Anisotropic Quantum Hall Droplets

Blagoje Oblak

(Univ Lyon 1)

Based on arXiv 2301.01726 (in PRX since yesterday)
with Lapierre, Moosavi, Stéphan, Estienne

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Intro

MOTIVATION

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Emergent phases of condensed matter

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- **Topological insulators**

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  emergent topological field theories

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[Gromov+, Read+, Son+, Wiegmann+]

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Here focus on **Quantum Hall droplets**

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Here focus on **Quantum Hall droplets**

- Reaction to deformed confining potential ?

Quantum Hall droplets

confining potential

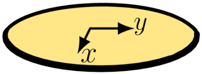
Quantum Hall droplets

Quantum Hall droplets

2D electron droplet

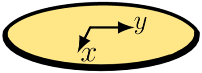
QUANTUM HALL DROPLETS

2D electron droplet



Quantum Hall droplets

2D electron droplet in strong magnetic field



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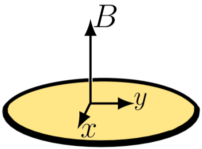

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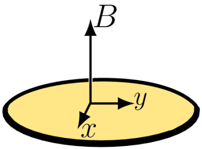
- Quantum Hall effect [since 1980]



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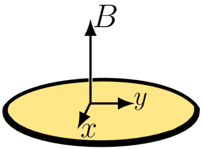
- Topological insulator [since 1980]



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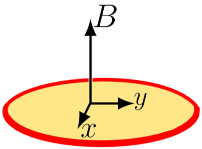

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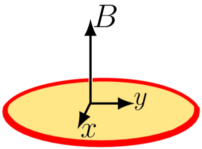

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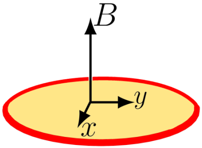

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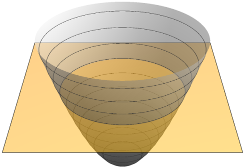

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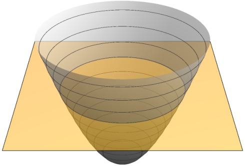

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[+ harmonic traps]

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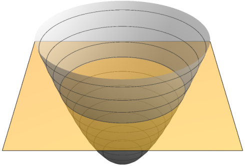

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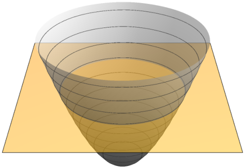

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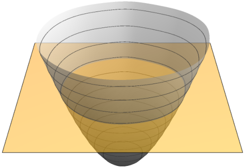

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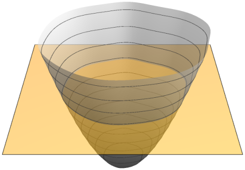

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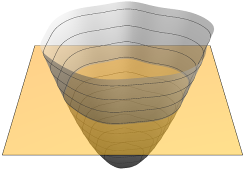

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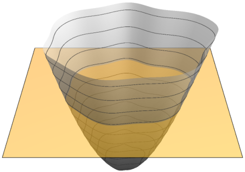

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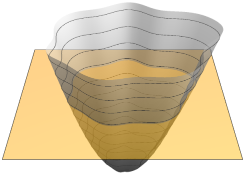

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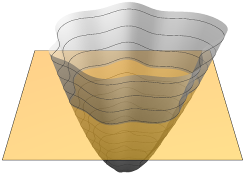

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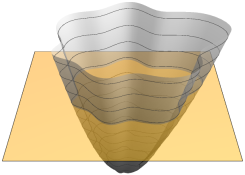

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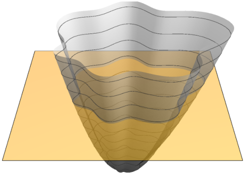

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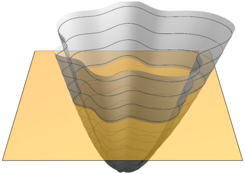

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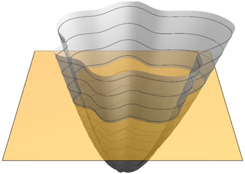

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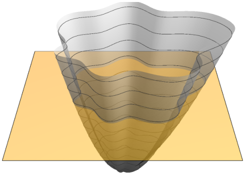

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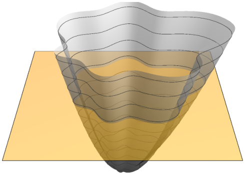

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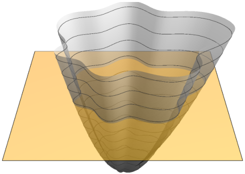

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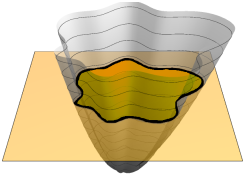

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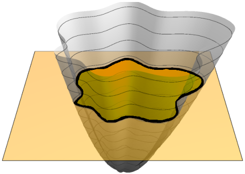

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But **anisotropic** are uncharted territory !

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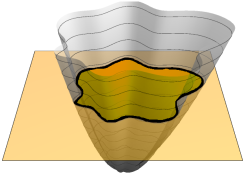

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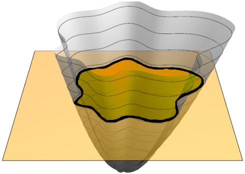

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- Signatures of **anisotropic edge modes** ?

THIS TALK IN A NUTSHELL

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QH droplets are incompressible

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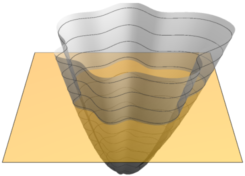

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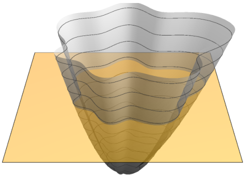

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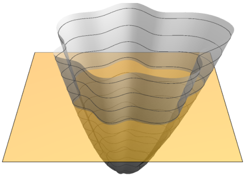

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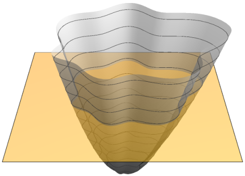

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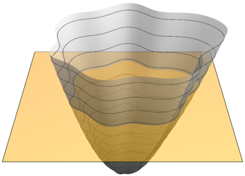

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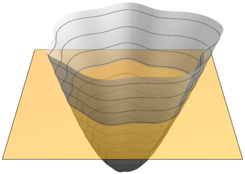

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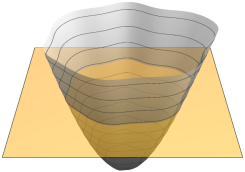

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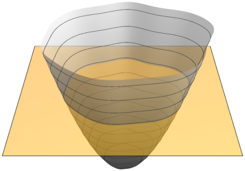

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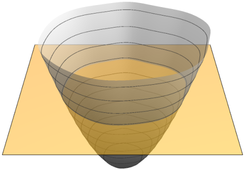

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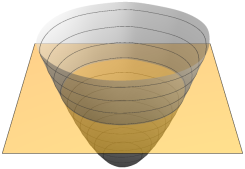

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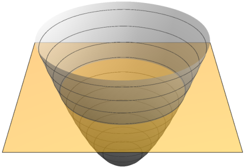

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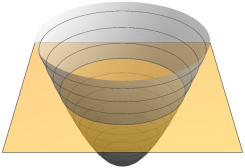

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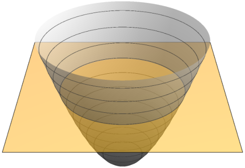

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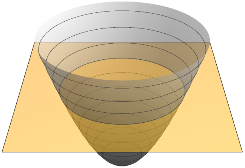

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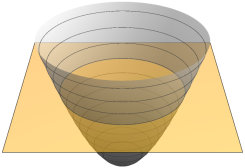

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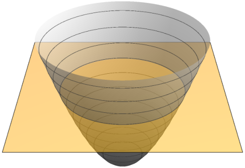
- Map isotropic $\rightarrow$ anisotropic states



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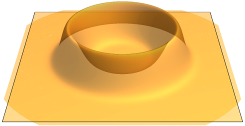

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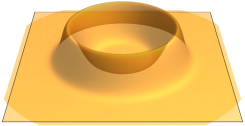

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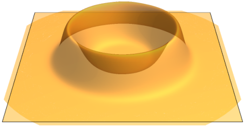

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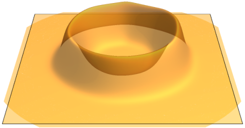

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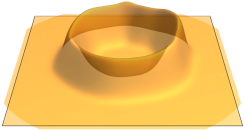

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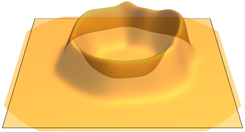

This talk in a nutshell

QH droplets are incompressible

- Use **area-preserving diffeo** to make $V(\mathbf{x})$ isotropic
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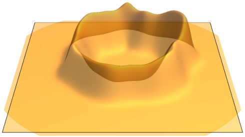

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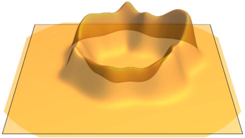

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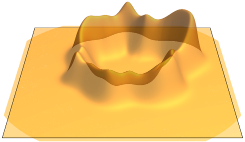

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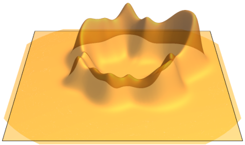

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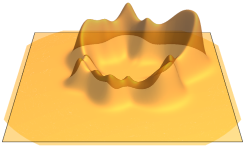

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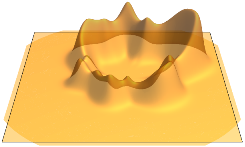

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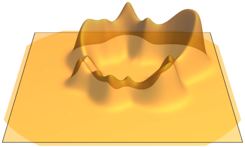

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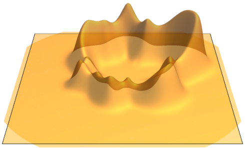

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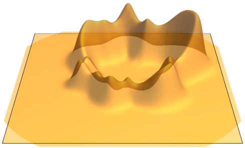
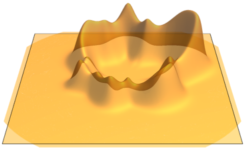


This talk in a nutshell

QH droplets are incompressible

- Use **area-preserving diffeo** to make $V(\mathbf{x})$ isotropic
- Map isotropic $\to$ anisotropic states
- Deduce many-body observables



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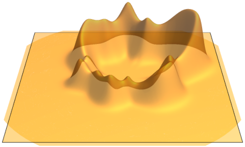

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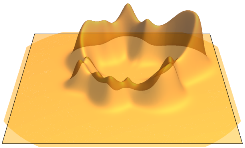

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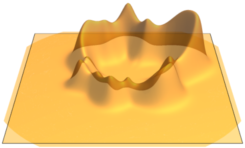

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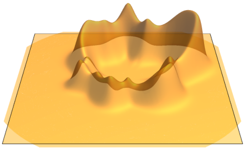
- Deduce many-body **density** **correlations** **edge modes** ...for **free electrons**



Plan

1. **Anisotropic wave functions**
2. Many-body observables
3. Probing anisotropy with microwaves

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1. Semiclassical states in lowest Landau level

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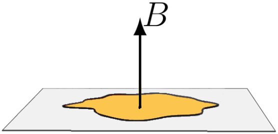
C. Energy spectrum

SETUP AND ASSUMPTIONS

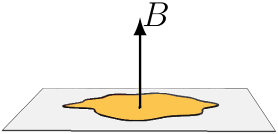
2D electrons + magnetic field



Setup and assumptions

2D electrons + magnetic field

- Hamiltonian $H = (\mathbf{p} - q\mathbf{A})^2 + V(\mathbf{x})$



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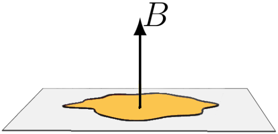

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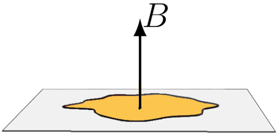

No exact solution

Setup and assumptions

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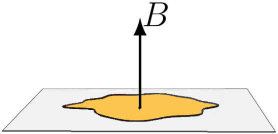

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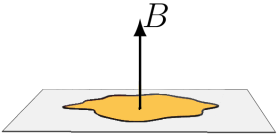

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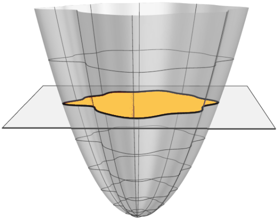

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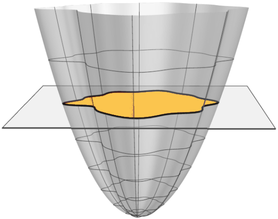

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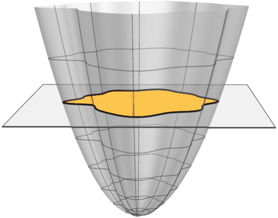

No exact solution ⇒ simplifying assumptions :

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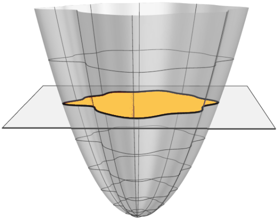
- Strong B

