tinfty_3 \tag{2.16}$$

$$-\frac{1}{8}\,Q2\,(\alpha 2 - 1 + t00)$$

$$\frac{1}{6}\,Q1\,(\alpha 2 - 1 + t00) + \frac{1}{96}\,(16\,P1 + tinfty_3)\,(\alpha 2 - 1 + t00)\,Q2$$

$$\alpha 2 := 1 - t00$$

$$0$$

$$0$$

$$0$$