Flat JT Gravity and the Schwarzian of BMS$_2$

Blagoje Oblak

École Polytechnique

arXiv 2112.14609 with Hamid Afshar

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Intro

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Quantum gravity ?

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- AdS/CFT

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- $\geq$ 3D has Bondi-Metzner-Sachs aspt symmetry

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$\Bigg\}$ **This talk**

THIS TALK IN A NUTSHELL

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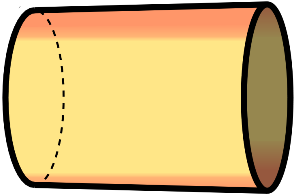

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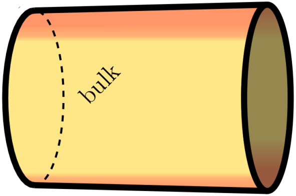

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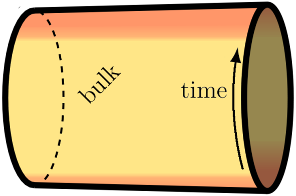

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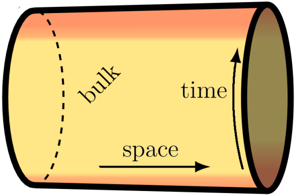

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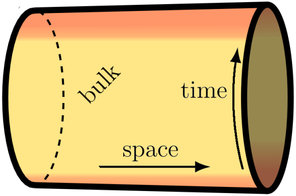

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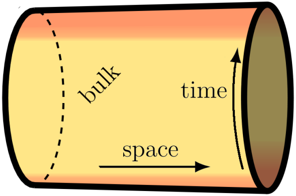

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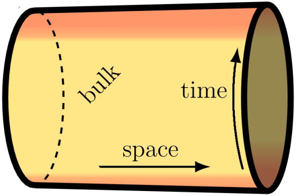

2D Jackiw-Teitelboim = 1D bdry theory

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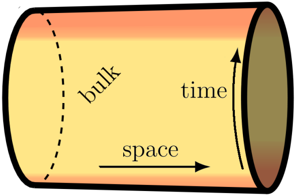

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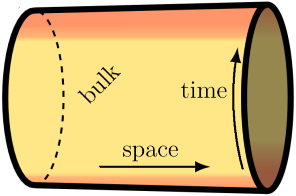

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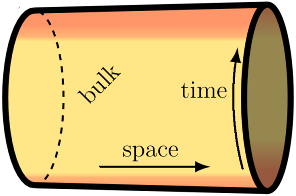

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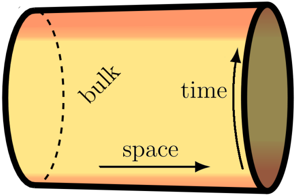

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- Similar to AdS/Schwarzian

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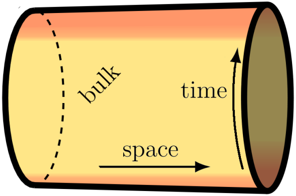

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2D Jackiw-Teitelboim = 1D bdry theory

- "BMS-Schwarzian" = Zero-mode of **stress tensor**
- One-loop exact partition fct
- Similar to AdS/Schwarzian, but group is "weird" !

PLAN

1. **Bondi gauge JT gravity = 1D theory**

2. BMS$_2$ group and its orbit(s)

3. One-loop exact partition function

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1. JT Gravity as a 1D theory

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A. Bondi gauge and fall-offs

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C. Boundary action

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2D space-time with **metric** g and **scalar** Φ

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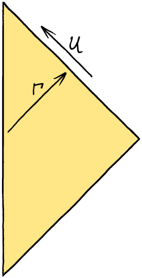

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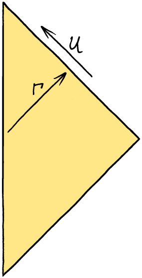

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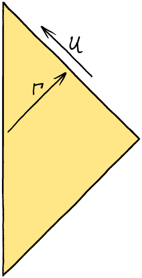

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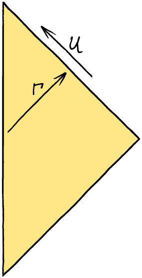

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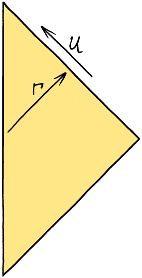

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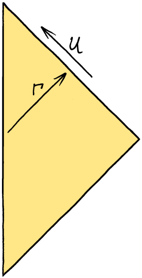

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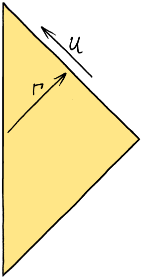

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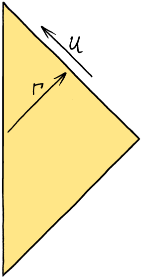

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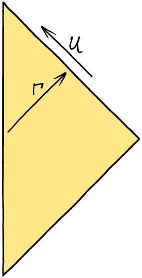