Setup and assumptions



2D electrons + magnetic field

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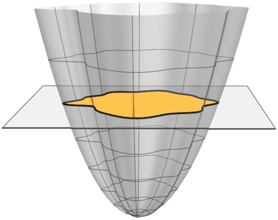

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  $\Rightarrow$ Project to **lowest Landau level**

Monotonic potentials

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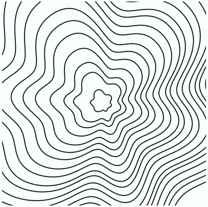

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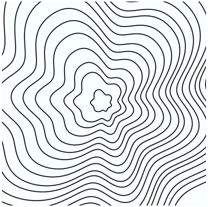

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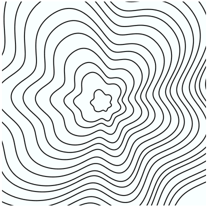

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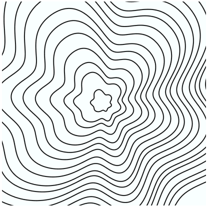

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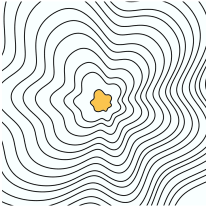

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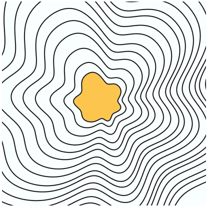

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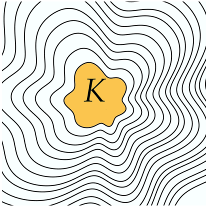

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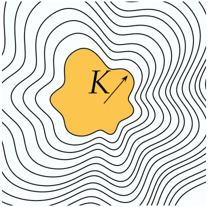

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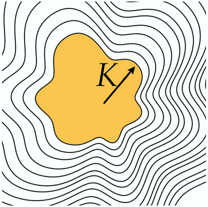

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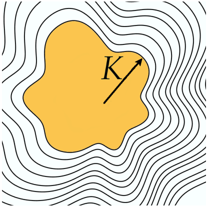

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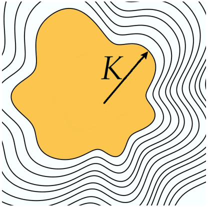

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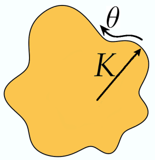

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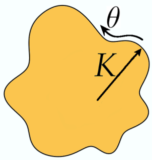

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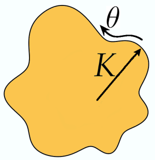

Example : Pick $s(\varphi) > 0$ and $V(\mathbf{x}) = \mathcal{V}\left(\frac{r^2}{s(\varphi)}\right)$

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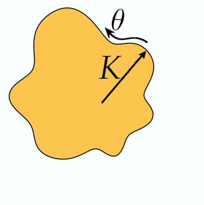

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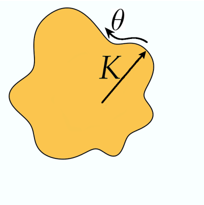

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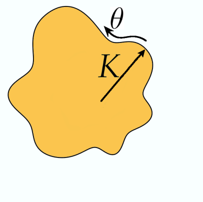

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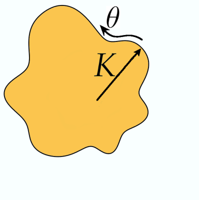

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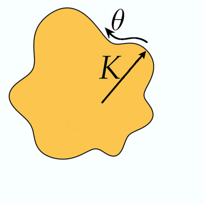

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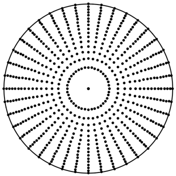

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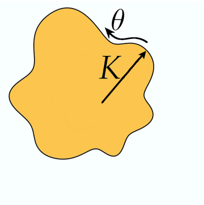

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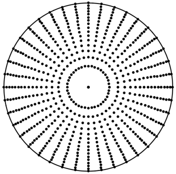

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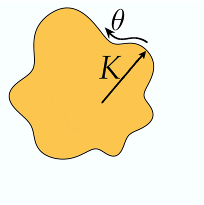

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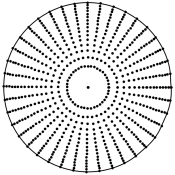

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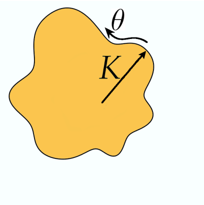

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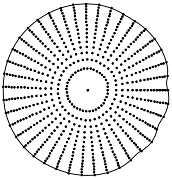

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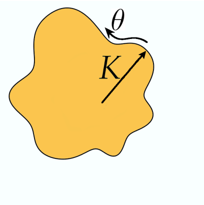

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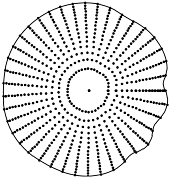

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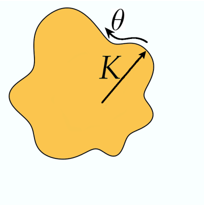

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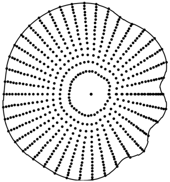

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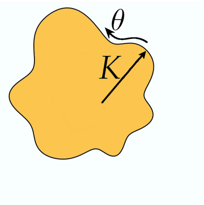

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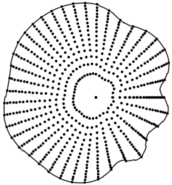

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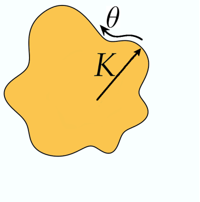

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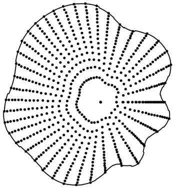

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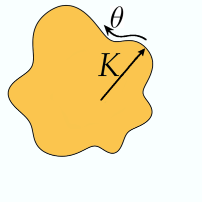

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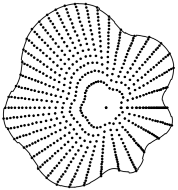

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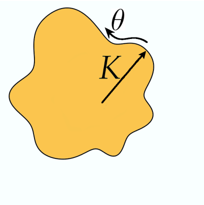

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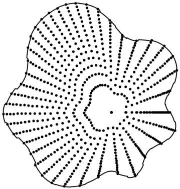

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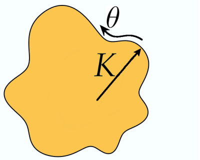

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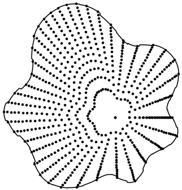

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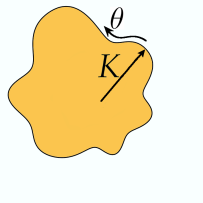

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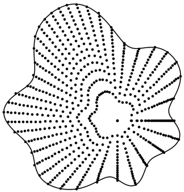

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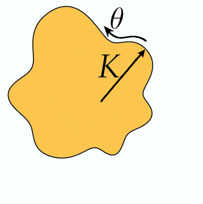

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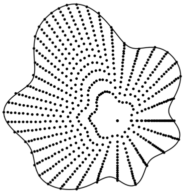

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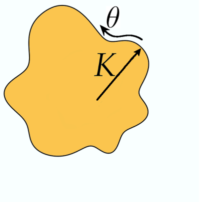

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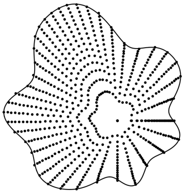

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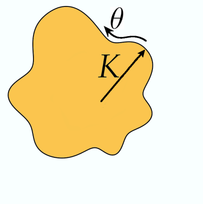

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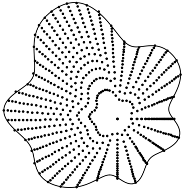

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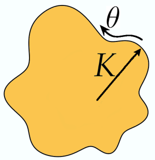

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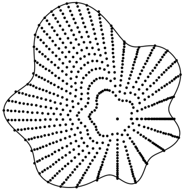

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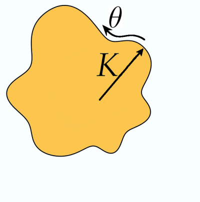

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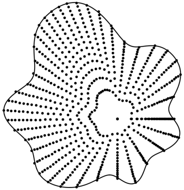

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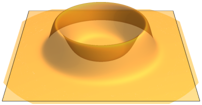

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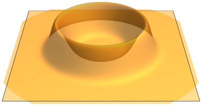

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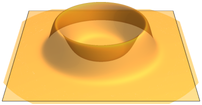

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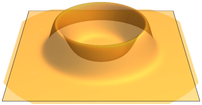

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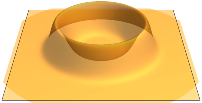

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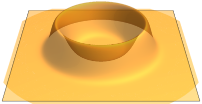

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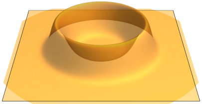

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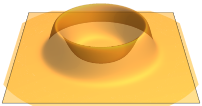

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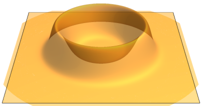

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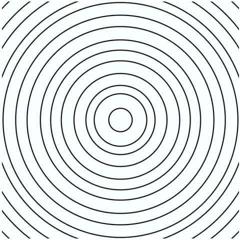

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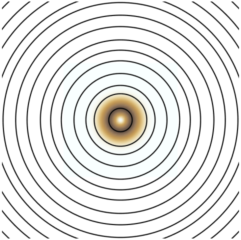

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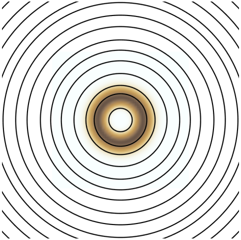

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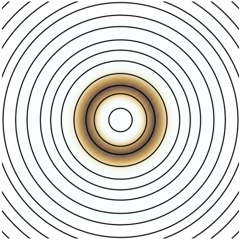

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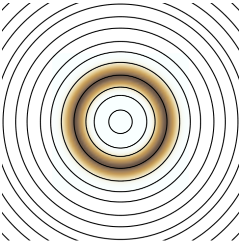

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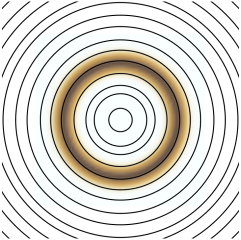

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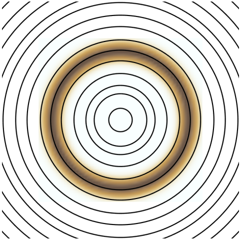

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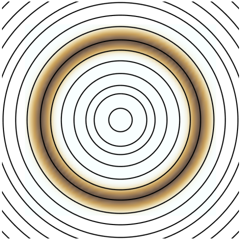

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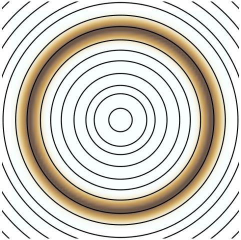

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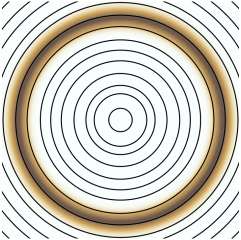

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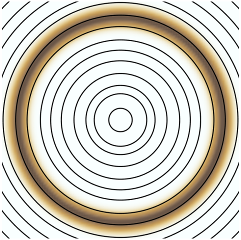

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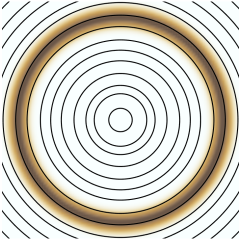

Semiclassical eigenstates

SEMICLASSICAL EIGENSTATES

Goal : Build eigenfunctions ψ_m of PVP [Charles 2003]

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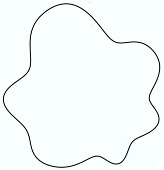

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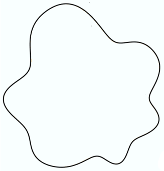

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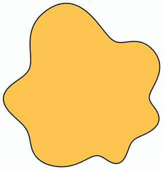

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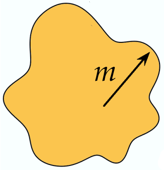

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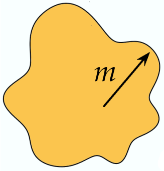

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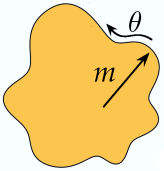

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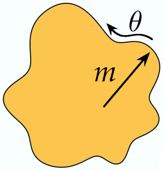

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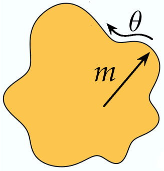

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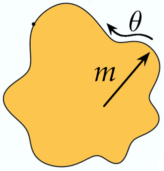

SEMICLASSICAL EIGENSTATES

Goal : Build eigenfunctions ψ_m of PVP at **small ℓ** [Charles 2003]

