Berry Phases and Drift in the KdV Equation

Blagoje Oblak

LPTHE (Sorbonne) & CPHT (Polytechnique)

October 2021

Based on arXiv 1703.06142 (*JHEP*)
1907.01438 (*J Math Fluid Mech*)
2002.01780 (*Chaos*) w/ G. Kozyreff

Berry Phases and Drift in the KdV Equation

Blagoje Oblak

LPTHE (Sorbonne) & CPHT (Polytechnique)

October 2021

Based on arXiv 1703.06142 (*JHEP*)
1907.01438 (*J Math Fluid Mech*)
2002.01780 (*Chaos*) w/ G. Kozyreff

Berry Phases and Drift in the KdV Equation

Berry Phases and Drift in the KdV Equation

Context :

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity
- 2016–2019 Fluids

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity
- 2016–2019 Fluids
- 2019–present Quantum Hall effect

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity
- 2016–2019 Fluids
- 2019–present Quantum Hall effect

Common thread : **diffeomorphisms** (continuous deformations)

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity [space-time diffeos]
- 2016–2019 Fluids
- 2019–present Quantum Hall effect

Common thread : **diffeomorphisms** (continuous deformations)

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity [space-time diffeos]
- 2016–2019 Fluids [flows = time-dependent diffeos]
- 2019–present Quantum Hall effect

Common thread : **diffeomorphisms** (continuous deformations)

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity [space-time diffeos]
- 2016–2019 Fluids [flows = time-dependent diffeos]
- 2019–present Quantum Hall effect [area-preserving diffeos]

Common thread : **diffeomorphisms** (continuous deformations)

Berry Phases and Drift in the KdV Equation

Context :

- 2012–2016 Quantum gravity [space-time diffeos]
- 2016–2019 Fluids [flows = time-dependent diffeos]
- 2019–present Quantum Hall effect [area-preserving diffeos]

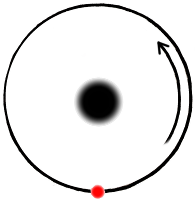
Common thread : **diffeomorphisms** (continuous deformations)

- This talk : **Berry phases** due to diffeos in **fluids**

MOTIVATION: THOMAS PRECESSION

[Thomas 1926]

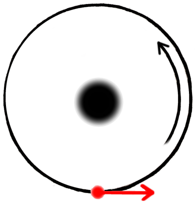
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

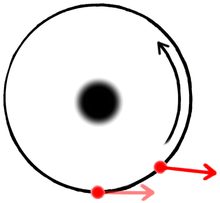
Electron orbiting atomic nucleus :



MOTIVATION: THOMAS PRECESSION

[Thomas 1926]

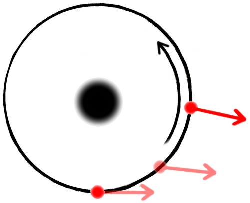
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

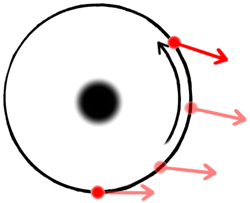
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

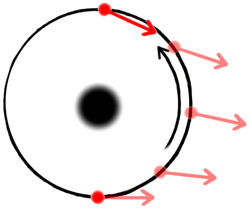
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

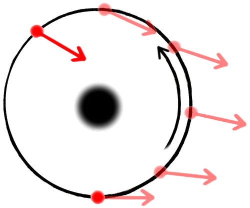
Electron orbiting atomic nucleus :



MOTIVATION: THOMAS PRECESSION

[Thomas 1926]

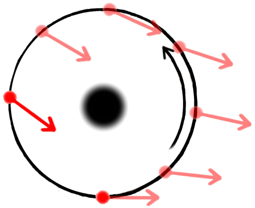
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

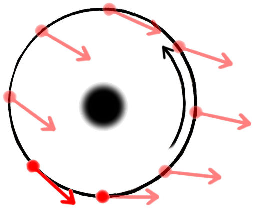
Electron orbiting atomic nucleus :



MOTIVATION: THOMAS PRECESSION

[Thomas 1926]

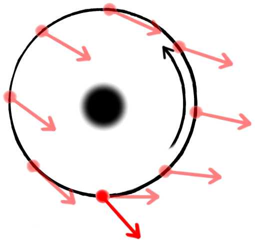
Electron orbiting atomic nucleus :



Motivation: Thomas precession

[Thomas 1926]

Electron orbiting atomic nucleus :

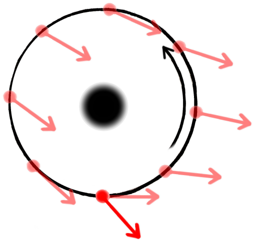


Motivation: Thomas precession

[Thomas 1926]

Electron orbiting atomic nucleus :

- Rotation after one period

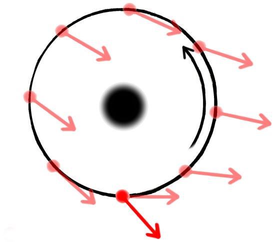


Motivation: Thomas precession

[Thomas 1926]

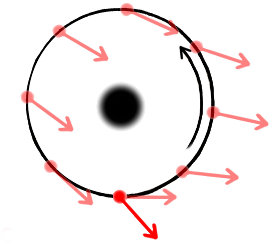
Electron orbiting atomic nucleus :

- Rotation after one period
- Actuallly **Berry phase**
  from adiabatic Lorentz boosts



MOTIVATION: THOMAS PRECESSION

[Thomas 1926]



Electron orbiting atomic nucleus :

- Rotation after one period
- Actuallly **Berry phase** from adiabatic Lorentz boosts

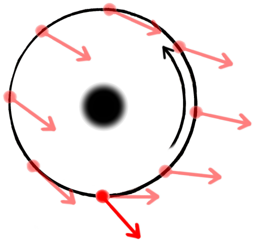
Question : ∃? Berry phases for *any* unitary group action ?

Motivation: Thomas precession

[Thomas 1926]

Electron orbiting atomic nucleus :

- Rotation after one period
- Actuallly **Berry phase** from adiabatic Lorentz boosts



Question : ∃? Berry phases for *any* unitary group action ?

- Yes !

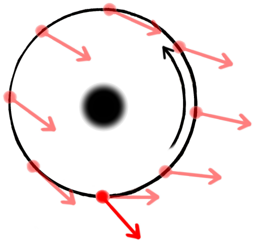
[Berry 1984, Jordan 1987]

Motivation: Thomas precession

[Thomas 1926]

Electron orbiting atomic nucleus :

- Rotation after one period
- Actuallly **Berry phase** from adiabatic Lorentz boosts



Question : ∃? Berry phases for *any* unitary group action ?

- Yes ! [Berry 1984, Jordan 1987]
- What about sample deformations ?

THIS TALK IN TWO STATEMENTS

This talk in two statements

1. Sample **deformations** produce **Berry phases**

THIS TALK IN TWO STATEMENTS

1. Sample **deformations** produce **Berry phases** (Virasoro Berry phases) [B.O. 2017]

THIS TALK IN TWO STATEMENTS

1. Sample **deformations** produce **Berry phases** (Virasoro Berry phases) [B.O. 2017]

2. **KdV drift velocity** yields dynamical realization [B.O. & Kozyreff 2020]

Plan of the talk

1. **Virasoro Berry phases**
2. Drift velocity in KdV
3. Berry phases in KdV

Plan of the talk

1. Virasoro Berry phases
2. **Drift velocity in KdV**
3. Berry phases in KdV

PLAN OF THE TALK

1. Virasoro Berry phases

2. Drift velocity in KdV

3. **Berry phases in KdV**

1. Virasoro Berry phases

1. Virasoro Berry phases

A. Reminder on Berry phases

1. Virasoro Berry phases

A. Reminder on Berry phases

B. Berry phases of Diff S^1

1. Virasoro Berry phases

A. Reminder on Berry phases

B. Berry phases of Diff S^1

C. Berry phases of Virasoro

Berry phases in general

Reminder on Berry phases :

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g

BERRY PHASES IN GENERAL

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow$ g_t

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow$ g_t
- Wavefct picks phase $\oint \mathrm{d}t\, E \quad - i \oint \mathrm{d}t\, \langle\psi(g_t)|\frac{\partial}{\partial t}|\psi(g_t)\rangle$

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow$ $\boldsymbol{g_t}$
- Wavefct picks phase $\underbrace{\oint \mathrm{d}t\, E}_{\text{Dynamical } \phi} - i \oint \mathrm{d}t \left\langle \psi(g_t) \middle| \frac{\partial}{\partial t} \middle| \psi(g_t) \right\rangle$

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow \boldsymbol{g_t}$
- Wavefct picks phase $\underbrace{\oint \mathrm{d}t\, E}_{\text{Dynamical } \phi} - \underbrace{i \oint \mathrm{d}t\, \langle \psi(g_t) | \frac{\partial}{\partial t} | \psi(g_t) \rangle}_{\textbf{Berry } \phi}$

Berry phases in general

Reminder on Berry phases :

- Quantum system depending on **parameters** g
- Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow$ $\boldsymbol{g_t}$
- Wavefct picks phase $\underbrace{\oint \mathrm{d}t\, E}_{\text{Dynamical } \phi} - \underbrace{i \oint \mathrm{d}t \, \langle \psi(g_t) | \frac{\partial}{\partial t} | \psi(g_t) \rangle}_{\textbf{Berry } \phi}$

What Berry phase is produced by sample diffeos ?

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1

BERRY PHASES OF DIFFEOS

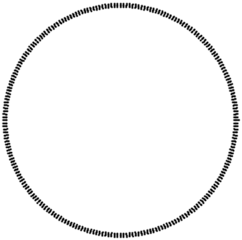
Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

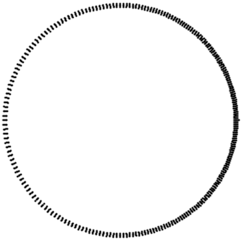
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

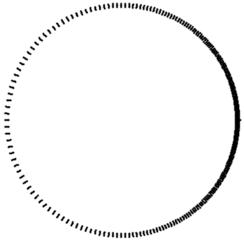
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

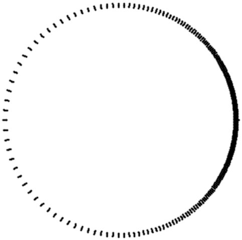
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

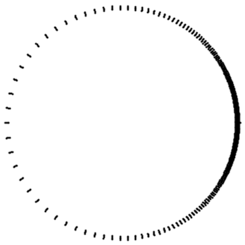
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

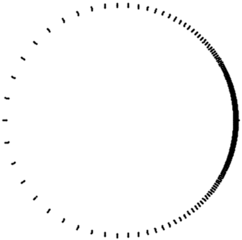
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

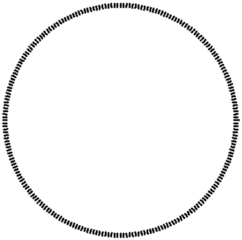
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

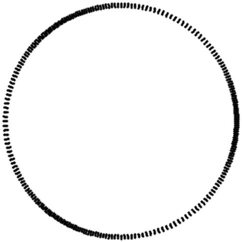
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

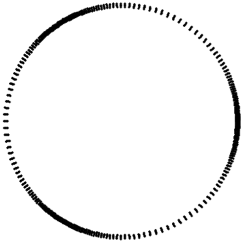
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

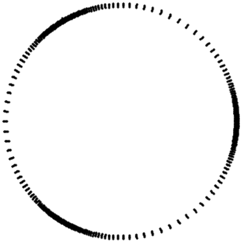
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