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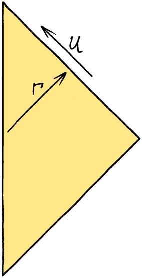

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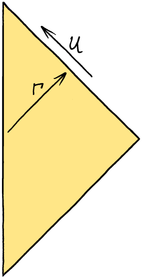

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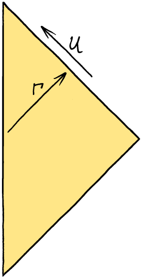

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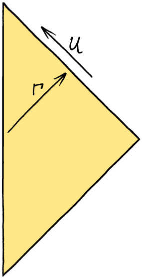
  $\Phi \approx x(u)r + y(u)$ with ODE $(x', y'') \approx$ stuff



ASPT SYMMETRIES

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r - \left(\frac{r^2}{\ell^2} + \quad \mathcal{O}(r) \quad\right)\mathrm{d}u^2 \ \text{ and } \ \Phi = \mathcal{O}(r)$$

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  ↓

  time **diffeo**

ASPT SYMMETRIES

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  time **diffeo** radial **translation**

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Set finite temperature

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Set finite temperature

- Euclidean time $\varphi \equiv 2\pi i u/\beta$

ASPT SYMMETRIES

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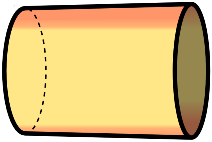

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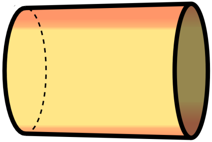

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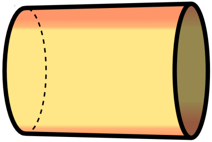

$$i[\ell_m, \ell_n] = (m-n)\ell_{m+n}$$
$$i[\ell_m, q_n] = -(m+n)q_{m+n}$$
$$i[q_m, q_n] = 0$$

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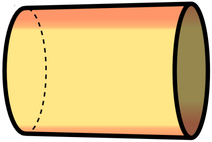

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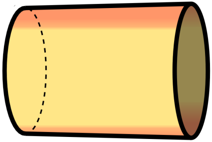
  $\downarrow$ time **diffeo** $\searrow$ radial **translation**

Set finite temperature



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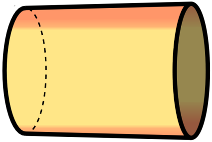

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- Tsf under BMS$_2$?

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$$\delta T = XT' + 2X'T - \alpha P - \alpha'$$
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- Eqns remain valid at finite temperature

BOUNDARY ACTION

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JT action $S_{\text{bulk}} = \frac{\kappa}{2}\int \mathrm{d}^2x\sqrt{|g|}\big(\Phi(R+2/\ell^2)-2\Lambda\big)$

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- Designed so that $\delta S = \; - \kappa \int \mathrm{d}u\, \delta\Big(\frac{1}{x}\Big) \times (\text{cst on-shell})$

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BOUNDARY ACTION

JT action $S_{\text{bulk}} = \frac{\kappa}{2}\int \mathrm{d}^2x\sqrt{|g|}\left(\Phi(R+2/\ell^2)-2\Lambda\right)$

- $\delta S_{\text{bulk}} = \int \mathrm{d}^2x(\text{EOM}) + \boldsymbol{\kappa\int \mathrm{d}u(x\,\delta T - y\,\delta P)}$
- Not differentiable ! Extra bdry term is needed :

$$S = S_{\text{bulk}} - \kappa\int \mathrm{d}u\left[Tx - Py + \frac{1}{x}\left(x'y - \Lambda y + y^2/2\ell^2\right)\right]$$

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Claim : $S_{\mathrm{E}} =$ **BMS-Schwarzian** $+ \, c \times \alpha_0$

2. BMS$_2$ group and its orbit

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A. Defining BMS_2

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C. Orbit and Schwarzian

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COADJOINT REP

[Action of group on $\underbrace{\text{dual of Lie algebra}}_{\text{"currents"}}$]

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Finite tsfs ?

ORBIT AND SCHWARZIAN

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- $\exists!$ f such that $\boldsymbol{\widetilde{Q} = 0}$

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Recall metric $\mathrm{d}s^2 \approx -2 \quad \mathrm{d}u \quad \mathrm{d}r - \big(\quad 2r \quad (c + Q') + 2T\big)\mathrm{d}u^2$

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- $\boldsymbol{c = -in}$ gives n-fold cover of **Euclidean plane**

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Real BMS$_2$ group has unique orbit

- What if we complexify ?
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B. Partition fct of BMS-Schwarzian + zero-mode

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Partition fct in **AdS_2/Schwarzian**

[Stanford-Witten 17]

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Smth similar with **Flat JT/BMS-Schwarzian** ?

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- To be continued...

Thanks for listening!