Calculus of variations and elliptic partial differential equations and systems

24 hours + classes and articles presentations

Petru Mironescu
Institut Camille Jordan
Université Claude Bernard Lyon 1
mironescu@math.univ-lyon1.fr
http://math.univ-lyon1.fr/~mironescu/

Description. This is an intermediate + course presenting some basic tools in the qualitative analysis, existence, and regularity theory for solutions of elliptic partial differential equations (PDEs) and, if time permits, systems. A first part, related to the direct method in the calculus of variations, goes beyond elliptic PDEs.

Prerequisites

1. Good knowledge of general measure theory and integration.
2. Reasonable knowledge of geometric aspects of the integration theory (Gauss-Ostrogradskii...) and of the local theory of submanifolds of $\mathbb{R}^n$.
3. Good knowledge of the basic results concerning the Laplace equation.

Syllabus

1. The direct method in the calculus of variations
   (a) Basic examples.
   (b) Notions of convexity.
   (c) Passing to the weak limits in nonlinear quantities. Compensation phenomena.
   (d) Gap phenomena.
   (e) Concentration-compactness.
2. Maximum principles and applications
   (a) Maximum principles for elliptic PDEs in non divergence and divergence form.
   (b) Iterative methods based on monotonicity (sub- and super-solutions).

    (c) Symmetry properties of solutions of semi-linear elliptic PDEs.

3. Other (non-)existence methods
    (a) Krasnoselskii's uniqueness result.
    (b) Pohozaev's identity.
    (c) Mountain pass solutions.
    (d) Other topological methods.
4. Regularity theory
    (a) Serrin's example.
    (b) Singular integrals.
    (c) L^p-theory for elliptic PDEs in non-divergence form.
    (d) A glimpse of the C^α-theory for elliptic PDEs in non-divergence form.
    (e) De Giorgi-Nash regularity theory for elliptic PDEs in divergence form.
    (f) Bootstrap. Regularity in the critical case.
    (g) A limiting case: Wente estimates. A glimpse of other compensation phenomena.
5. A glimpse of phase-transition problems
    (a) A glimpse of the BV space.
    (b) Abstract Γ-convergence.
    (c) The Modica-Mortola functional in the limit $\varepsilon \to 0$.
    (d) Vector-valued variants.