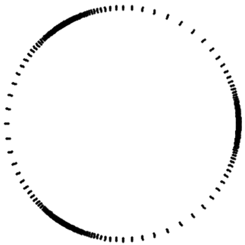
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

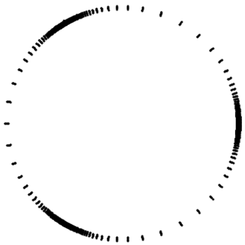
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

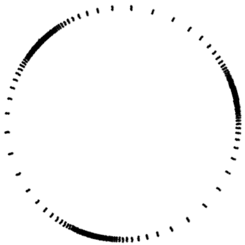
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

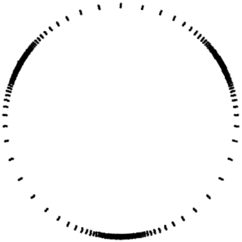
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

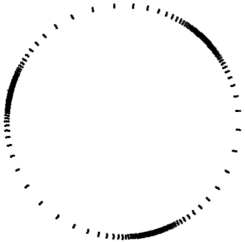
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

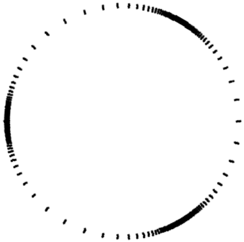
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

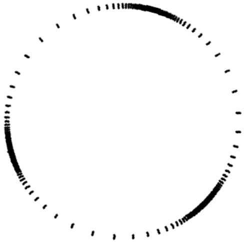
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

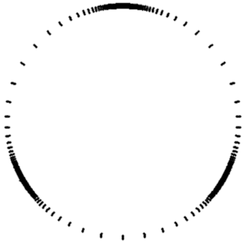
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

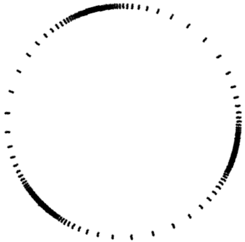
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

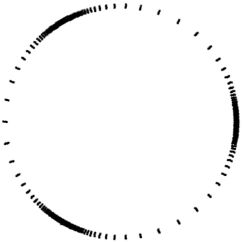
- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**



BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\mathcal{U}[g]\psi$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\big(\mathcal{U}[g]\psi\big)\big(g(x)\big)$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\left(\mathcal{U}[g]\psi\right)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $(\mathcal{U}[g]\psi)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $(\mathcal{U}[g]\psi)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $(\mathcal{U}[g]\psi)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\big(\mathcal{U}[g]\psi\big)\big(g(x)\big) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \mathrm{d}t \, \langle \psi | \mathcal{U}[g_t]^\dagger \dfrac{\partial}{\partial t} \mathcal{U}[g_t] | \psi \rangle$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\left(\mathcal{U}[g]\psi\right)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \mathrm{d}t \, \langle\psi|\mathcal{U}[g_t]^\dagger \dfrac{\partial}{\partial t} \mathcal{U}[g_t]|\psi\rangle = \oint \mathrm{d}t \, \mathrm{d}x \, \dfrac{\dot{g}}{g'} k$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $(\mathcal{U}[g]\psi)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \mathrm{d}t \, \langle \psi | \mathcal{U}[g_t]^\dagger \dfrac{\partial}{\partial t} \mathcal{U}[g_t] | \psi \rangle = \oint \mathrm{d}t \, \mathrm{d}x \, \dfrac{\dot{g}}{g'} \, k$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\left(\mathcal{U}[g]\psi\right)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \mathrm{d}t \, \langle\psi|\mathcal{U}[g_t]^\dagger \dfrac{\partial}{\partial t}\mathcal{U}[g_t]|\psi\rangle = \oint \mathrm{d}t \, \mathrm{d}x \, \dfrac{\dot{g}}{g'} k$

BERRY PHASES OF DIFFEOS

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\left(\mathcal{U}[g]\psi\right)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \langle \psi | \ \mathfrak{u}[\, g^{-1} \mathrm{d} g \,] \ | \psi \rangle = \oint \mathrm{d}t \, \mathrm{d}x \, \dfrac{\dot{g}}{g'} k$

Berry phases of diffeos

Particle on a circle $\Rightarrow$ position $x \sim x + 2\pi$

- Wavefct $\psi(x)$ on S^1
- Let $x \mapsto g(x)$ be a **diffeo**
- Rotations $g(x) = x + \theta$
- Unitary action of Diff S^1 : $\left(\mathcal{U}[g]\psi\right)(g(x)) = \dfrac{\psi(x)}{\sqrt{g'(x)}}$

Choose adiabatic, cyclic $g_t(x)$

- Berry phases ?
- Choose $\psi(x) = e^{ikx}$
- Berry $\phi = i \oint \underbrace{\langle\psi| \;\, \mathfrak{u}[\, g^{-1}\, \mathrm{d}\, g\,] \;\, |\psi\rangle}_{\textbf{Berry connection}} = \oint \mathrm{d}t\, \mathrm{d}x\, \dfrac{\dot{g}}{g'}\, k$

BERRY PHASES OF VIRASORO

$$\text{For Diff}\, S^1 : \text{Berry} = \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\, k$$

Berry phases of Virasoro

$$\text{For Diff}\,S^1:\ \text{Berry} = \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\,k$$

- **Virasoro group** = Diff S^1 with **central charge**

Berry phases of Virasoro

$$\text{For Diff}\, S^1 : \text{Berry} = \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\, k$$

- **Virasoro group** = Diff S^1 with **central charge** (recall CFT_2)

Berry phases of Virasoro

$$\text{For Diff}\,S^1 : \text{Berry} = \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\,k$$

- **Virasoro group** = Diff S^1 with **central charge** (recall CFT_2)
- Extra term in Berry phase :

Berry phases of Virasoro

$$\text{For Diff}\,S^1: \text{Berry} = \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\,k$$

- **Virasoro group** = Diff S^1 with **central charge** (recall CFT_2)
- Extra term in Berry phase :

$$\text{Berry } \phi = \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\left[k + c\left(\frac{g''}{g'}\right)'\right]$$

[B.O. 2017]

Berry phases of Virasoro

$$\text{For Diff } S^1 : \text{Berry} = \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\, k$$

- **Virasoro group** = Diff S^1 with **central charge** (recall CFT_2)
- Extra term in Berry phase :

$$\text{Berry } \phi = \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'} \left[k + c \left(\frac{g''}{g'} \right)' \right]$$

[B.O. 2017]

Berry phases of Virasoro

For Diff S^1 : Berry $= \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\,k$

- **Virasoro group** = Diff S^1 with **central charge** (recall CFT_2)
- Extra term in Berry phase :

$$\text{Berry } \phi = \oint \mathrm{d}t\,\mathrm{d}x\,\frac{\dot{g}}{g'}\left[k + c\left(\frac{g''}{g'}\right)'\right]$$

[B.O. 2017]

Question : ∃? dynamical system exhibiting such phases ?

2. Drift velocity in KdV

2. Drift velocity in KdV

A. Reconstruction of KdV

2. Drift velocity in KdV

A. Reconstruction of KdV

B. Digression on fluids

2. Drift velocity in KdV

A. Reconstruction of KdV

B. Digression on fluids

C. Periodic waves and drift

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos**

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$:

$$\dot{x}(t) = p(x(t),t)$$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$:
- Write $x(t) = g_t \circ g_0^{-1}(x_0)$

$$\dot{x}(t) = p(x(t),t)$$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$:
- Write $x(t) = g_t \circ g_0^{-1}(x_0)$:

$$\dot{g}_t \circ g_t^{-1}(x) = p(x,t)$$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$:
- Write $x(t) = g_t \circ g_0^{-1}(x_0)$:

$$\dot{g}\,g^{-1} = p$$

Reconstruction of KdV

$p(x,t)$: wave profile on S^1

- **Korteweg-de Vries (KdV)** : $\dot{p} = -3\,pp' + c\,p'''$
- $p(x,t)$ **sources its own diffeos** $g_t(x)$

How to find $g_t(x)$?

- Consider "fluid particle" @ $x(t)$:
- Write $x(t) = g_t \circ g_0^{-1}(x_0)$:

$$\dot{g}\,g^{-1} = p$$

- $g_t(x) =$ **reconstruction** of $p(x,t)$

Reconstruction of KdV

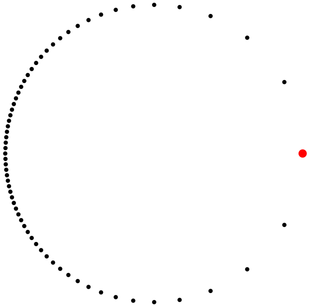
Simulate $\dot{x} = p(x,t)$

Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:

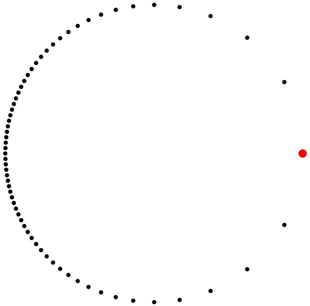
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



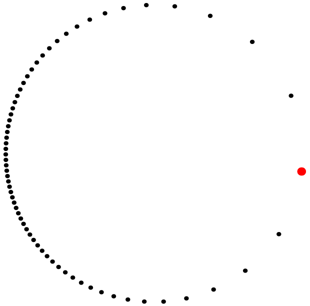
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



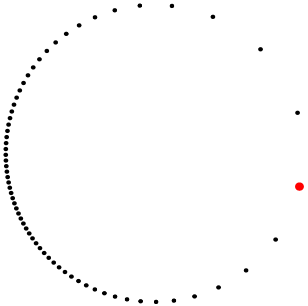
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



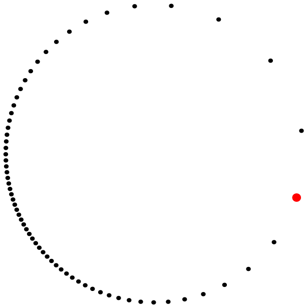
Reconstruction of KdV

Simulate $\dot{x} = p(x, t)$ or $\dot{g}\, g^{-1} = p$:



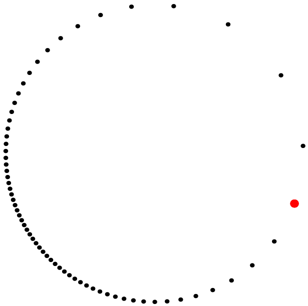
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



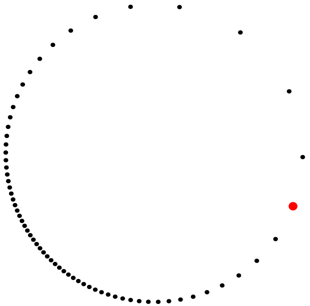
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\, g^{-1} = p$:



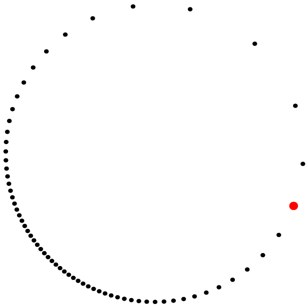
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



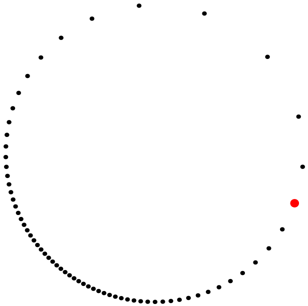
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