- Use area-angle coordinates in LLL

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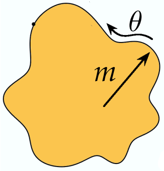

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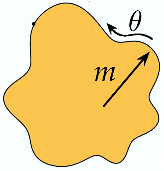

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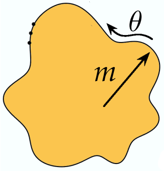

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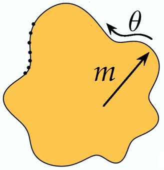

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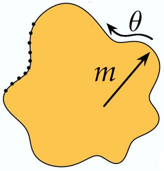

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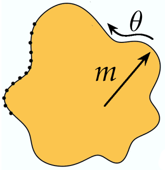

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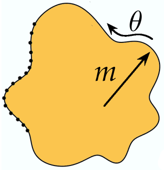

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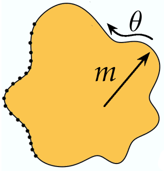

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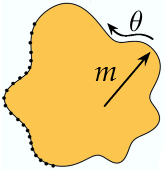

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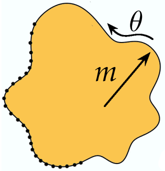

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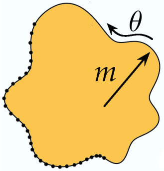

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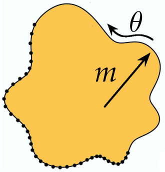

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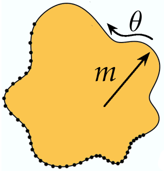

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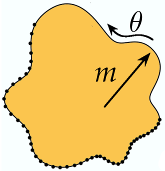

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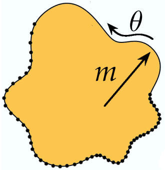

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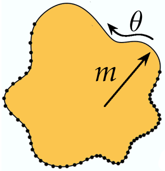

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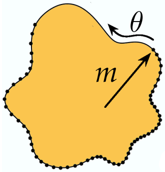

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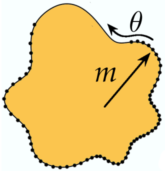

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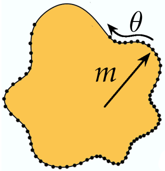

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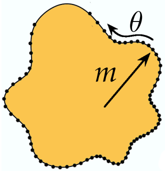

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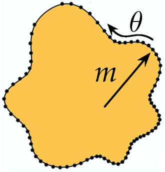

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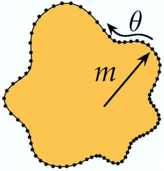

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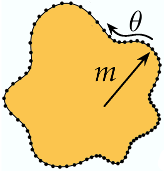

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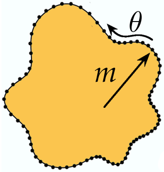

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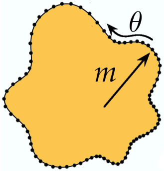

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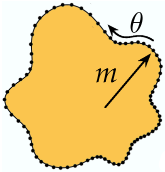

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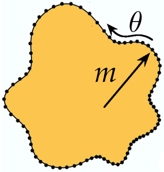

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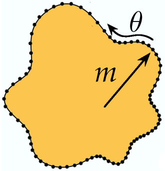

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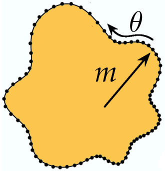

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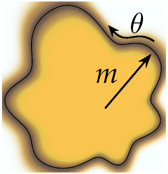

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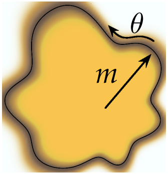

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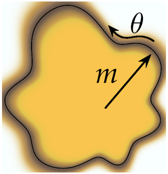
- Let $|\psi_m\rangle = P \oint \mathrm{d}\theta\, e^{im\theta}\, \boldsymbol{u(\theta)}\, |\mathbf{x}_{m,\theta}\rangle$
- Schrödinger = equation for $\boldsymbol{u(\theta)}$



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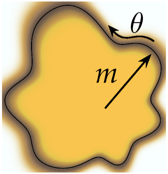

SEMICLASSICAL EIGENSTATES

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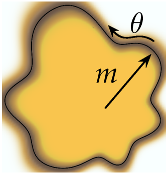
Guess wave function ?



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$$|\psi_m\rangle = P \oint \mathrm{d}\theta \, e^{im\theta} \, u(\theta) \, |\mathbf{x}_{m,\theta}\rangle$$

Guess wave function :

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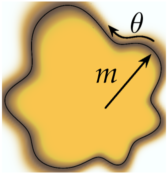

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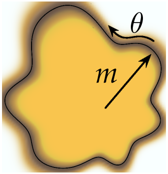

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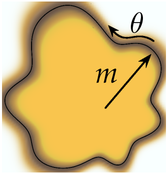

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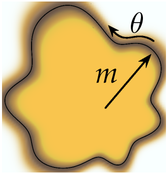

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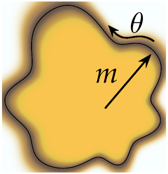

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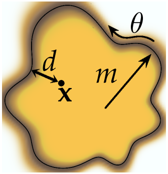

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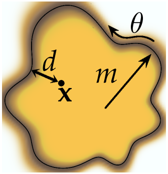

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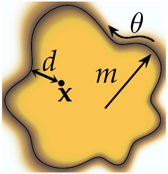

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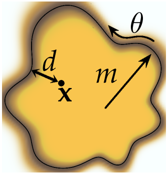

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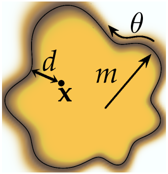

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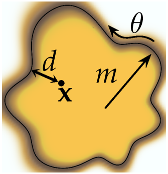

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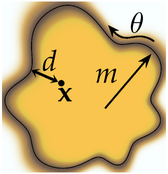

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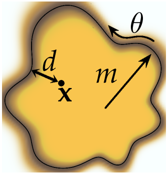

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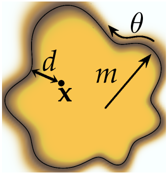

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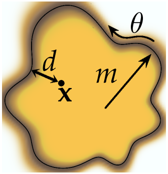

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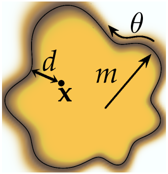

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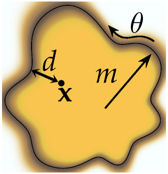

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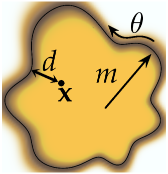

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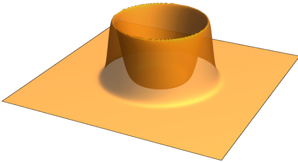

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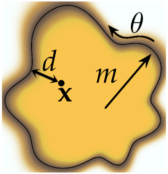

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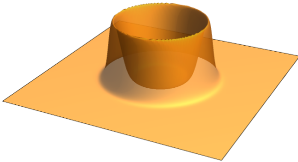

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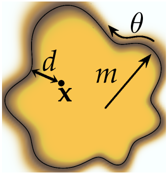

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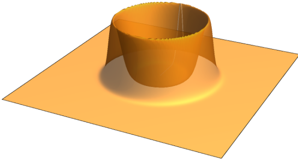

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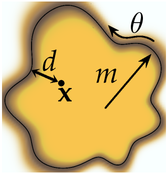

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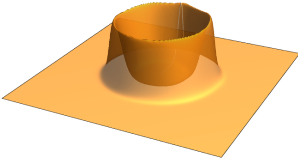

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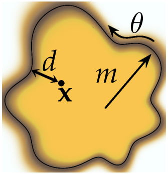

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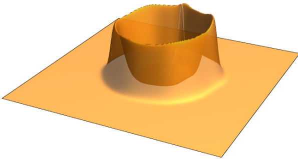

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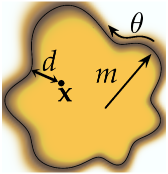

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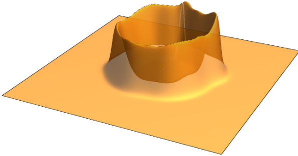

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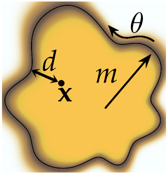

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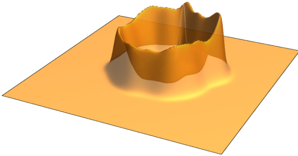

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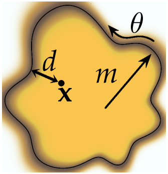

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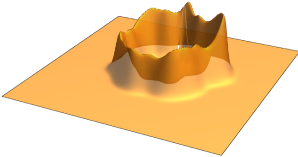

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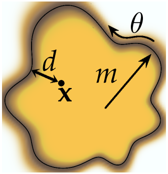

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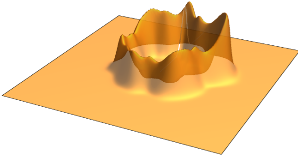

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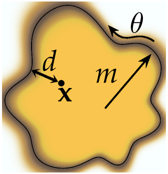

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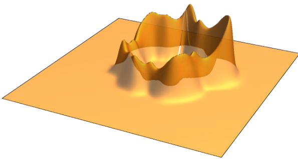

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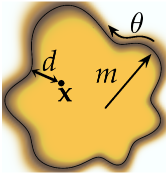

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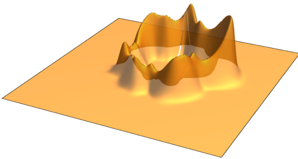

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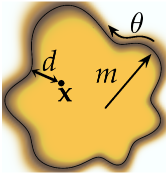

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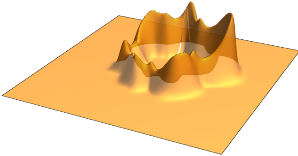

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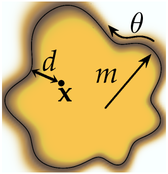

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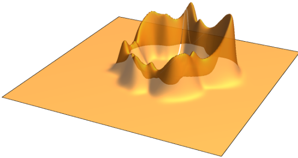

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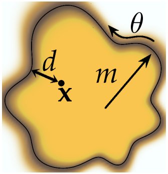

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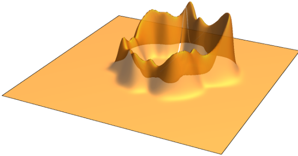

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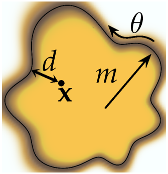

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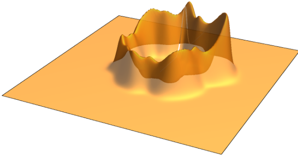

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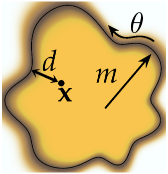

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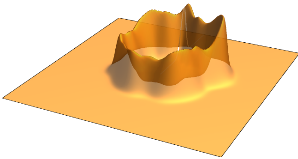

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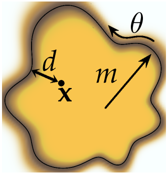

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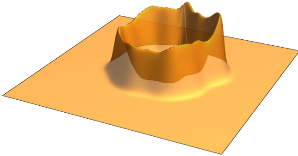

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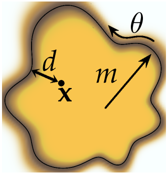

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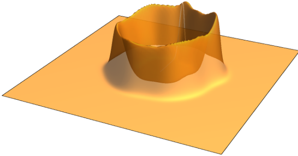

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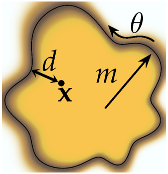

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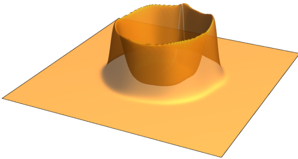

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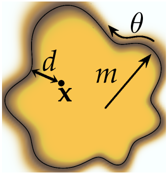

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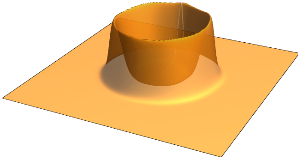

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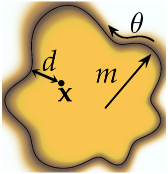

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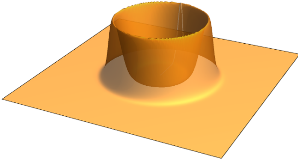

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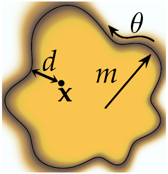

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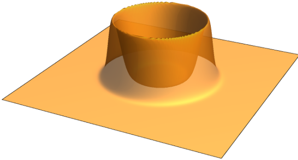

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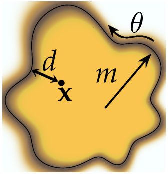

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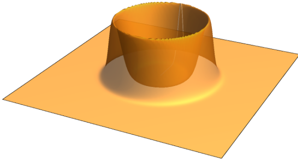

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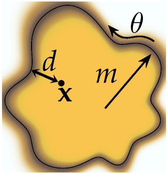

