

Université Claude Bernard Lyon 1 & ENS de Lyon
2nd year Master program Advanced Mathematics
Calculus of variations and elliptic partial differential equations

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Description. This is an intermediate + course presenting some basic tools in the qualitative analysis, existence, and regularity theory for solutions of elliptic partial differential equations (PDEs). A first part, related to the direct method in the calculus of variations, goes beyond elliptic PDEs.

http://math.univ-lyon1.fr/~mironescu/enseignement/edp_MA2.html

Prerequisites

1. Good knowledge of general measure theory and integration.
2. Reasonable knowledge of geometric aspects of the integration theory (Gauss-Ostrogradskii...) and of the local theory of submanifolds of \mathbb{R}^n .
3. Good knowledge of the basic results concerning the Laplace equation.

Syllabus

1. The direct method in the calculus of variations
 - (a) Basic examples.
 - (b) Notions of convexity.
 - (c) Passing to the weak limits in nonlinear quantities. Compensation phenomena.
 - (d) Gap phenomena.
2. Maximum principles and applications
 - (a) Maximum principles for elliptic partial differential equations (PDEs) in non divergence and divergence form.
 - (b) Iterative methods based on monotonicity (sub- and supersolutions).
 - (c) Symmetry properties of solutions of semilinear elliptic PDEs.

- (d) Krasnoselskii's uniqueness result.
- 3. Regularity theory
 - (a) Serrin's example.
 - (b) Singular integrals.
 - (c) L^p -theory for elliptic equations in non-divergence form.
 - (d) A glimpse of the C^α -theory for elliptic equations in non-divergence form.
 - (e) De Giorgi-Nash regularity theory for elliptic equations in divergence form.
 - (f) Bootstrap. Regularity in the critical case.
 - (g) A limiting case : Wentz estimates. A glimpse of other compensation phenomena.
- 4. Other (non-)existence methods
 - (a) Concentration-compactness.
 - (b) Mountain pass solutions.
 - (c) Other topological methods.
 - (d) Pohozaev's identity.
- 5. A glimpse of phase-transition problems
 - (a) A glimpse of the BV space.
 - (b) Abstract Γ -convergence.
 - (c) The Modica-Mortola functional in the limit $\varepsilon \rightarrow 0$.
 - (d) Vector-valued variants.

Evaluations

1. Article presentation : 20 %.
2. Partial examination (2 hours) : 30 %.
3. Final examination (3 hours) : 50 %.

Schedule. All the courses are on the LyonTech-la Doua campus.

1. 2 hours courses on Fridays 2–4 PM : 15/09, 22/09, 29/09, 10/11, 17/11, 24/11.
2. 2 hours courses on Wednesdays 9–11 AM : 04/10, 11/10, 18/10, 25/10, 08/11, 22/11.

3. Office hours (1 h) : Fri 29/09 4–5 PM, Wed 11/10 11–12 AM, Wed 25/10 11–12 AM, Wed 08/11 11–12 AM, Fri 10/11 4–5 PM, Fri 24/11 4–5 PM.
4. Partial examination : Fri 27/10 2–4 PM.
5. Articles presentation : Fri 08/12 2–5 PM.
6. Final examination : Wed 13/12, 9–12 AM.