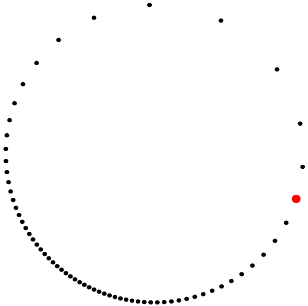
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



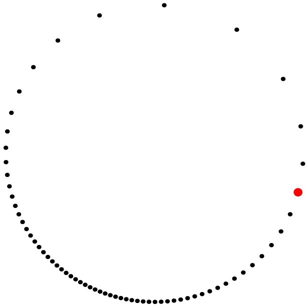
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



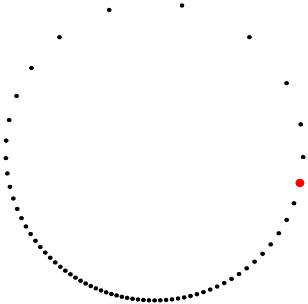
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



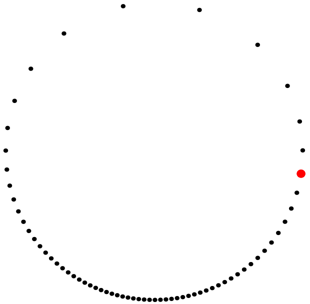
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



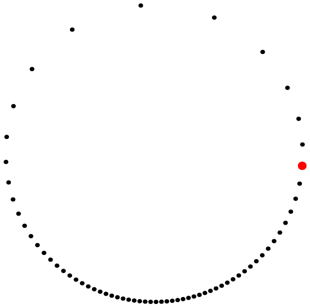
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



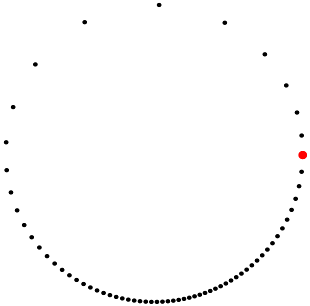
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



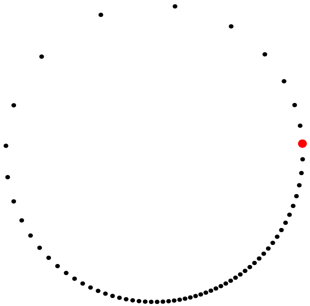
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



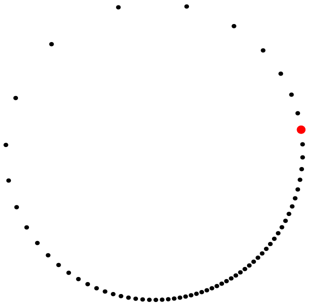
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



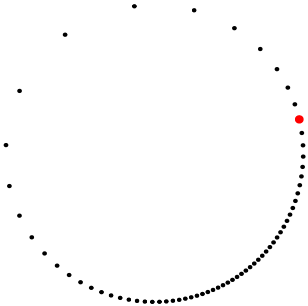
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



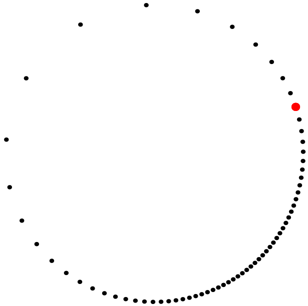
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



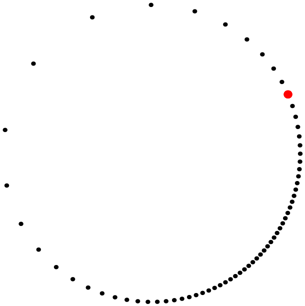
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



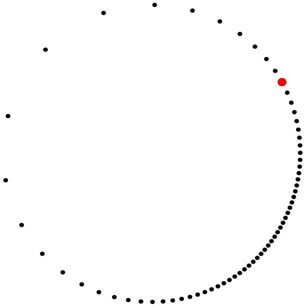
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



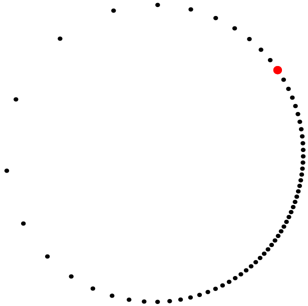
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



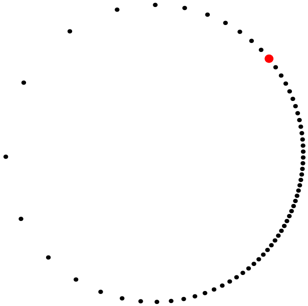
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



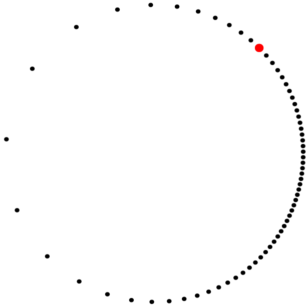
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



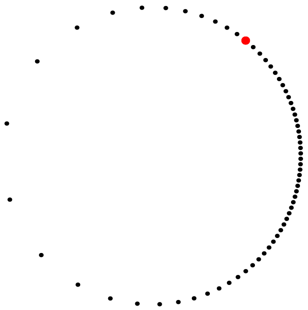
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



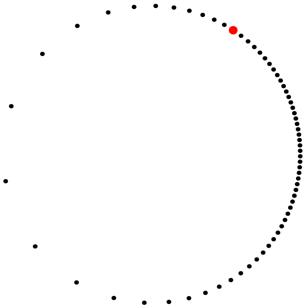
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



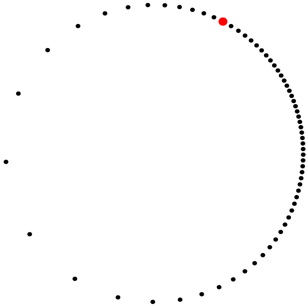
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



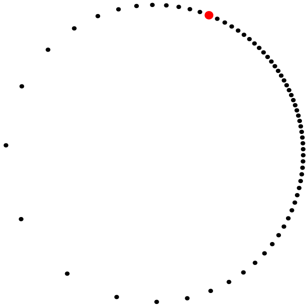
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



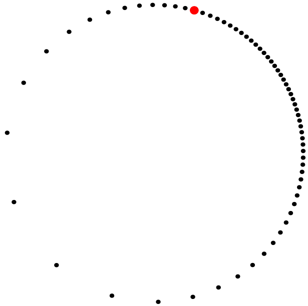
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



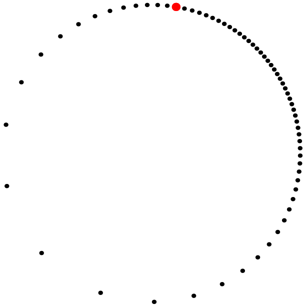
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



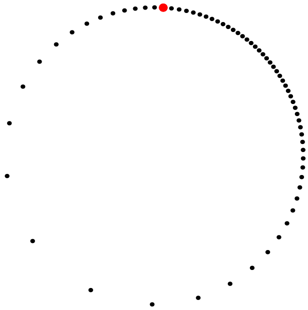
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



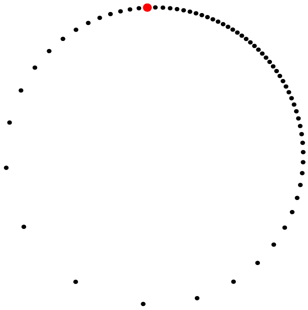
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



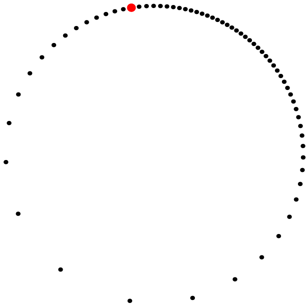
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



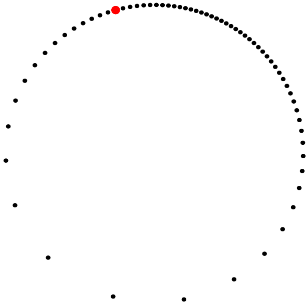
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



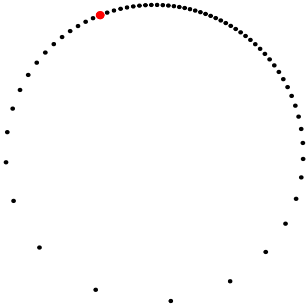
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



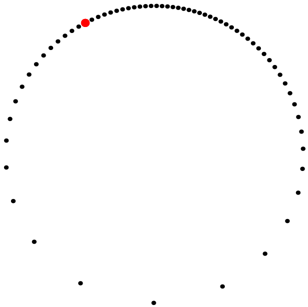
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



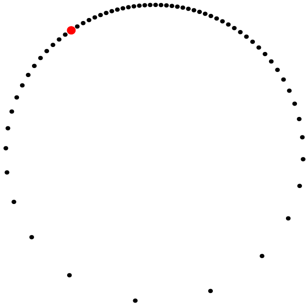
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



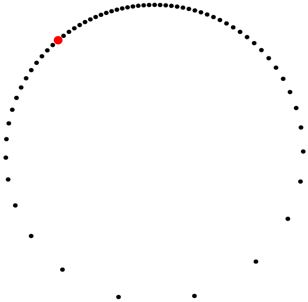
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



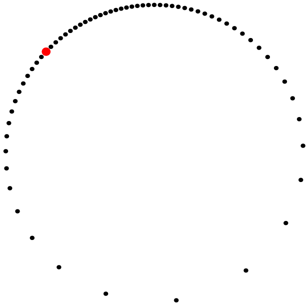
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



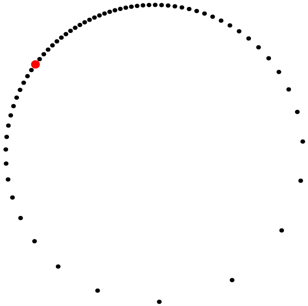
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



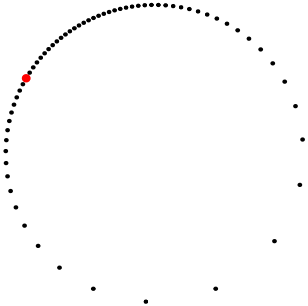
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



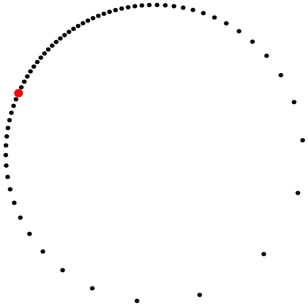
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\, g^{-1} = p$:



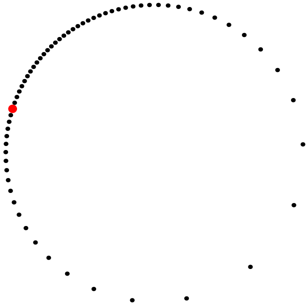
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



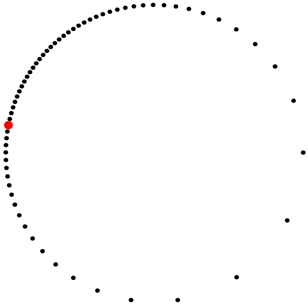
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



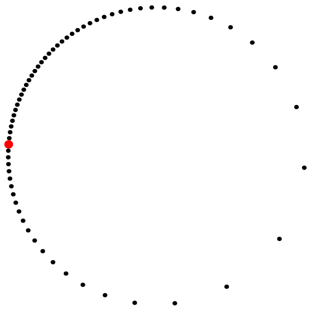
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



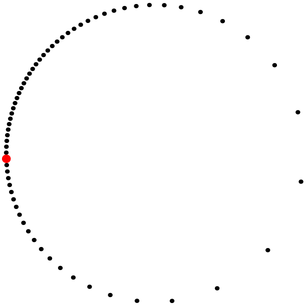
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