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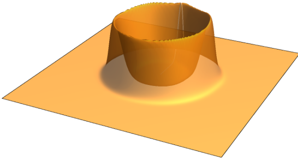

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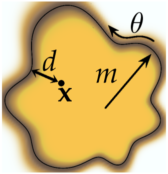

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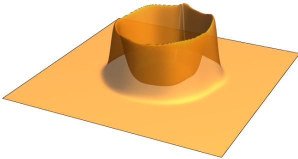

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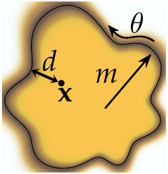

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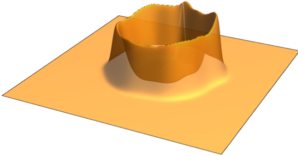

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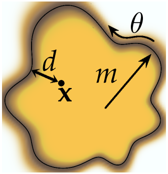

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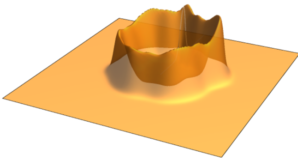

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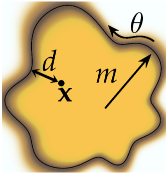

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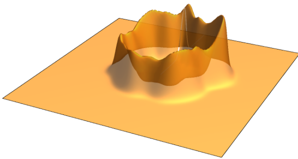

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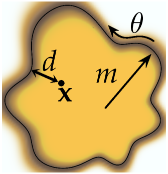

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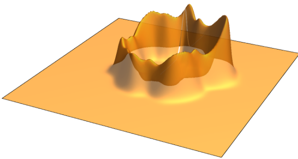

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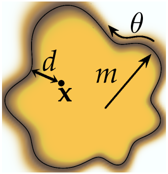

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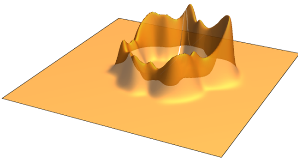

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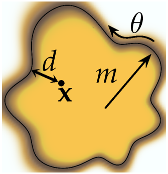

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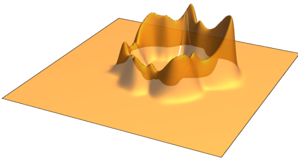

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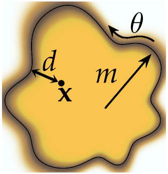

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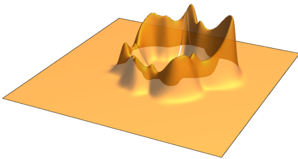

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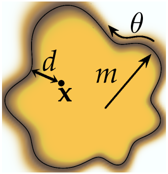

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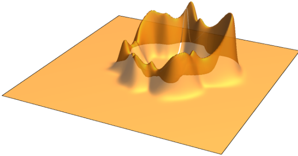

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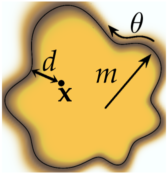

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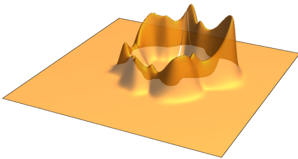

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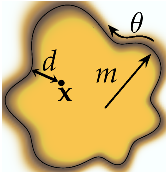

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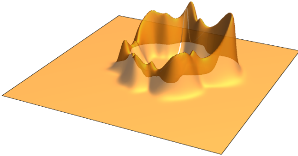

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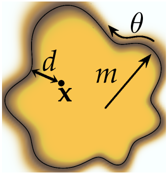

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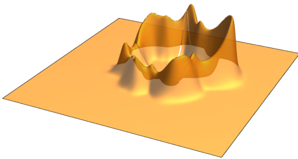

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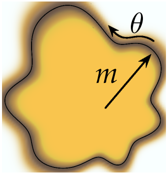

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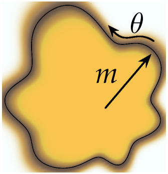

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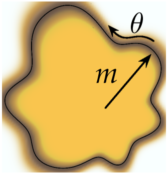

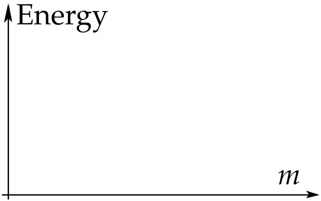

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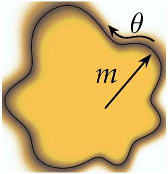

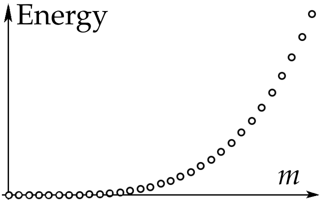

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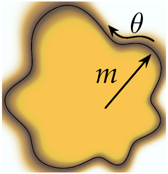

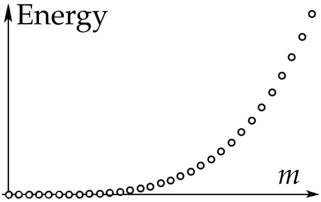

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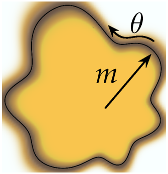

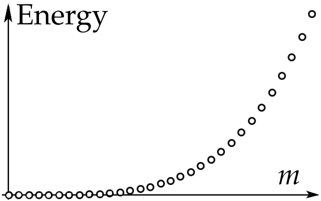

2. Many-body observables

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A. Ground state and density

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A. Ground state and density

B. Correlations

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A. Ground state and density

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C. Edge modes

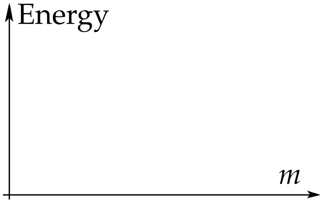
Many-body ground state

Many-body ground state

Free particles !

Many-body ground state

Free particles !



MANY-BODY GROUND STATE

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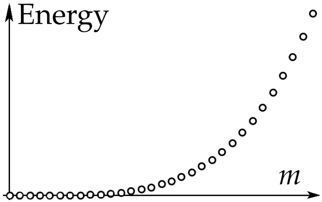

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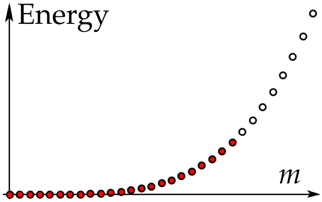

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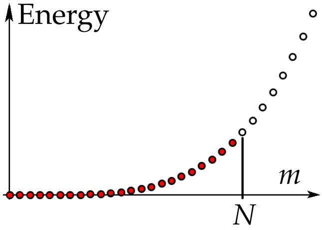

Many-body ground state

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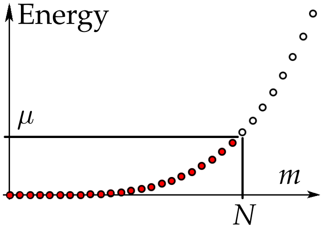

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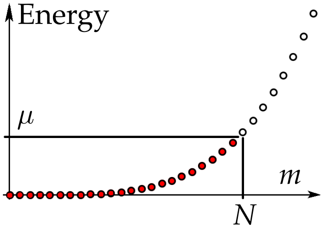

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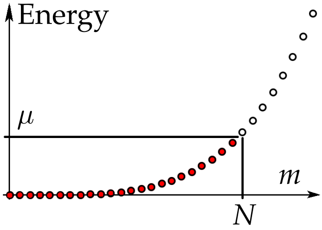
  (Slater determinant of ψ_m's)



Many-body ground state

Free particles !

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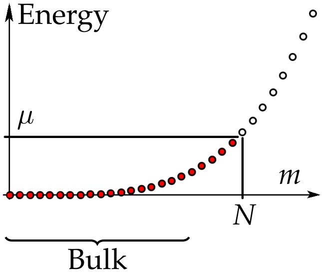

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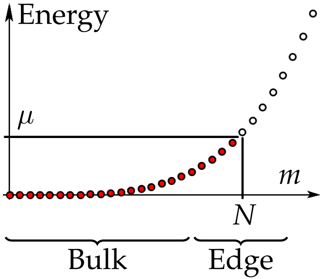

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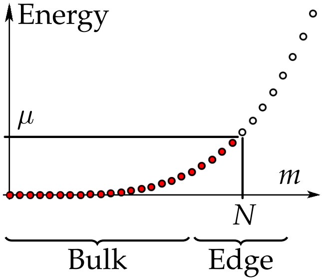

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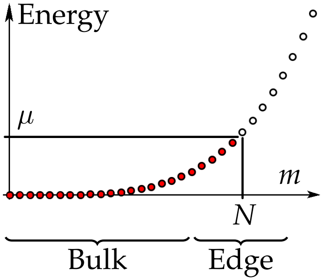
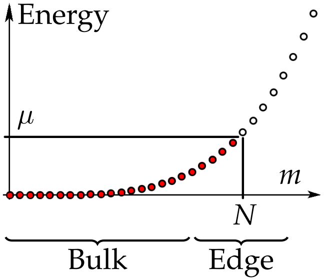


Many-body $= \sum$ One-body

Many-body ground state



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- Compute everything !

Many-body ground state

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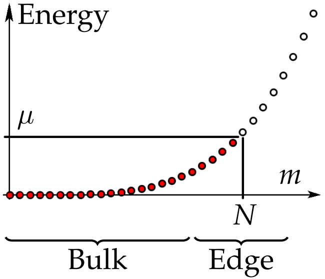

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- Compute everything !
- **Density** $\rho(\mathbf{x}) = \langle\Omega|c^\dagger(\mathbf{x})c(\mathbf{x})|\Omega\rangle$

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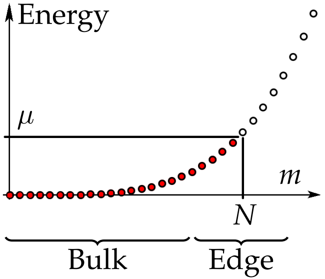

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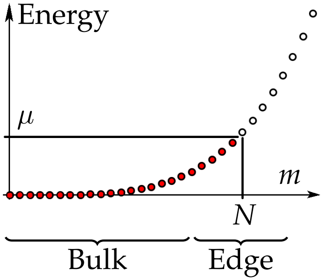

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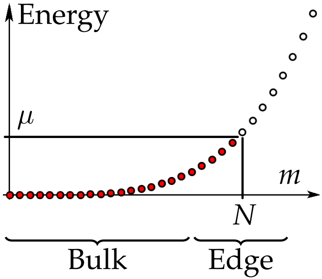

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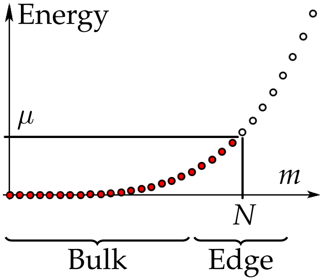

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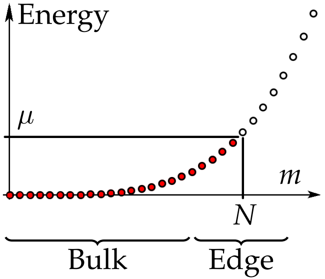

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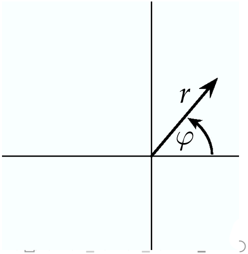

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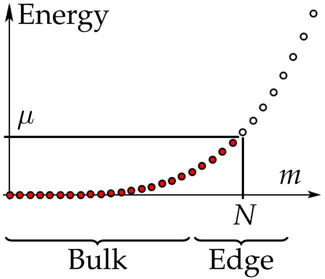

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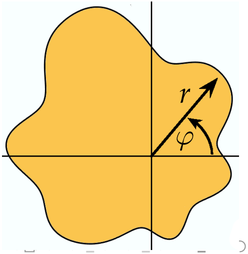

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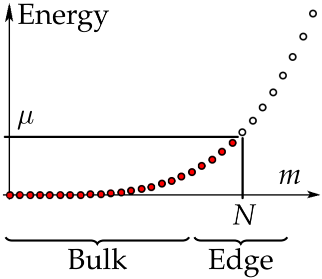

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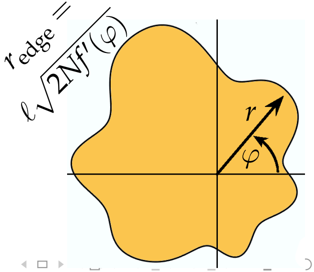
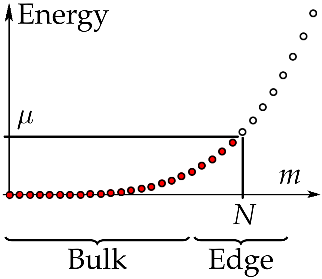


Many-body ground state



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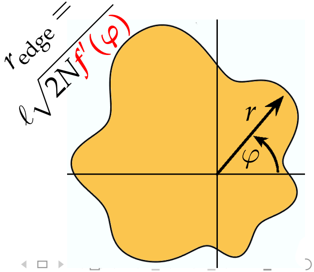

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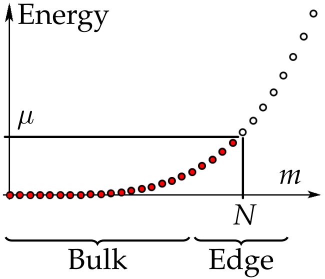

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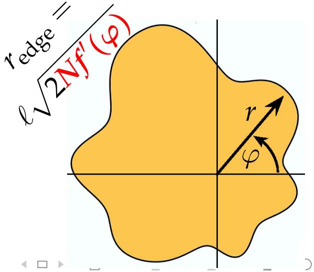
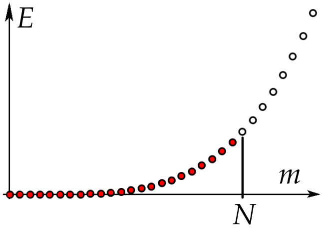
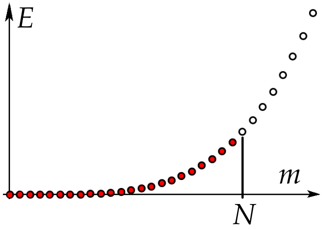


DENSITY



DENSITY $\rho(\mathbf{x}) = \sum_{m=0}^{N-1} |\psi_m(\mathbf{x})|^2$



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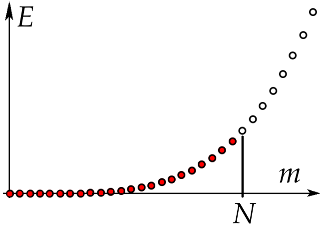

$$\text{Edge-deformed}: \ |\psi_m|^2 \sim \frac{\exp\left[-\frac{1}{v^2}\left(\frac{r}{\ell\sqrt{f'}}-\sqrt{2m}\right)^2\right]}{2\pi\ell^2\, v(\varphi)\,\sqrt{2\pi m}}$$

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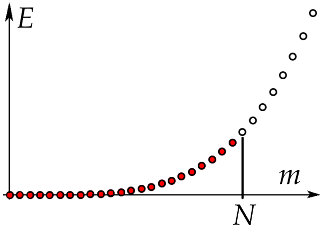

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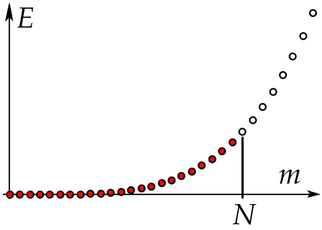

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- Fix (r, φ) and convert $\sum_m \sim \int_0^{\sqrt{N}} \mathrm{d}\sqrt{m}$

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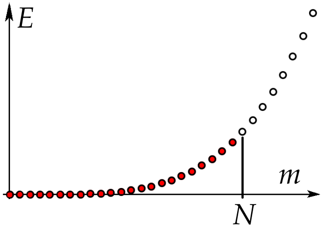

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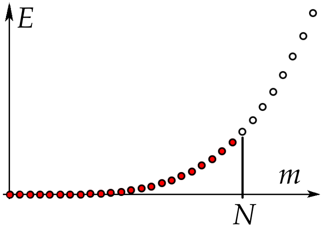

Bulk ?

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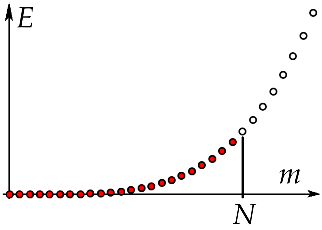

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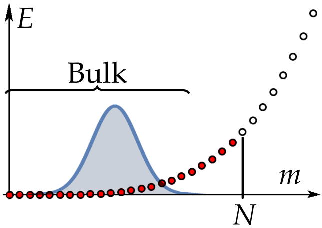

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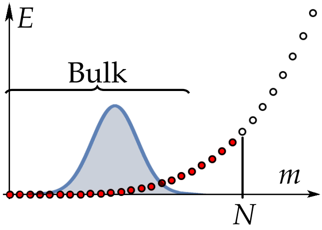

Bulk $r \ll \ell\sqrt{2Nf'}$

- $\rho \sim \dfrac{1}{2\pi\ell^2}$

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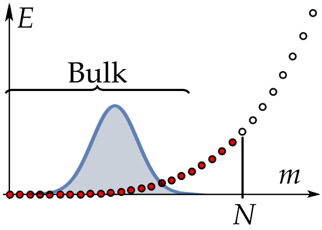

Bulk $r \ll \ell\sqrt{2Nf'}$

- $\rho \sim \dfrac{1}{2\pi\ell^2}$ **uniform** !

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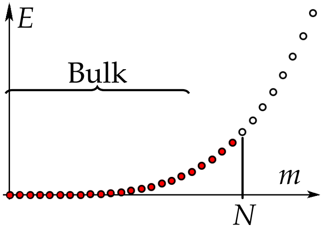

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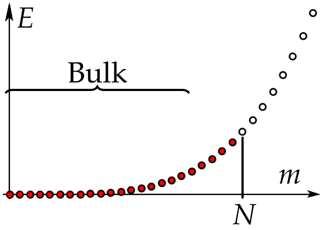

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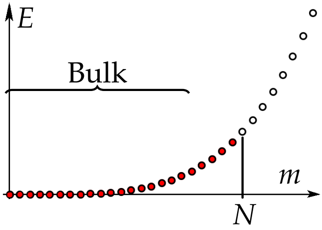

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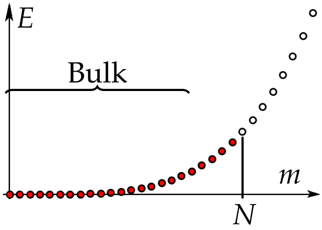

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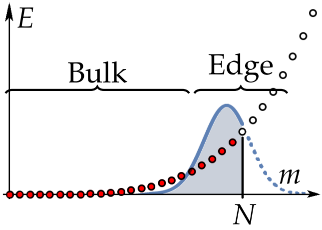

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Edge $r \sim \ell\sqrt{2Nf'}$

Edge-deformed : $|\psi_m|^2 \sim \dfrac{\exp\left[-\frac{1}{v^2}\left(\frac{r}{\ell\sqrt{f'}} - \sqrt{2m}\right)^2\right]}{2\pi\ell^2\, v(\varphi)\, \sqrt{2\pi m}}$

- Fix (r, φ) and convert $\sum_m \sim \int_0^{\sqrt{N}} \mathrm{d}\sqrt{m}$ **Gaussian integral** !

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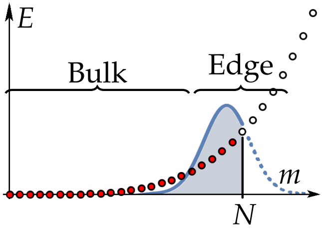

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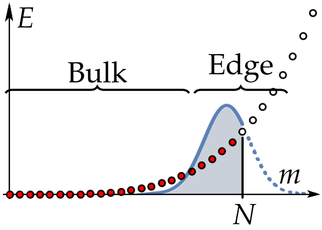

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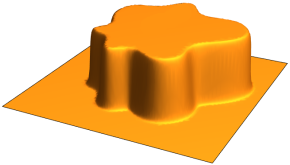

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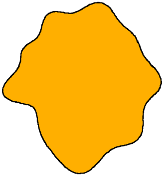

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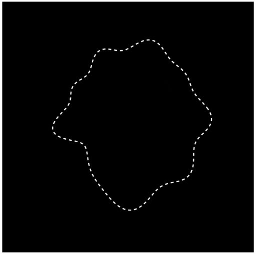

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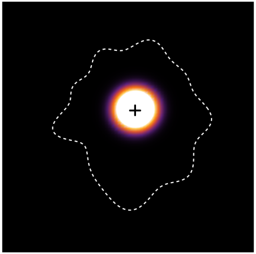

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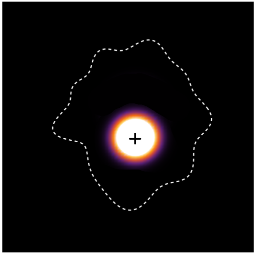

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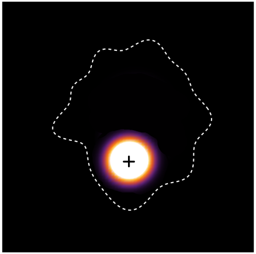

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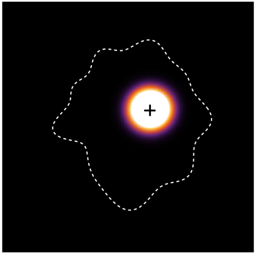

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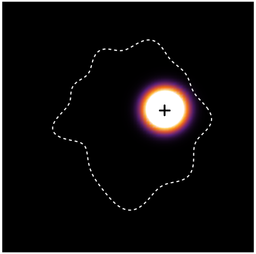

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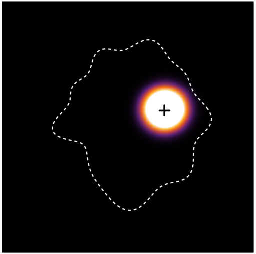

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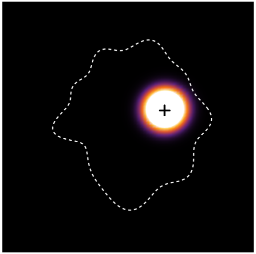

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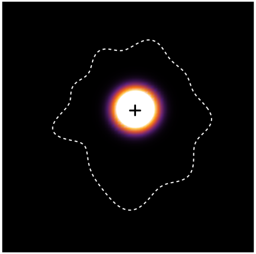

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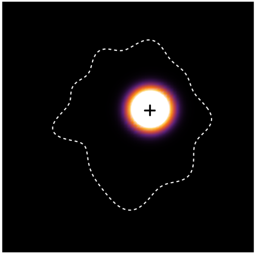

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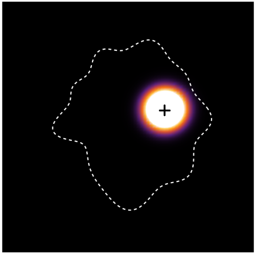

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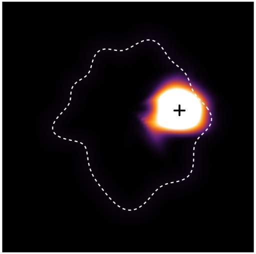

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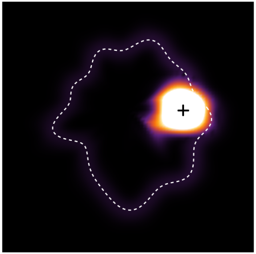

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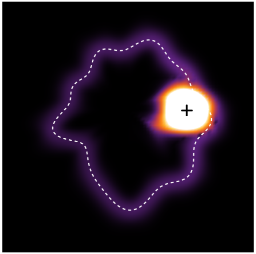

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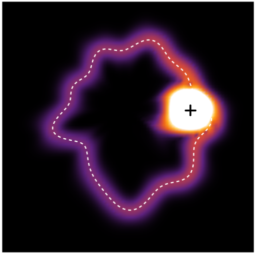

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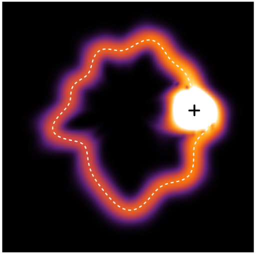

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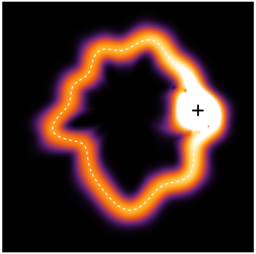

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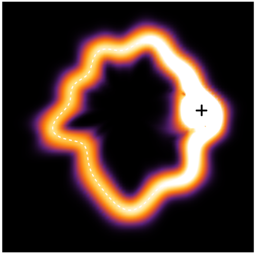

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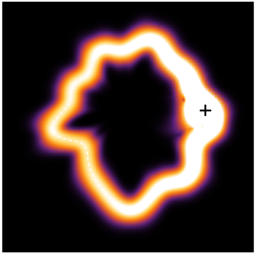

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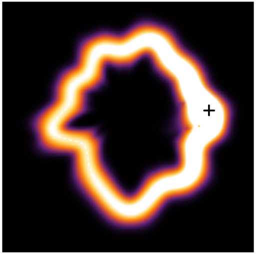

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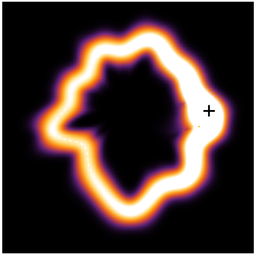

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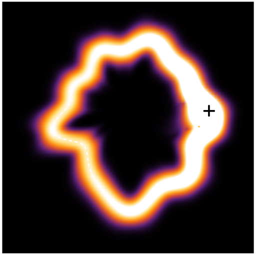

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Edge :

- Upper bound crucial !