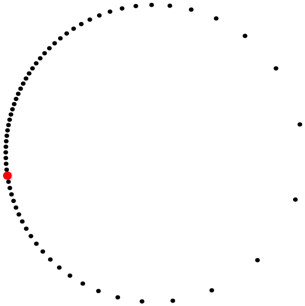
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



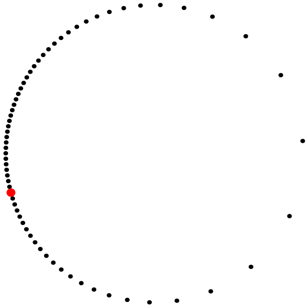
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



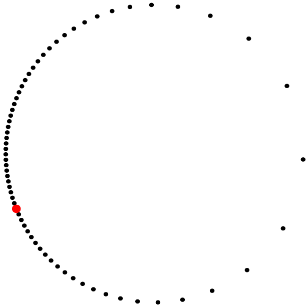
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\, g^{-1} = p$:



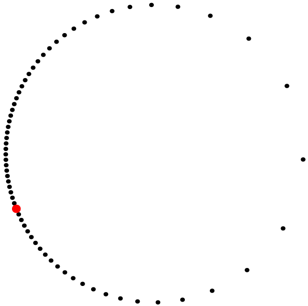
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



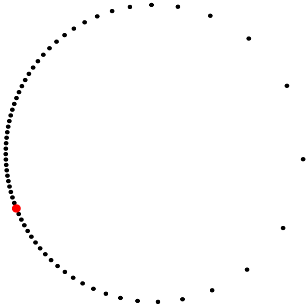
RECONSTRUCTION OF KDV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



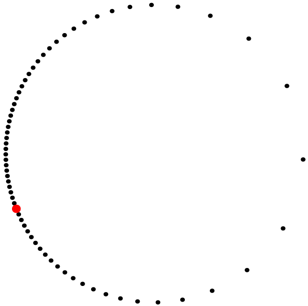
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



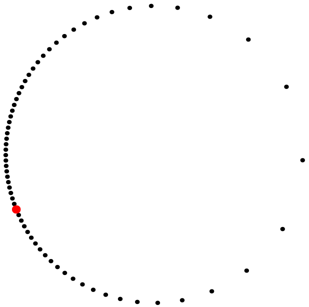
Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



Reconstruction of KdV

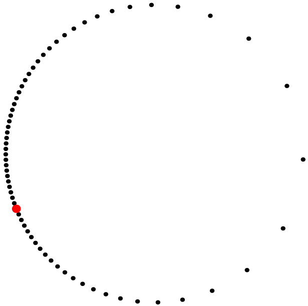
Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



Similar to hydrodynamics !

Reconstruction of KdV

Simulate $\dot{x} = p(x,t)$ or $\dot{g}\,g^{-1} = p$:



Similar to hydrodynamics !

- Is it really the same ?

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate

Digression on fluids

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth

Digression on fluids

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt$ = "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

Is this **reconstruction** ?

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt$ = "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

Is this **reconstruction** ? [$\dot{x} = p$]

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt$ = "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

Is this **reconstruction** ? [$\dot{x} = p$]

- Shift by V

Digression on fluids

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

Is this **reconstruction** ? $[\dot{x} = p]$

- Shift by V

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt$ = "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - \boldsymbol{V}$

Is this **reconstruction** ? $[\dot{x} = p]$

- Shift by V absorbed by redefining Virasoro

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = 2\,p(x(t),t) - V$

Is this **reconstruction** ? [$\dot{x} = p$]

- Shift by V absorbed by redefining Virasoro
- Factor 2 mismatch

Digression on fluids

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt$ = "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = \mathbf{2}\, p(x(t),t) \;\mathbf{-\,V}$

Is this **reconstruction** ? [$\dot{x} = p$]

- Shift by V absorbed by redefining Virasoro
- Factor 2 mismatch

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = \mathbf{2}\,p(x(t),t) \mathbf{- V}$

Is this **reconstruction** ? [$\dot{x} = p$]

- Shift by V absorbed by redefining Virasoro
- Factor 2 mismatch is unsolved...

DIGRESSION ON FLUIDS

KdV $\sim$ **shallow water** [Korteweg & de Vries, 1895]

- $x = X - Vt =$ "light-cone" coordinate
- $p(x,t) \sim$ deviation wrt average depth
- $\dot{x}(t) = \mathbf{2}\,p(x(t),t) - \boldsymbol{V}$

Is this **reconstruction** ? [$\dot{x} = p$]

- Shift by V absorbed by redefining Virasoro
- Factor 2 mismatch is unsolved...
- Similar, but not the same

PERIODIC WAVES

Think of $p(x,t)$ as ∞ time-dependent **parameters**

Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+T)=p(x,t)$?

PERIODIC WAVES

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\boldsymbol{T}) = p(x,t)$?

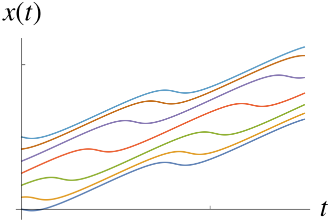
- Solve $\dot{x} = p(x,t)$ numerically

PERIODIC WAVES

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\boldsymbol{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically

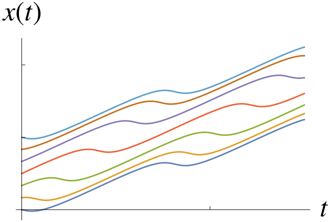


Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\boldsymbol{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"

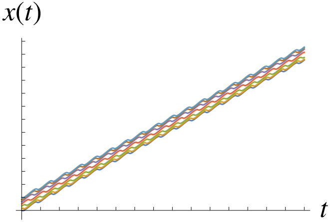


Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+T) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"

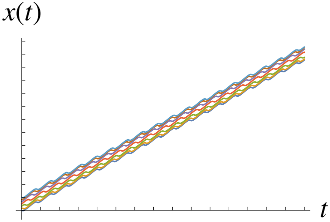


Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\boldsymbol{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**



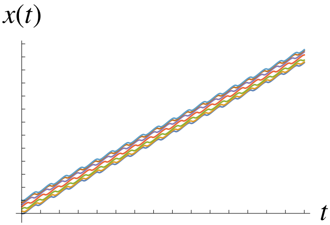
Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\mathbf{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**

Why ? And how fast ?

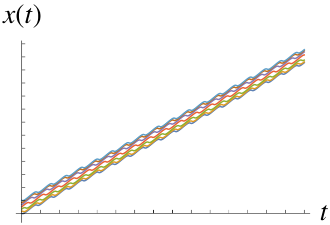


Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\boldsymbol{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

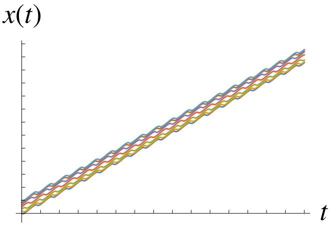
- Recon. *not* periodic : $g_T \neq g_0$

Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+T)=p(x,t)$?

- Solve $\dot{x}=p(x,t)$ numerically
- $x(t)\propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

- Recon. *not* periodic : $g_T \neq g_0$

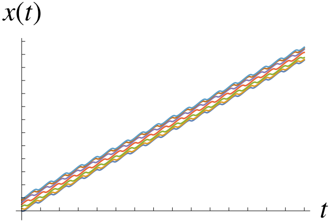
$$v_{\text{Drift}}=\lim_{t\to\infty}\frac{x(t)}{t}$$

Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+\mathbf{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

- Recon. *not* periodic : $\boldsymbol{g_T \neq g_0}$

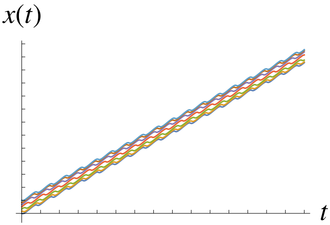
$$v_{\text{Drift}} = \lim_{t\to\infty} \frac{x(t)}{t} = \frac{1}{T} \lim_{n\to\infty} \frac{\left(g_T\, g_0^{-1}\right)^n (x_0)}{n}$$

Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x,t+T)=p(x,t)$?

- Solve $\dot{x}=p(x,t)$ numerically
- $x(t)\propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

- Recon. *not* periodic : $g_T \neq g_0$

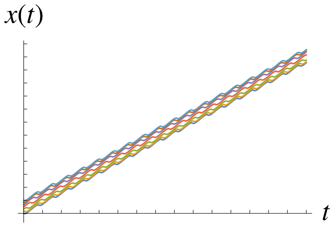
$$v_{\text{Drift}}=\lim_{t\to\infty}\frac{x(t)}{t}=\frac{1}{T}\lim_{n\to\infty}\frac{\left(g_T\,g_0^{-1}\right)^n(x_0)}{n}\equiv\frac{\Delta\phi}{T}$$

Periodic waves

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x, t + T) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

- Recon. *not* periodic : $g_T \neq g_0$

Rotation nbr of $g_T\, g_0^{-1}$

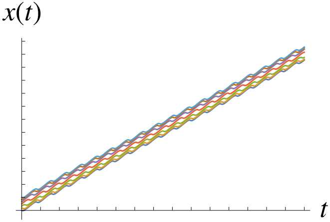
$$v_{\text{Drift}} = \lim_{t\to\infty} \frac{x(t)}{t} = \frac{1}{T} \lim_{n\to\infty} \frac{\left(g_T\, g_0^{-1}\right)^n (x_0)}{n} \equiv \frac{\Delta\phi}{T}$$

PERIODIC WAVES

Think of $p(x,t)$ as ∞ time-dependent **parameters**

What if $p(x, t + \mathbf{T}) = p(x,t)$?

- Solve $\dot{x} = p(x,t)$ numerically
- $x(t) \propto t$ "seen from far away"
- Particle **drifts**



Why ? And how fast ?