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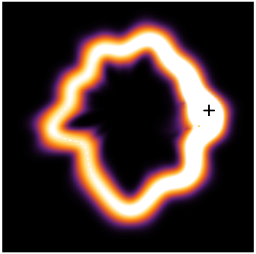

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Edge :

- Upper bound gives geometric series $\sim \sum_{m=0}^{N-1} e^{im(\theta_y - \theta_x)}$

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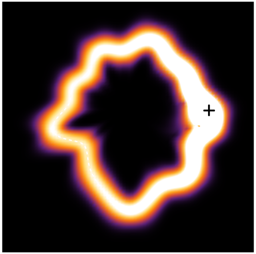

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Edge :

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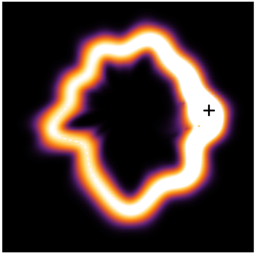

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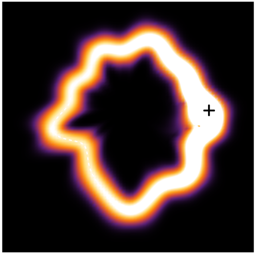

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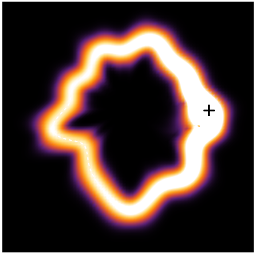

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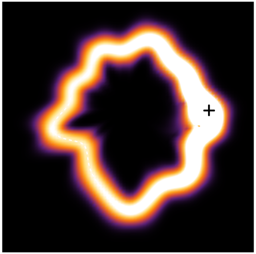

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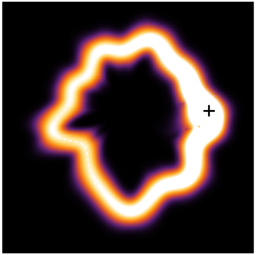

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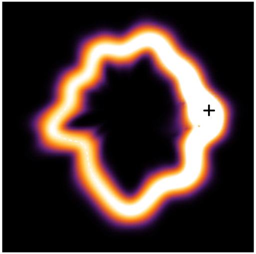

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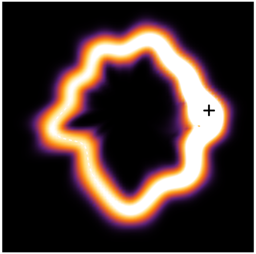

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- **Long-ranged** (gapless free fermion) but **inhomogeneous** ?

EDGE MODES

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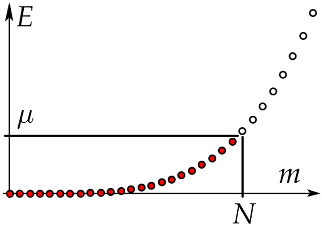
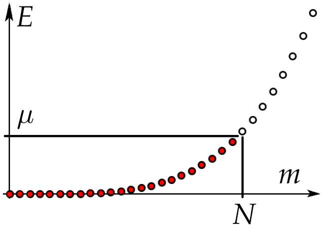


EDGE MODES

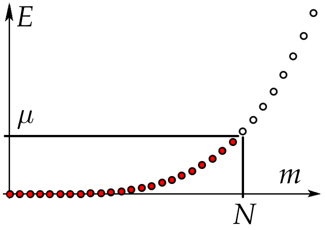
Low-energy dynamics ?



Edge modes

Low-energy Hamiltonian :

$$\mathcal{H} \sim \sum_{m=0}^{\infty} (E_m - \mu) a_m^\dagger a_m$$



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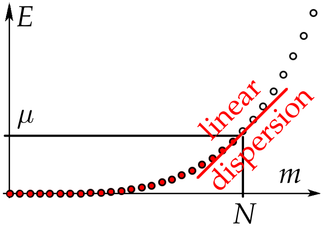

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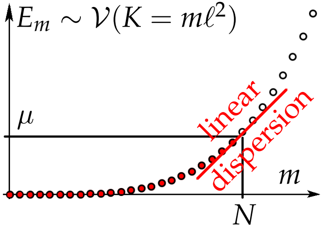

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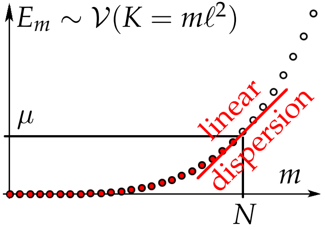

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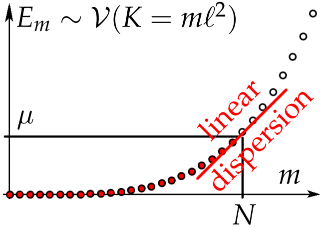

EDGE MODES

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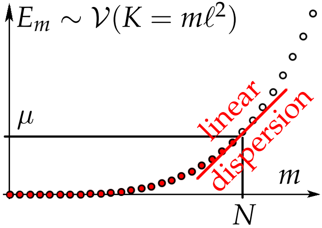

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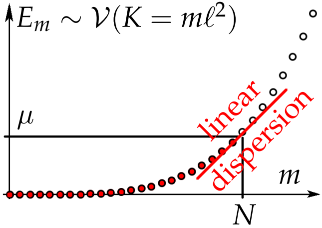

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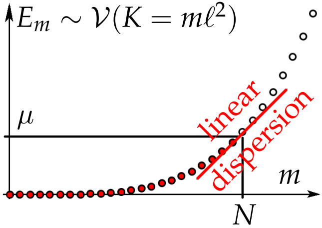

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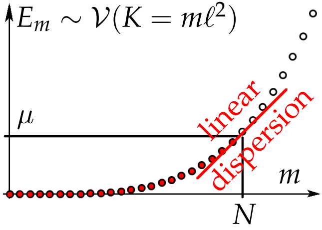

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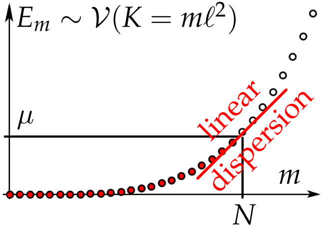

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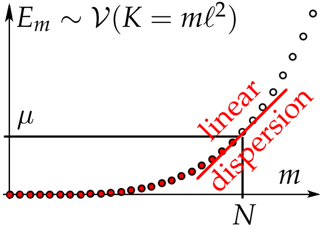

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- Where is inhomogeneity ?

EDGE MODES FROM BULK

Edge modes from bulk

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despite inhomogeneity in lab coord !

3. Microwave absorption

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A. Microwave absorption rates

3. Microwave absorption

A. Microwave absorption rates

B. Conclusion

Microwave absorption

Microwave absorption

Observable predictions for anisotropic droplets ?

MICROWAVE ABSORPTION

Observable predictions for anisotropic droplets ?

Local imaging feasible but tough...

Microwave absorption

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- Global measurement :
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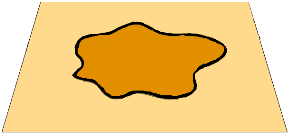

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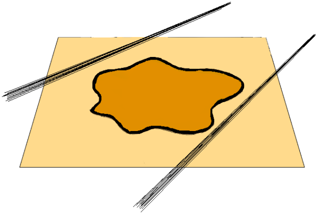

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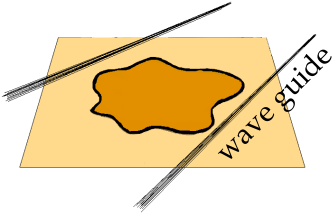

MICROWAVE ABSORPTION

Observable predictions for anisotropic droplets ?

Local imaging feasible but tough...

- Global measurement :
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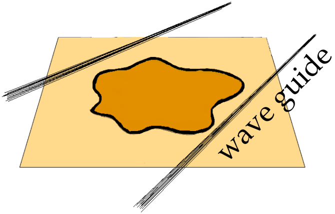

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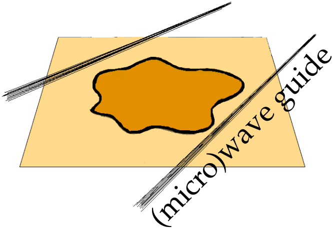

MICROWAVE ABSORPTION

Microwave absorption

Recall **time-dependent perturbations**

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- Prepare state $|\psi_m\rangle$

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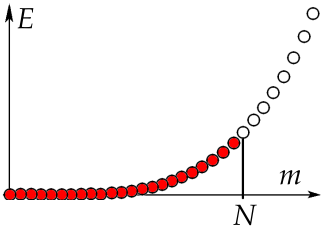

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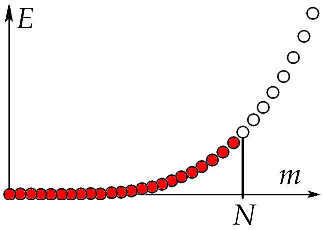

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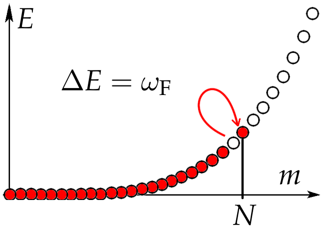

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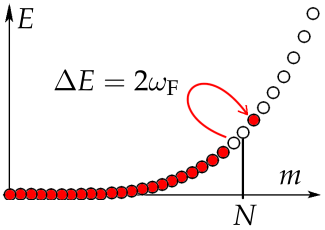

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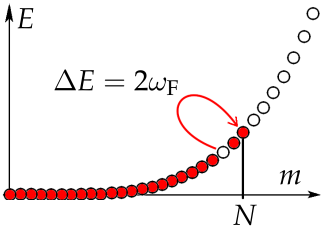

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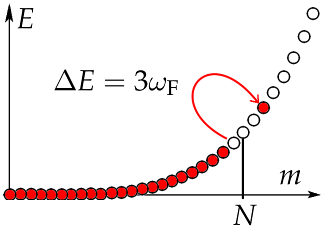

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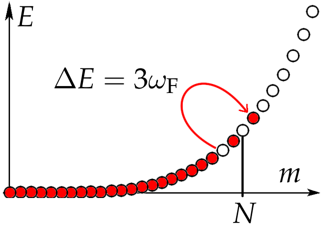

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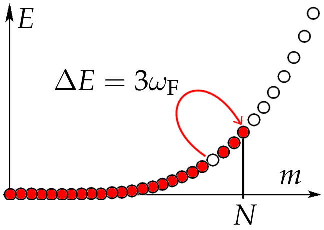

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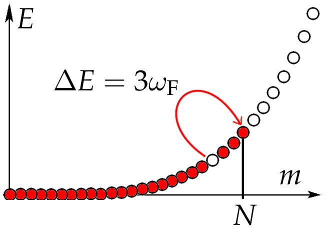

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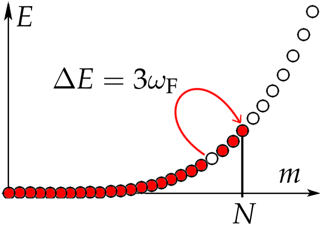

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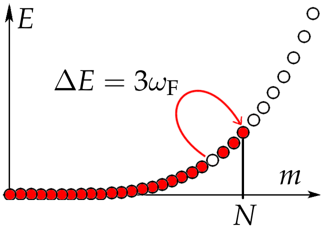

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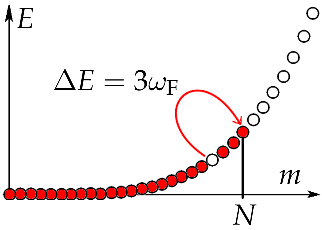

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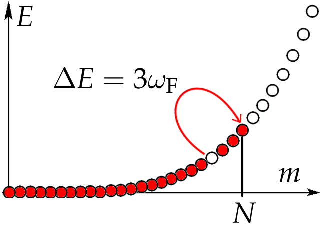

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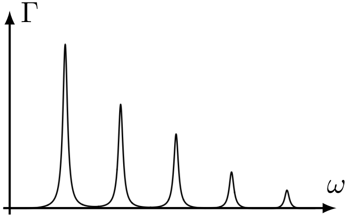

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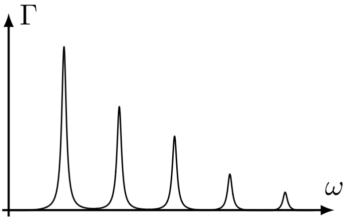

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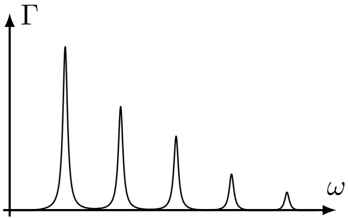

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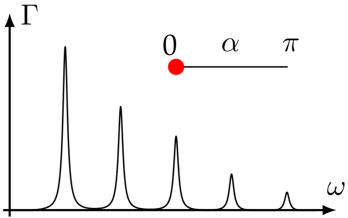