- Recon. *not* periodic : $\mathbf{g_T \neq g_0}$

Rotation nbr of $g_T\, g_0^{-1}$

$$v_{\text{Drift}} = \lim_{t\to\infty} \frac{x(t)}{t} = \frac{1}{T} \lim_{n\to\infty} \frac{\left(g_T\, g_0^{-1}\right)^n (x_0)}{n} \equiv \frac{\Delta\phi}{T}$$

Claim : $\boxed{\Delta\phi = \text{Dynamical } \phi + \textbf{Berry } \boldsymbol{\phi} + \text{Anomalous } \phi}$

3. Berry phases in KdV

Adiabatic diffeos
↘
Berry ϕ

3. Berry phases in KdV

KdV reconstruction

$\downarrow$

Adiabatic diffeos

$\searrow$

Berry ϕ

3. Berry phases in KdV

KdV reconstruction

$\downarrow$

Adiabatic diffeos

$\swarrow \qquad \searrow$

Drift $\qquad$ Berry ϕ

3. Berry phases in KdV

KdV reconstruction

$\downarrow$

Adiabatic diffeos

$\swarrow \qquad \searrow$

Drift $\xleftrightarrow{\;?\;}$ Berry ϕ

3. Berry phases in KdV

KdV reconstruction

↓

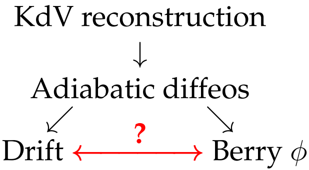
Adiabatic diffeos

↙ ↘

Drift ←?→ Berry ϕ

3. Berry phases in KdV

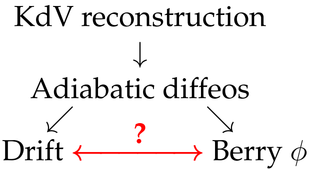
A. Euler equations



3. Berry phases in KdV

A. Euler equations

B. Phases in Euler equations



3. Berry phases in KdV

A. Euler equations

B. Phases in Euler equations

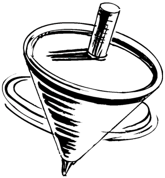
C. Phases in KdV

EULER EQUATIONS

Simple example : **free rigid body**

Euler equations

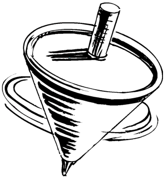
Simple example : **free rigid body**



Euler equations

Simple example : **free rigid body**

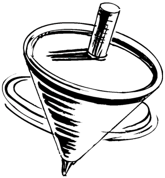
- Inertial frame : cst ang. momentum



EULER EQUATIONS

Simple example : **free rigid body**

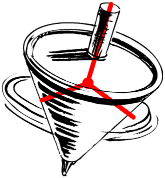
- Inertial frame : cst ang. momentum k



EULER EQUATIONS

Simple example : **free rigid body**

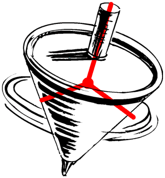
- Inertial frame : cst ang. momentum k



Euler equations

Simple example : **free rigid body**

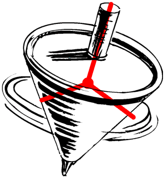
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t



Euler equations

Simple example : **free rigid body**

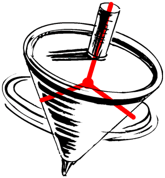
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t)$



Euler equations

Simple example : **free rigid body**

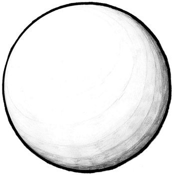
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

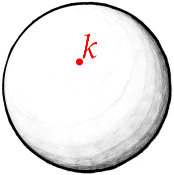
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

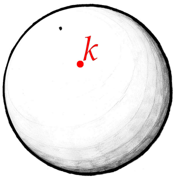
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

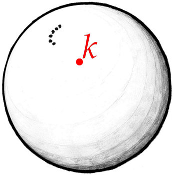
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

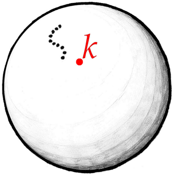
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

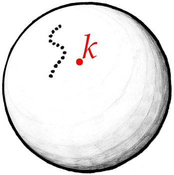
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

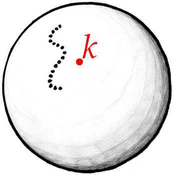
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

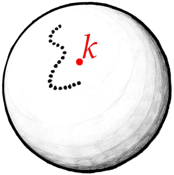
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

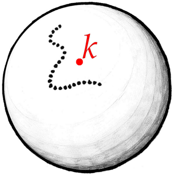
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

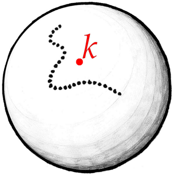
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

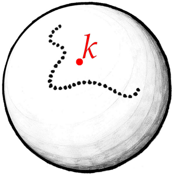
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

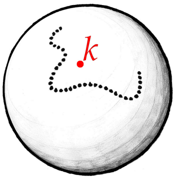
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

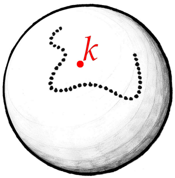
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

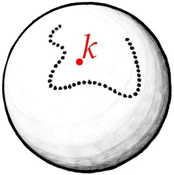
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



EULER EQUATIONS

Simple example : **free rigid body**

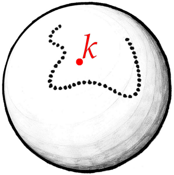
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Euler equations

Simple example : **free rigid body**

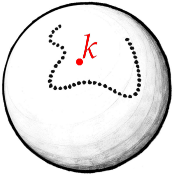
- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Dynamics :

EULER EQUATIONS

Simple example : **free rigid body**



- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$

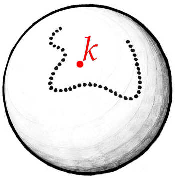
Dynamics :

- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$

Euler equations

Simple example : **free rigid body**

- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$



Dynamics :

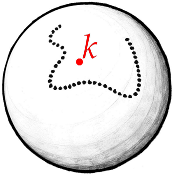
- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$

  $\downarrow$ inertia

Euler equations

Simple example : **free rigid body**

- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$

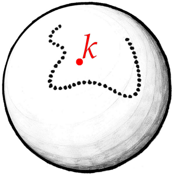


Dynamics :

- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$ ($\mathcal{I}$ ↓ inertia)
- Angular velocity $\sim$ momentum

Euler equations

Simple example : **free rigid body**



- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$

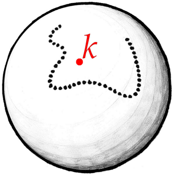
Dynamics :

- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$ ($\mathcal{I}$ ↓ inertia)
- Angular velocity ∼ momentum : $\dot{g}\,g^{-1} = \mathcal{I}^{-1}(p)$

EULER EQUATIONS

Simple example : **free rigid body**

- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$

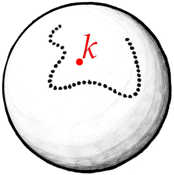


Dynamics :

- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$
  ($\mathcal{I}$ ↓ inertia)
- Angular velocity ∼ momentum : $\dot{g}\, g^{-1} = \mathcal{I}^{-1}(p)$
- **Reconstruct** g_t

Euler equations

Simple example : **free rigid body**



- Inertial frame : cst ang. momentum k
- Orientation = rotation g_t
- Attached frame : momentum $p(t) = g_t \cdot k$

Dynamics :

- **Euler equation** $\dot{p} = \mathcal{I}^{-1}(p) \wedge p$ ($\mathcal{I}$ ↓ inertia)
- Angular velocity ∼ momentum : $\dot{g}\, g^{-1} = \mathcal{I}^{-1}(p)$
- **Reconstruct** g_t

Generalization to any group ?

Euler equations

Take G Lie group

EULER EQUATIONS

Take G Lie group

- algebra $\mathfrak{g}$

EULER EQUATIONS

Take G Lie group

- algebra $\mathfrak{g}$
- dual $\mathfrak{g}^*$

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$
- dual $\mathfrak{g}^*$

Euler equations

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

Euler equations

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation**

Euler equations

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$

Euler equations

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction**

Euler equations

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction** : $\dot{g}g^{-1} \sim p$
- Find g_t

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$
- Find g_t
- $p(t) = g_t \cdot k$ with fixed k

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$
- Find g_t
- $p(t) = g_t \cdot k$ with fixed k

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$
- **Reconstruction** : $\dot{g}g^{-1} \sim p$
- Find g_t
- $p(t) = g_t \cdot k$ with fixed k

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$
- Find g_t
- $p(t) = g_t \cdot k$ with fixed k

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution $=$ path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t
- $p(t) = g_t \cdot k$ with fixed k

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t $(g_t(x))$
- $p(t) = g_t \cdot k$ with fixed k

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t $(g_t(x))$
- $p(t) = g_t \cdot k$ with fixed k (waves related by diffeos)

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t $(g_t(x))$
- $p(t) = g_t \cdot k$ with fixed k (waves related by diffeos)

Time-dep. changes of frames

EULER EQUATIONS

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution = path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t $(g_t(x))$
- $p(t) = g_t \cdot k$ with fixed k (waves related by diffeos)

Time-dep. changes of frames ($\sim$ Thomas precession)

Euler equations

Take G Lie group $\sim$ positions/configs g (Virasoro)

- algebra $\mathfrak{g}$ $\sim$ velocities $\dot{g}\,g^{-1}$
- dual $\mathfrak{g}^*$ $\sim$ momenta p (wave profiles)

Time evolution $=$ path in G

- **Euler equation** : $\dot{p} \sim p \wedge p$ (KdV)
- **Reconstruction** : $\dot{g}\,g^{-1} \sim p$ $(\dot{x} = p(x,t))$
- Find g_t $(g_t(x))$
- $p(t) = g_t \cdot k$ with fixed k (waves related by diffeos)

Time-dep. changes of frames ($\sim$ Thomas precession)

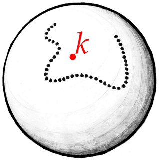
- What if $p(t+T) = p(t)$?

EULER PHASES

$$p(t+T) = p(t)$$

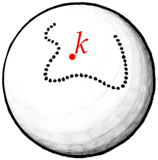
EULER PHASES

$$p(t+T) = p(t)$$



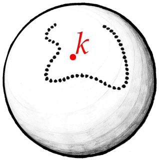
EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$



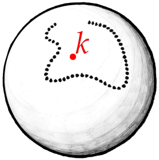
EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$



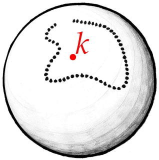
EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$



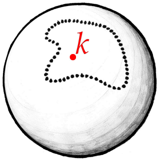
EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$



Euler phases

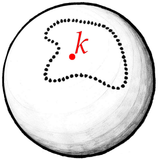
$$p(t+T) = p(t) = g_t \cdot k$$



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

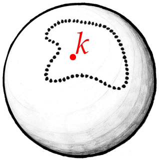
- $g_T \cdot k = g_0 \cdot k$



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

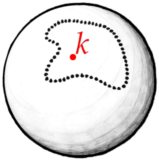
- $g_0^{-1} g_T \cdot k = k$



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

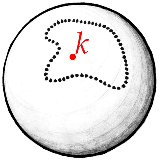
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

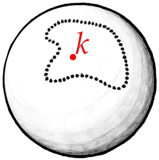
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

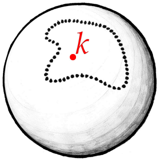
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

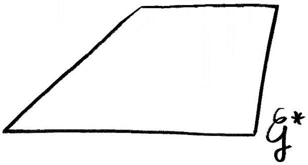
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

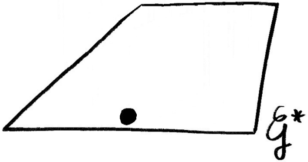
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

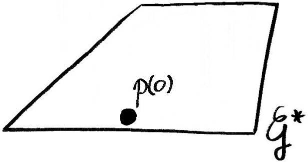
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

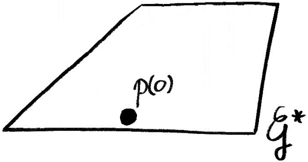
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

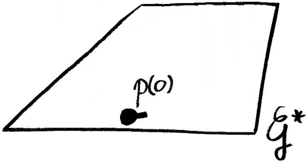
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

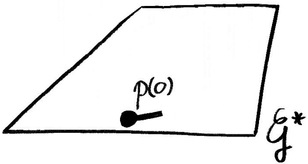
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

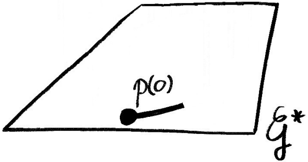
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

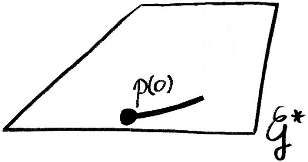
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

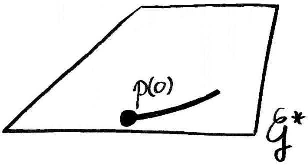
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