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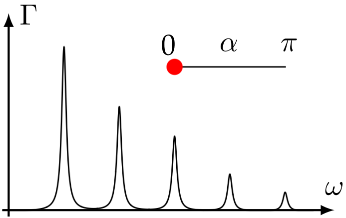

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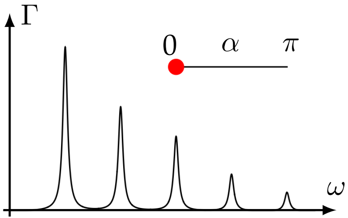

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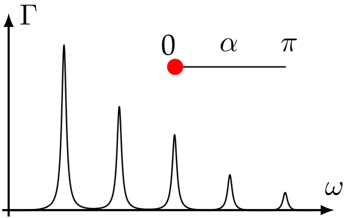

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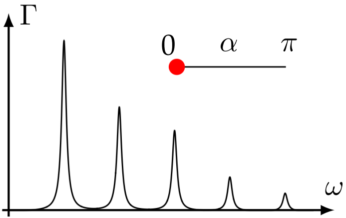

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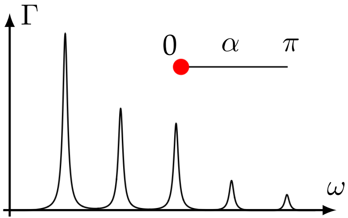

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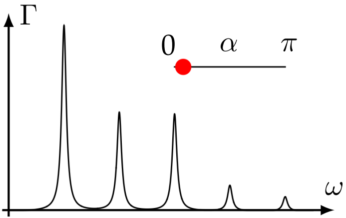

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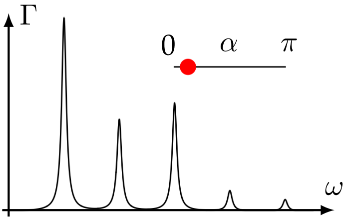

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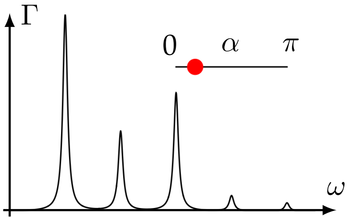

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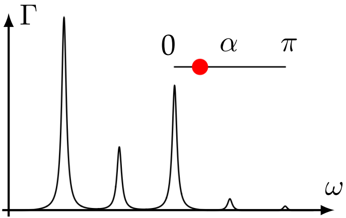

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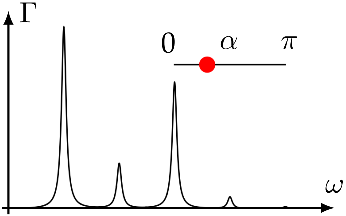

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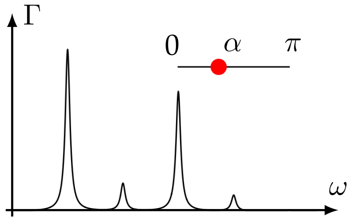

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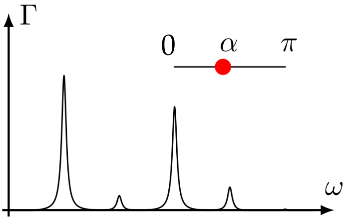

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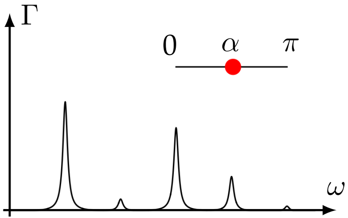

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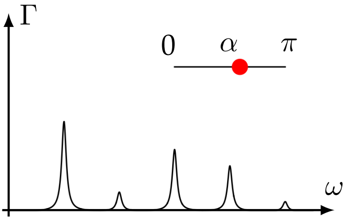

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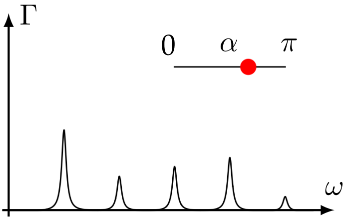

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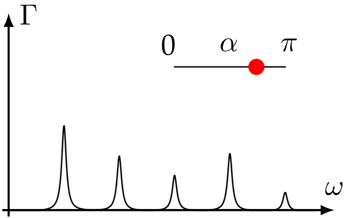

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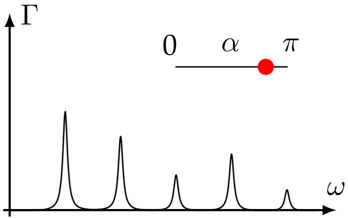

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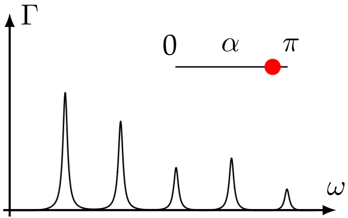

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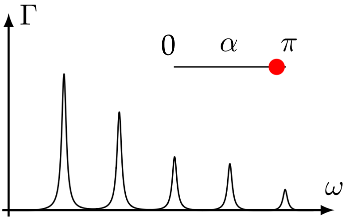

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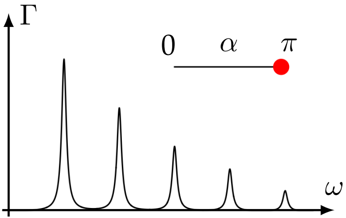

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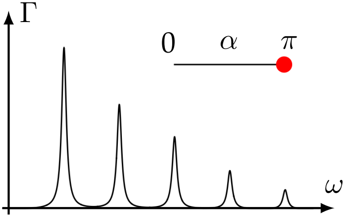

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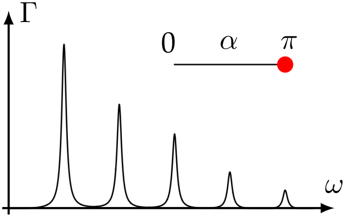

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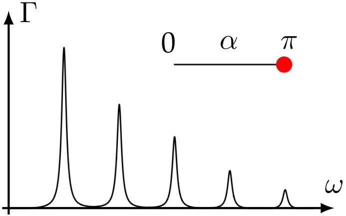

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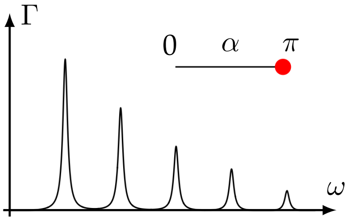

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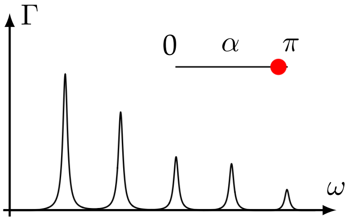

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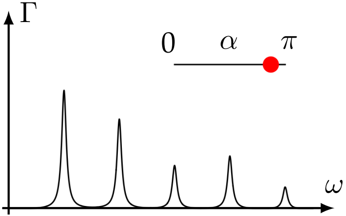

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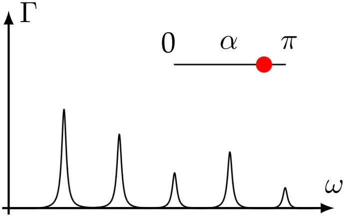

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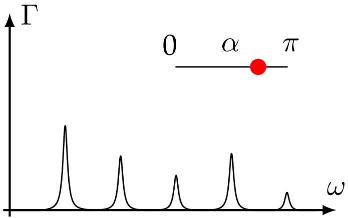

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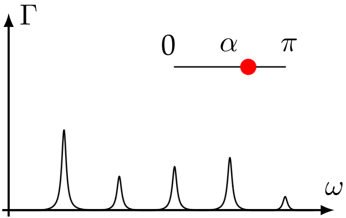

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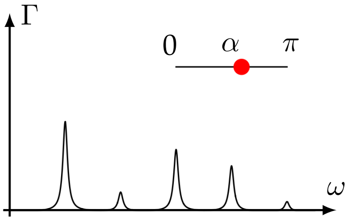

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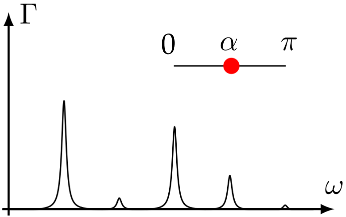

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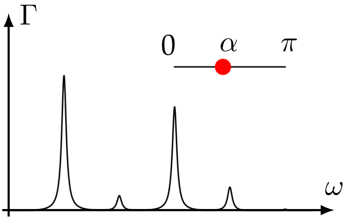

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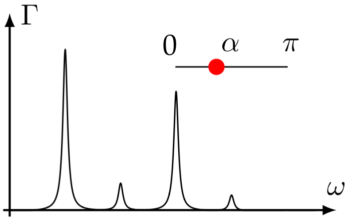

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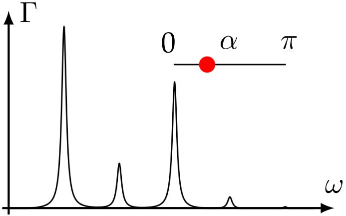

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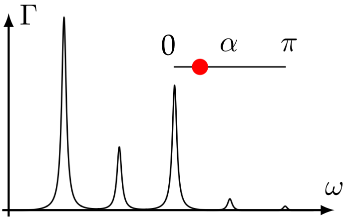

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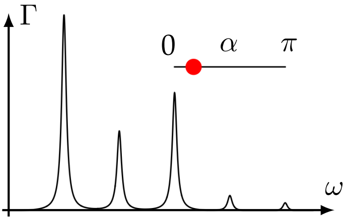

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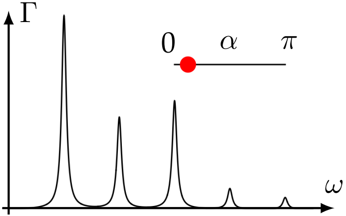

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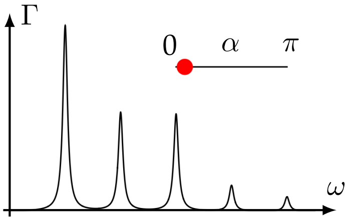

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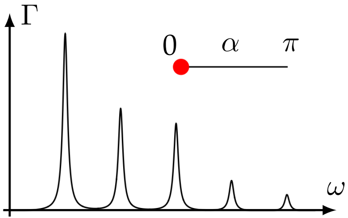

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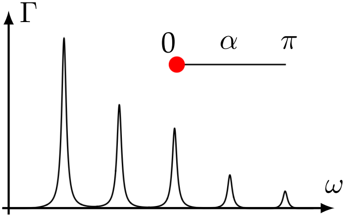

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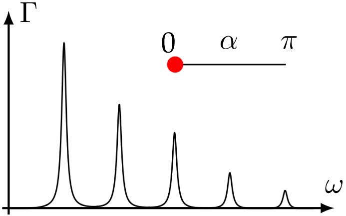

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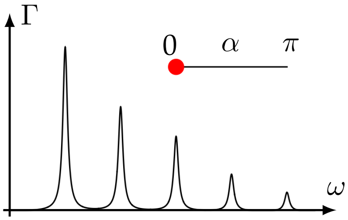

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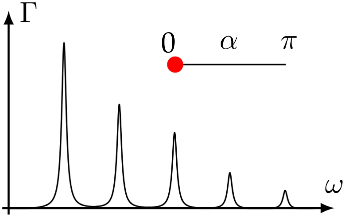

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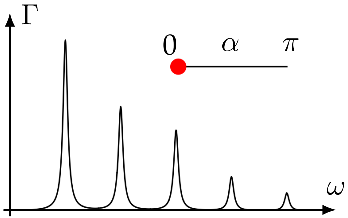

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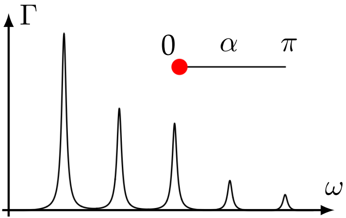

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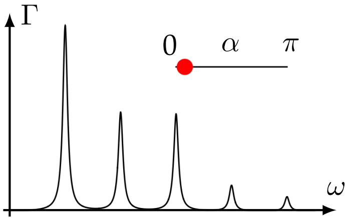

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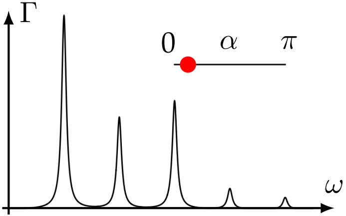

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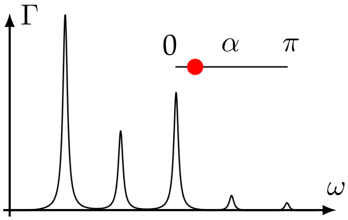

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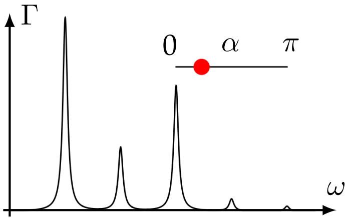