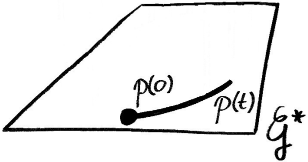
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

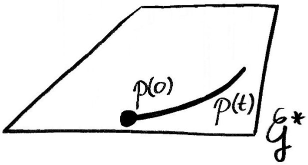
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

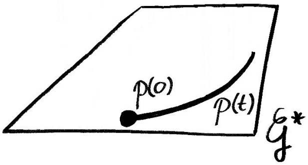
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

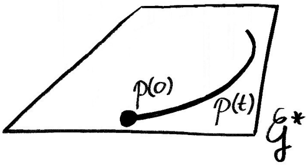
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

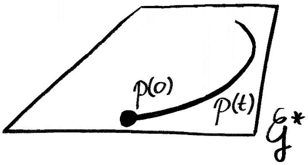
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

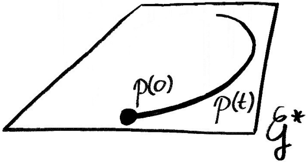
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

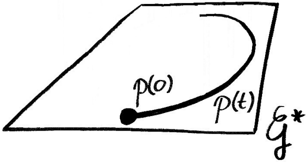
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

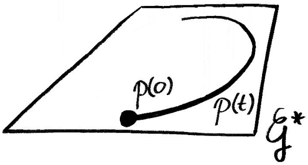
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

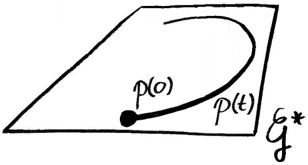
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

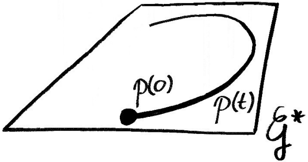
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

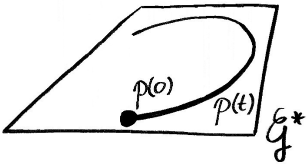
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

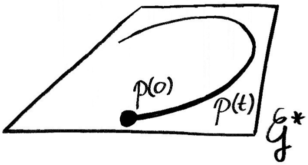
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

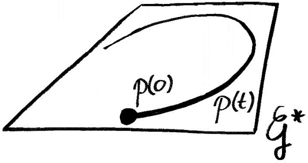
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

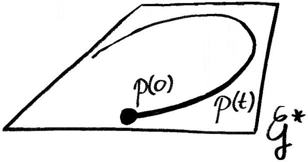
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

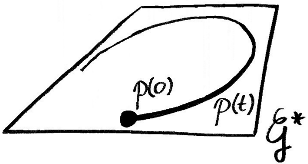
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

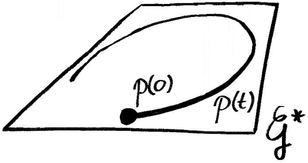
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

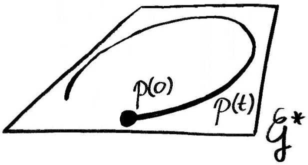
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

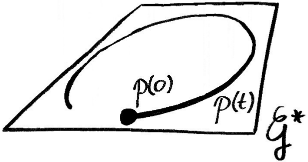
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

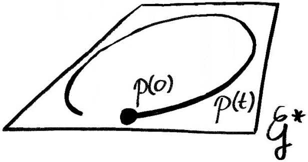
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

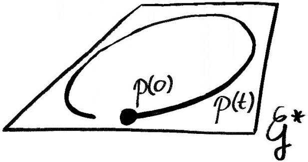
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

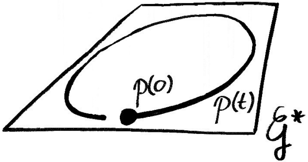
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

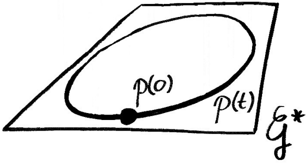
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

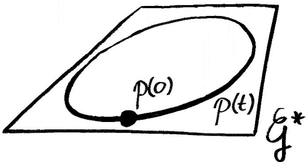
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k$

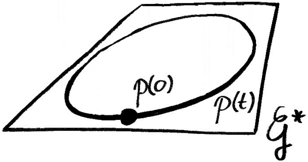
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k$

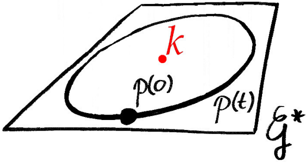
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k$$

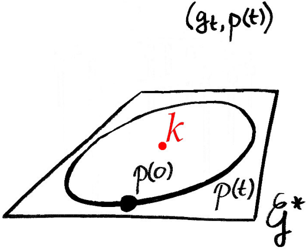
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

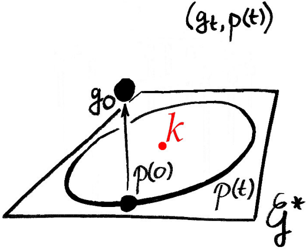
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k$$

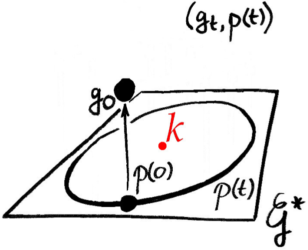
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

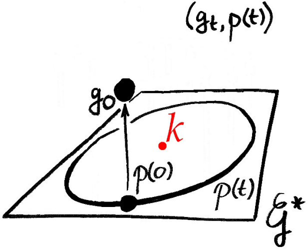
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

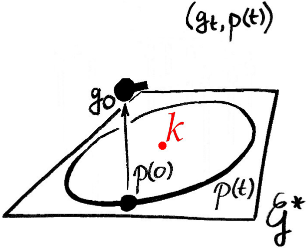
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

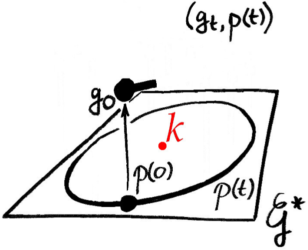
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

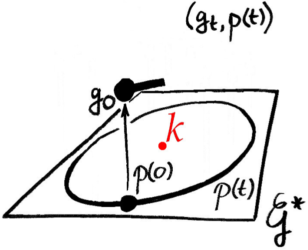
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

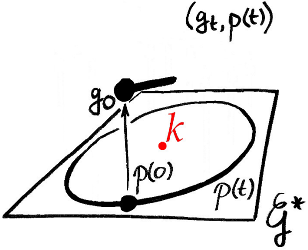
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

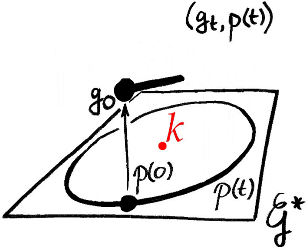
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

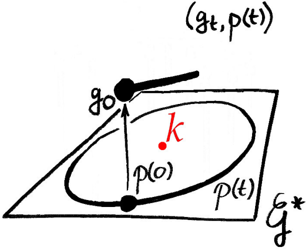
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

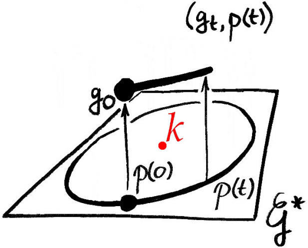
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

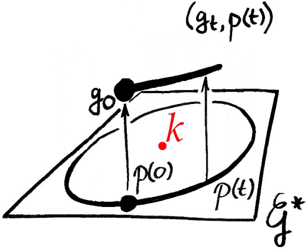
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

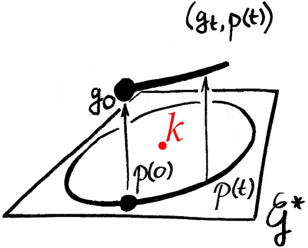
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

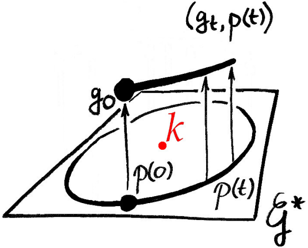
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T$ = rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

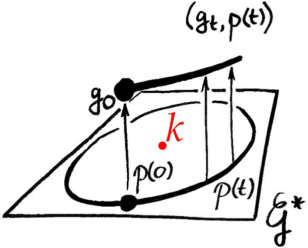
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

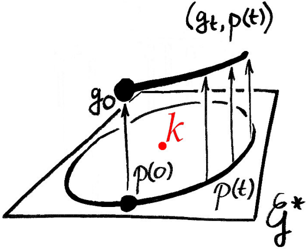
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

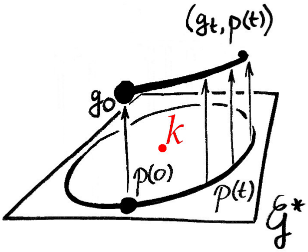
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

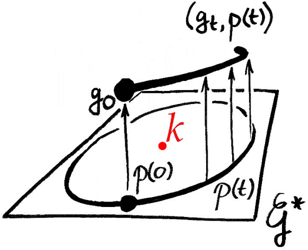
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

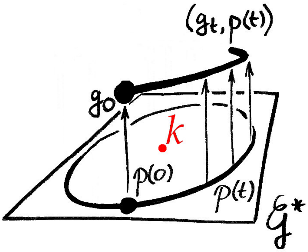
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

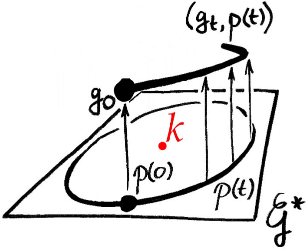
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

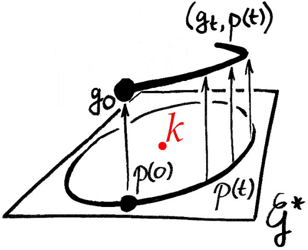
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

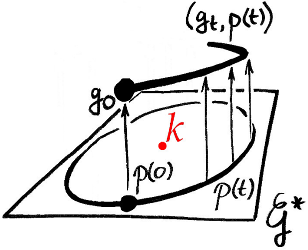
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

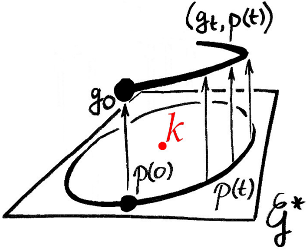
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

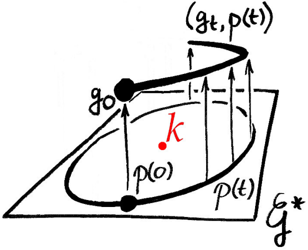
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

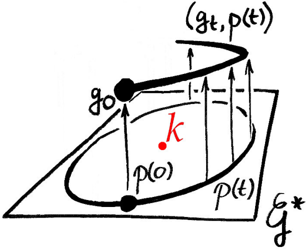
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

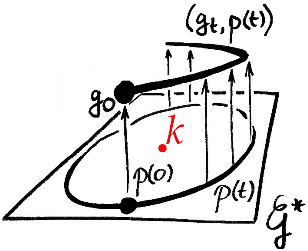
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

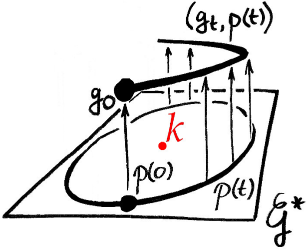
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

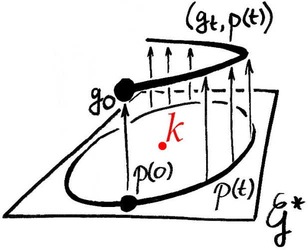
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T$ = rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