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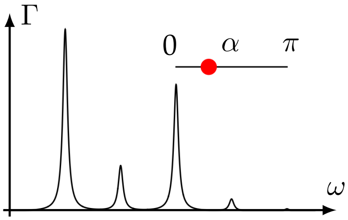

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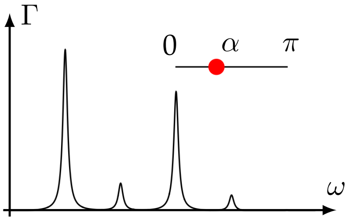

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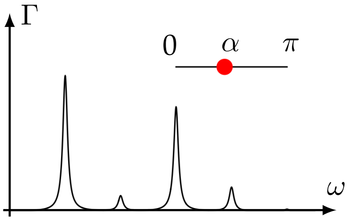

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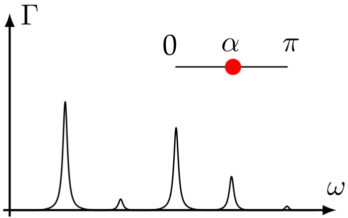

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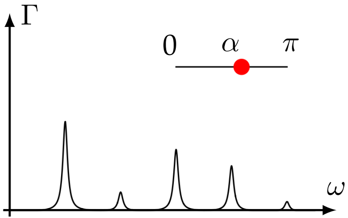

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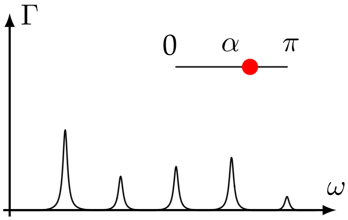

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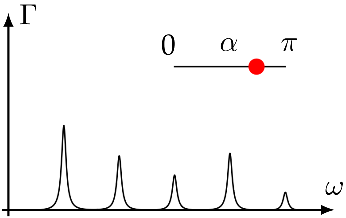

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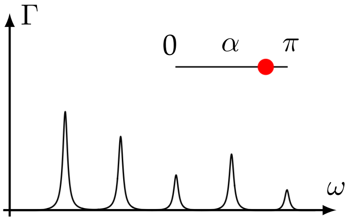

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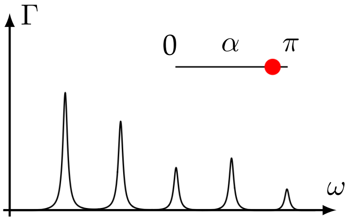

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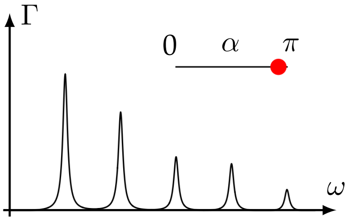

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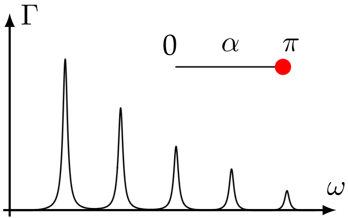

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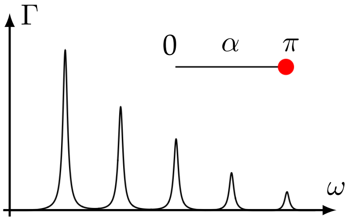

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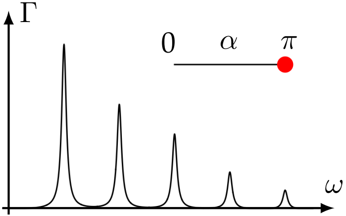

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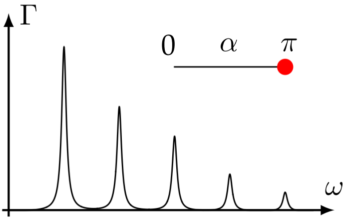

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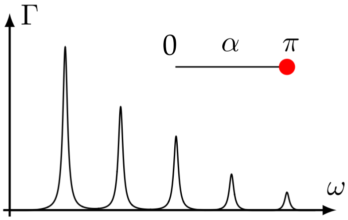

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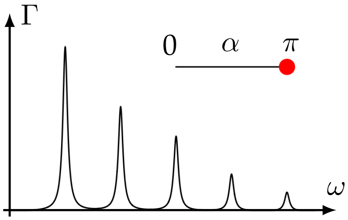

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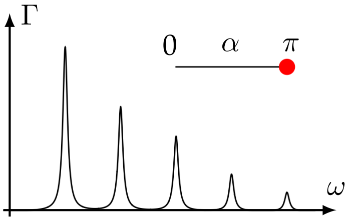

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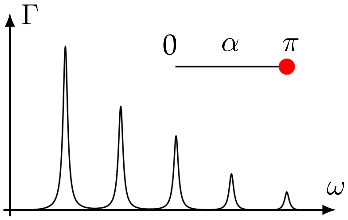
Recall **time-dependent perturbations** :

- Prepare state $|\psi_m\rangle$ (anisotropic LLL state)
- Perturb by $W\cos(\omega t)$ (electrostatic potential)

$$\Gamma(\omega) \sim \sum_{p=1}^{\infty} \frac{p}{2} \left| \oint \frac{\mathrm{d}\theta}{2\pi}\, e^{ip\theta}\, \boldsymbol{W}\big|_{\mathbf{edge}}(\boldsymbol{\theta}) \right|^2 \delta(\omega - p\,\omega_\mathrm{F})$$

- Use polarized microwaves :
  $W = qE[x\cos(\alpha) + y\sin(\alpha)]$



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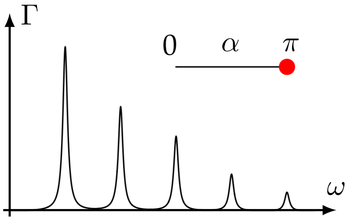

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Note :
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Merci !

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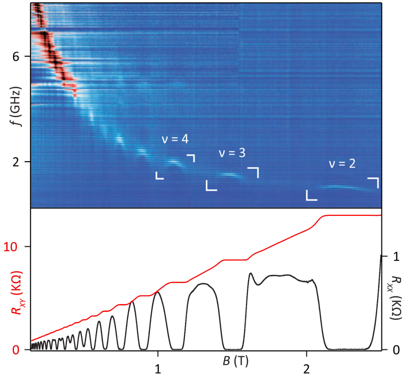
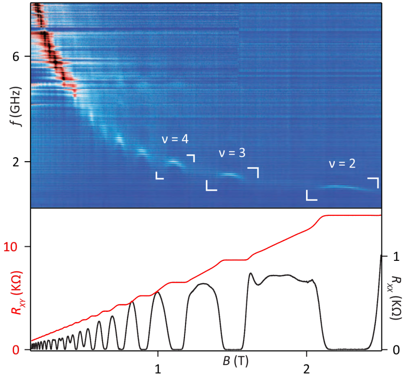


[Mahoney-Reilly+ PRX 2017]

Bonus : microwave experiments



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Isotropic droplet

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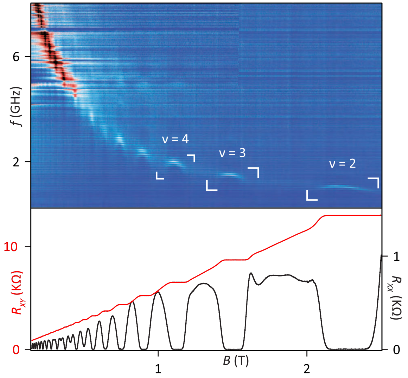

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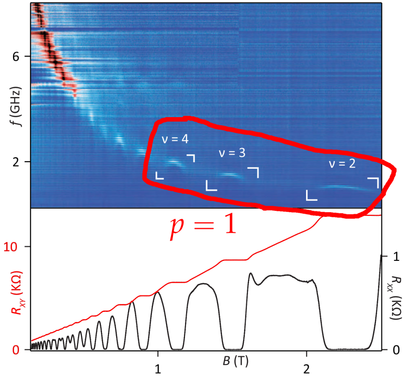

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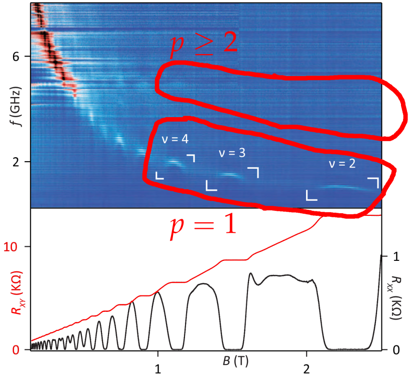

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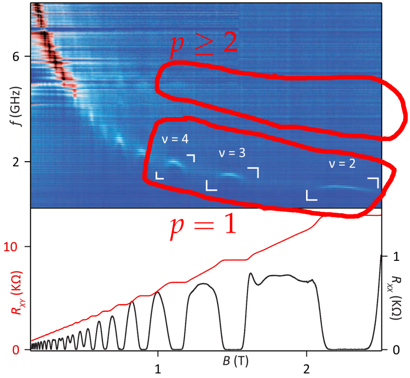

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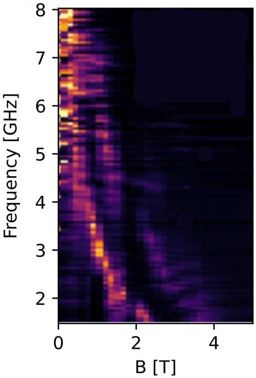

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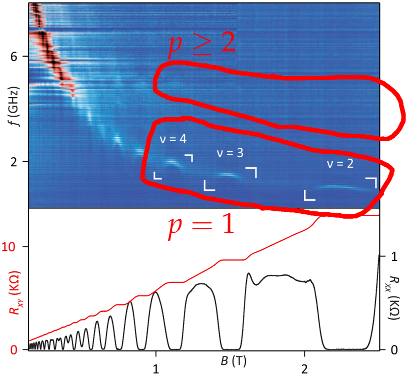

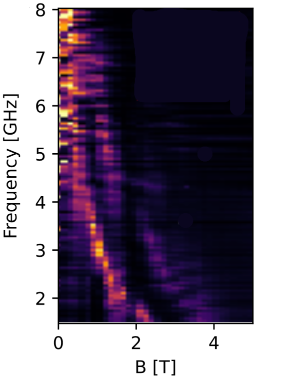

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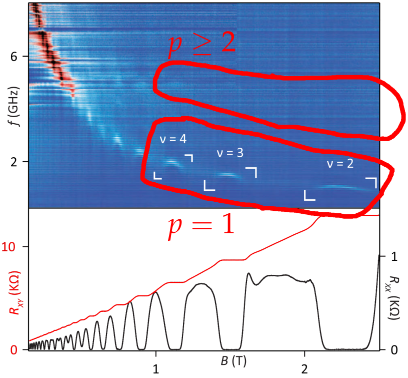

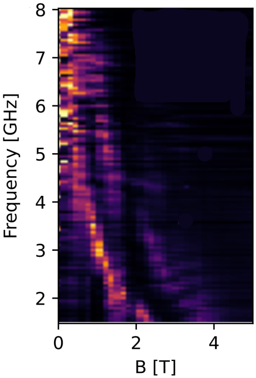

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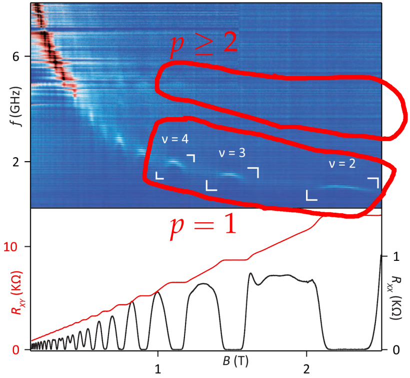

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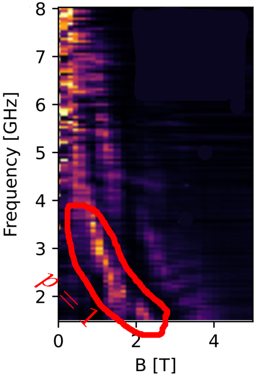

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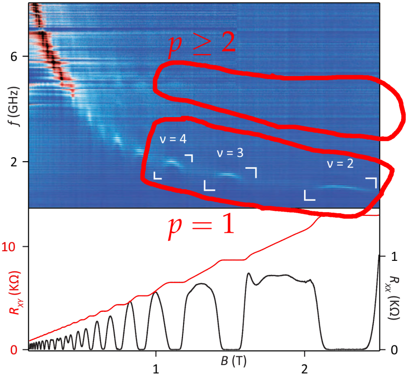

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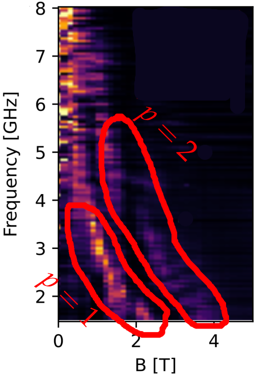

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