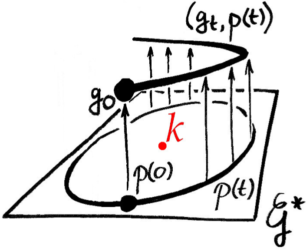
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

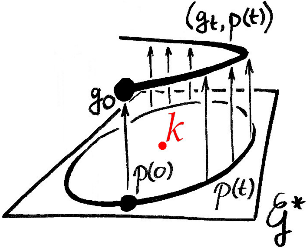
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

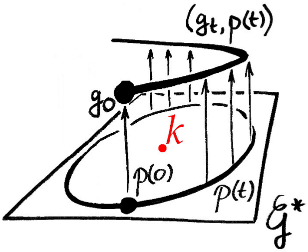
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

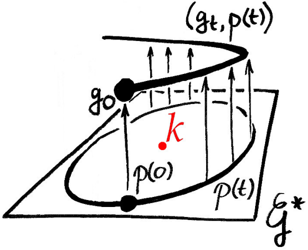
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

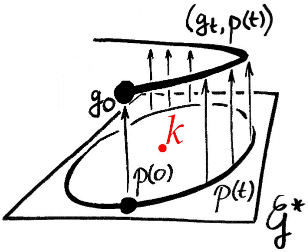
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

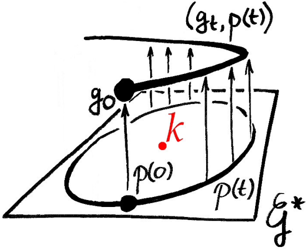
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

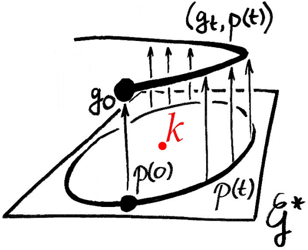
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

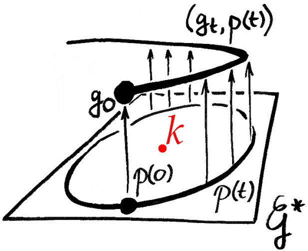
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

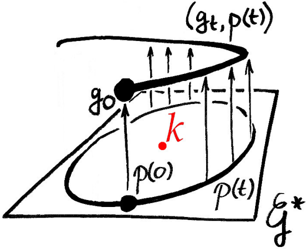
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

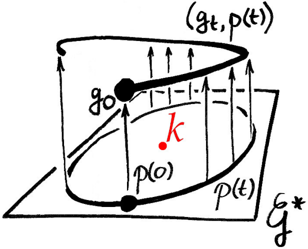
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

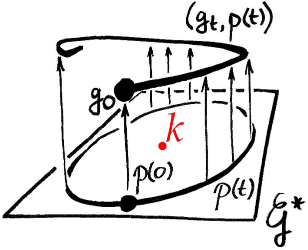
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

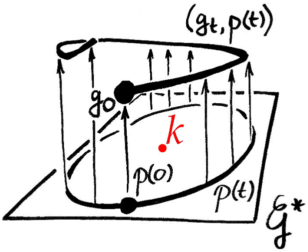
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

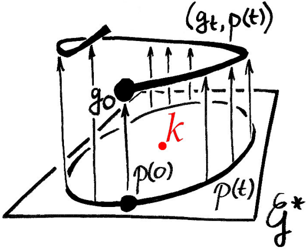
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

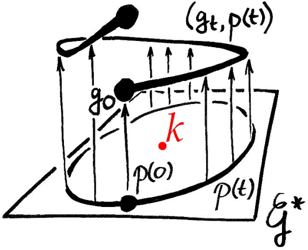
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

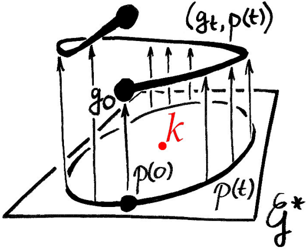
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

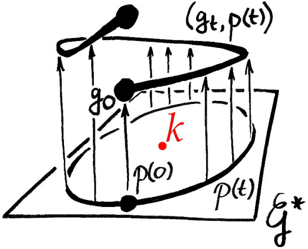
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

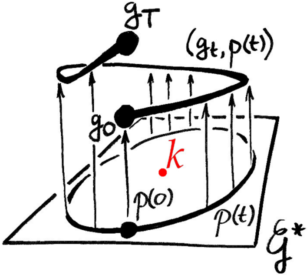
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

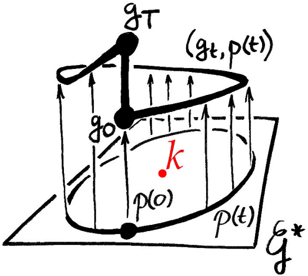
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

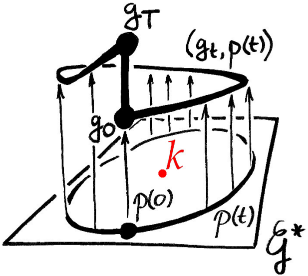
- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

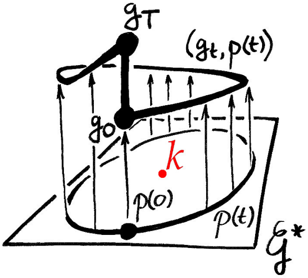
- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !

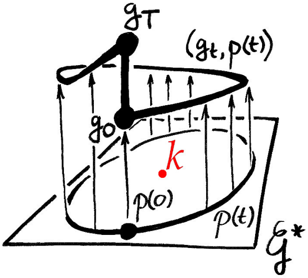


Recall **Berry connection** $\mathcal{A} \sim \langle\psi|\,g^{-1}\,\mathrm{d}\,g|\psi\rangle$

EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\Delta\phi$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !

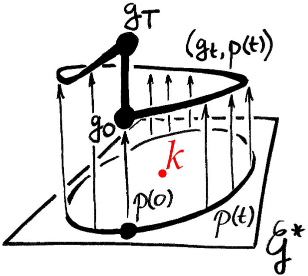


Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1}\, \mathrm{d} g | k \rangle$

Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



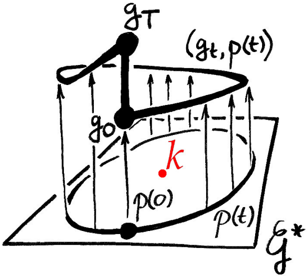
Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1} \mathrm{d} g | k \rangle$

- Classical version $\mathcal{A} = \langle k, g^{-1} \mathrm{d}g \rangle$

EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



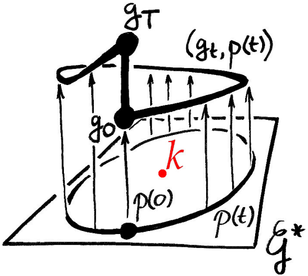
Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1} \mathrm{d} g | k \rangle$

- Classical version $\boldsymbol{\mathcal{A} = \langle k, g^{-1} \mathrm{d}g \rangle}$
- Integrate : $\oint \mathcal{A} = \int_{\text{helix}} \mathcal{A} + \int_{\text{straight}} \mathcal{A}$

Euler phases

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



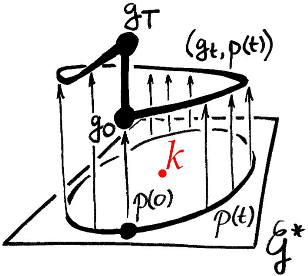
Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1} \mathrm{d} g | k \rangle$

- Classical version $\boldsymbol{\mathcal{A} = \langle k, g^{-1} \mathrm{d}g \rangle}$
- Integrate : $\displaystyle\oint \mathcal{A} = \underbrace{\int_{\text{helix}} \mathcal{A}}_{2ET} + \int_{\text{straight}} \mathcal{A}$

EULER PHASES

$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$

- $g_0^{-1} g_T \cdot k = k \qquad$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



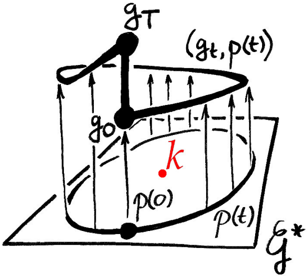
Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1} \,\mathrm{d}\, g | k \rangle$

- Classical version $\mathcal{A} = \langle k, g^{-1}\,\mathrm{d}g \rangle$
- Integrate : $\oint \mathcal{A} = \underbrace{\int_{\text{helix}} \mathcal{A}}_{2ET} + \underbrace{\int_{\text{straight}} \mathcal{A}}_{\propto \Delta\phi}$

EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\, g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1} \mathrm{d}\, g | k \rangle$

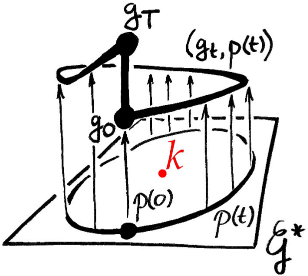
- Classical version $\boldsymbol{\mathcal{A} = \langle k, g^{-1}\, \mathrm{d}g \rangle}$
- Integrate : $\oint \mathcal{A} = \underbrace{\int_{\text{helix}} \mathcal{A}}_{2ET} + \underbrace{\int_{\text{straight}} \mathcal{A}}_{\propto \Delta\phi}$
- $\boldsymbol{\Delta\phi \propto 2ET - \oint \mathcal{A}}$

[Montgomery 1991]

EULER PHASES

$$p(t+T) = p(t) = g_t \cdot k \qquad (\dot{g}\,g^{-1} \sim p)$$

- $g_0^{-1} g_T \cdot k = k$ (stabilizes k)
- $g_0^{-1} g_T =$ rotation by $\boldsymbol{\Delta\phi}$
- How to compute $\Delta\phi$?
- In KdV, gives drift velocity !



Recall **Berry connection** $\mathcal{A} \sim \langle k | g^{-1}\,\mathrm{d}g | k \rangle$

- Classical version $\boldsymbol{\mathcal{A} = \langle k, g^{-1}\,\mathrm{d}g \rangle}$
- Integrate : $\displaystyle\oint \mathcal{A} = \underbrace{\int_{\text{helix}} \mathcal{A}}_{2ET} + \underbrace{\int_{\text{straight}} \mathcal{A}}_{\propto \Delta\phi}$
- $\boldsymbol{k\,\Delta\phi = 2ET - \oint \mathcal{A}}$

[Montgomery 1991]

Phases in KdV

PHASES IN KDV

Periodic KdV wave $p(x,t)$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

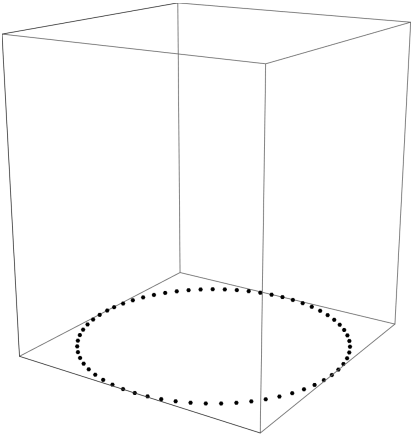
(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

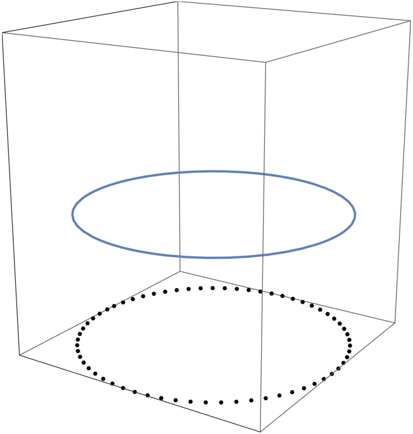
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

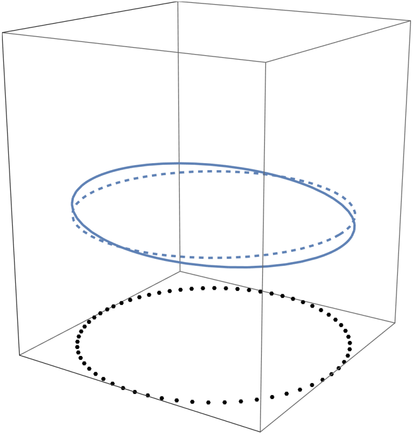
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

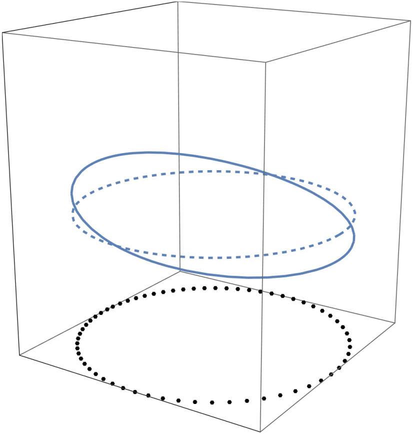
(conformal tsf of CFT stress tensors : $g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

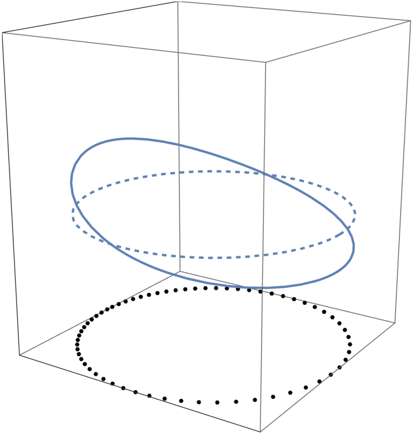
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

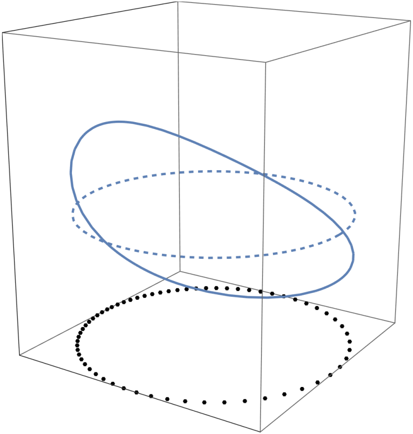
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

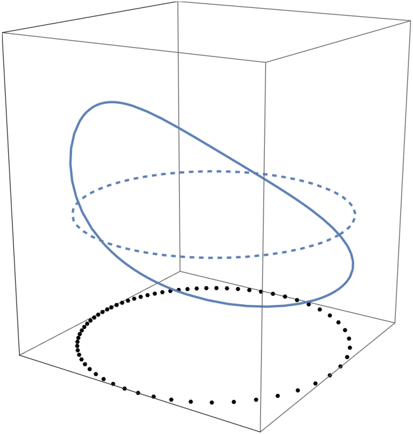
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

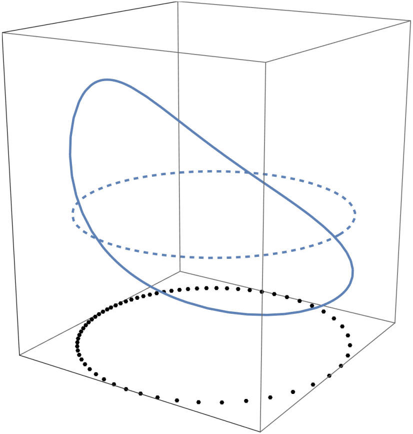
(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

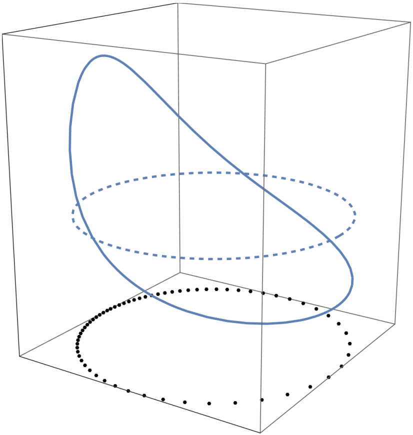
(conformal tsf of CFT stress tensors : $g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



PHASES IN KDV

Periodic KdV wave $p(x,t)=(g_t\cdot k)(x)$

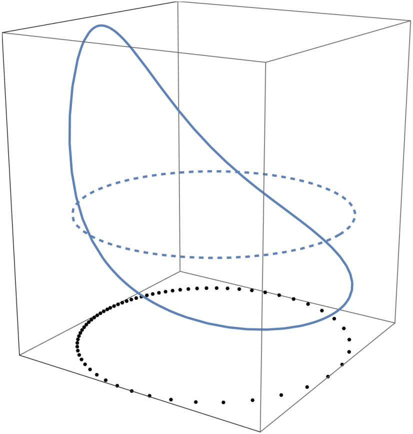
(conformal tsf of CFT stress tensors :
$g^{-1}\cdot k=k(g')^2-c[g'''/g'-\frac{3}{2}(g''/g')^2]$.)



PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)



PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)

- Assumption : **$k =$ constant** (flat profile)

PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].$)

- Assumption : **k = constant** (flat profile)
- k determined by p [Kirillov, Witten ~1987]

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

- Assumption : **k = constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

- Assumption : **$k =$ constant** (flat profile)
- k determined by p [Kirillov, Witten ~1987]
- k stabilized by rotations $\Rightarrow$ $g_0^{-1} g_T(x) = x + \Delta\phi$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

- Assumption : $k =$ **constant** (flat profile)
- k determined by p [Kirillov, Witten ~1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].$)

- Assumption : **k = constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \boldsymbol{\Delta\phi}$

Phase formula :

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors : $g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].$)

- Assumption : $k = \text{constant}$ (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A}$$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors : $g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].$)

- Assumption : $\mathbf{k = constant}$ (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \boldsymbol{\Delta\phi}$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T \int \mathrm{d}x\, p^2$$

PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

- Assumption : **k = constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T \int \mathrm{d}x\, p^2 - \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'} \left[k + c\,(g''/g')'\right]$$

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :

$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2].)$

- Assumption : k **= constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T \int \mathrm{d}x\, p^2 - \oint \mathrm{d}t\,\mathrm{d}x\, \frac{\dot{g}}{g'}\left[k + c\,(g''/g')'\right]$$

- Compare with $v_{\text{Drift}} = \Delta\phi / T$

PHASES IN KDV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors : $g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)

- Assumption : **k = constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \boldsymbol{\Delta\phi}$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T\int \mathrm{d}x\, p^2 - \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\left[k + c\,(g''/g')'\right]$$

- Compare with $v_{\text{Drift}} = \Delta\phi/T$ **... and fail !**

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)

- Assumption : $k =$ **constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T \int \mathrm{d}x\, p^2 - \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'} \left[k + c\,(g''/g')' \right]$$

- Compare with $v_{\text{Drift}} = \Delta\phi/T$ **... and fail !**
- Add central entry to reconstruction

Phases in KdV

Periodic KdV wave $p(x,t) = (g_t \cdot k)(x)$

(conformal tsf of CFT stress tensors :
$g^{-1} \cdot k = k(g')^2 - c[g'''/g' - \frac{3}{2}(g''/g')^2]$.)

- Assumption : $k =$ **constant** (flat profile)
- k determined by p [Kirillov, Witten ∼1987]
- k stabilized by rotations $\Rightarrow g_0^{-1} g_T(x) = x + \Delta\phi$

Phase formula :

$$k\Delta\phi = 2ET - \oint \mathcal{A} = T \int \mathrm{d}x\, p^2 - \oint \mathrm{d}t\, \mathrm{d}x\, \frac{\dot{g}}{g'}\left[k + c\,(g''/g')'\right]$$

- Compare with $v_{\text{Drift}} = \Delta\phi/T$ **... and fail !**
- Add central entry to reconstruction
- **Anomalous phase !**

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$

PHASES IN KDV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $v_{\text{Drift}} = \dfrac{\Delta\phi}{T}$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $v_{\mathrm{Drift}} = \dfrac{\Delta\phi}{T} = \underbrace{\dfrac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}}$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}}$$

PHASES IN KDV

Travelling wave $p(x,t) = p(x-vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p-v}}_{\text{Anomalous}}$$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$
- Perfect **match with observed drift !** [B.O. & Kozyreff 2020]

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$
- Perfect **match with observed drift !** [B.O. & Kozyreff 2020]

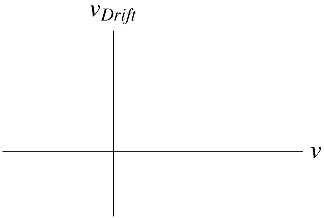
Note :
Some p's cannot be mapped on cst k

Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$
- Perfect **match with observed drift !** [B.O. & Kozyreff 2020]

Note :
Some p's cannot be mapped on cst k

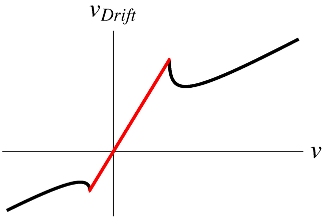


Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$
- Perfect **match with observed drift !** [B.O. & Kozyreff 2020]

Note :
Some p's cannot be mapped on cst k



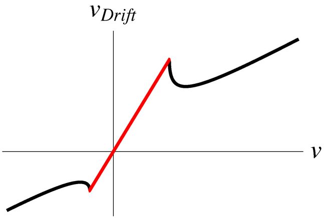
Phases in KdV

Travelling wave $p(x,t) = p(x - vt)$

- Period $T = 2\pi/|v|$
- $$v_{\text{Drift}} = \frac{\Delta\phi}{T} = \underbrace{\frac{1}{k}\int \mathrm{d}x\, p^2}_{\text{Dynamical}} + \underbrace{v - \frac{v}{k}\int \mathrm{d}x\, p}_{\text{Berry}} + \underbrace{\frac{c}{k}\int \mathrm{d}x\, \frac{p'^2}{p - v}}_{\text{Anomalous}}$$
- Perfect **match with observed drift !** [B.O. & Kozyreff 2020]

Note :
Some p's cannot be mapped on cst k

- Bifurcations of drift velocity



SUMMARY

Summary

Fluids generate **deformations**

SUMMARY

Fluids generate **deformations**
producing **geometric phases**

Summary

Fluids generate **deformations** producing **geometric phases** which affect particles' **drift**.

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

- In practice, use Virasoro group

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

- In practice, use Virasoro group (∼ conformal field theory)

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

- In practice, use Virasoro group (∼ conformal field theory)
  & Euler equations

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

- In practice, use Virasoro group (∼ conformal field theory)
  & Euler equations (∼ symplectic geometry)

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

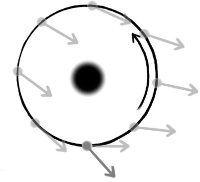
- In practice, use Virasoro group (∼ conformal field theory)
  & Euler equations (∼ symplectic geometry)
- "Anomalous phases"

SUMMARY

Fluids generate **deformations**
producing **geometric phases**
which affect particles' **drift**.

- In practice, use Virasoro group (∼ conformal field theory)
  & Euler equations (∼ symplectic geometry)
- "Anomalous phases"
- Topological bifurcations

Thank you for listening !



$=$