Gyroscopic Memory Effects in Gravity and Electrodynamics

Blagoje Oblak
(ULB Brussels)

October 2023

arXiv : 2112.04535 (JHEP)
2203.16216 (PRL)
2304.12348 (PRD)
} with Ali Seraj

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Intro

(How spinning tops see gravitational waves)

Two motivations

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1. Observable **gravitational waves** !

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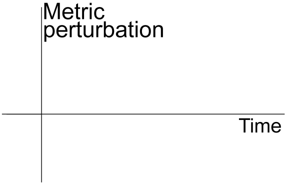

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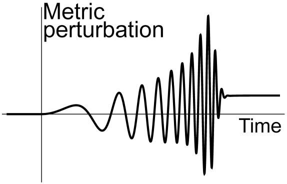

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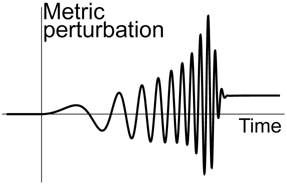

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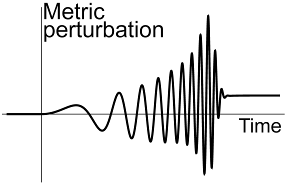

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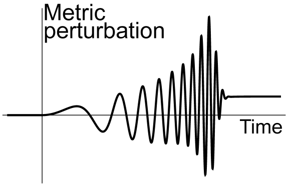

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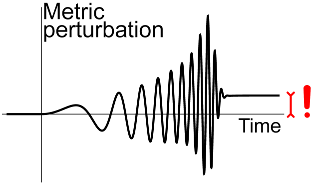

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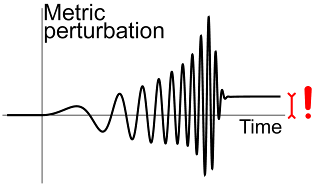

2. Semiclassical gravity ?

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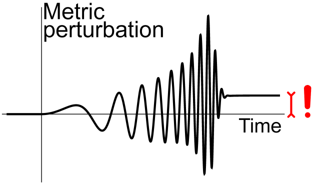

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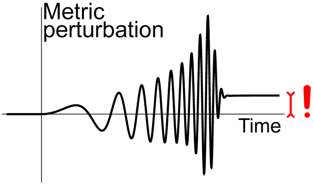

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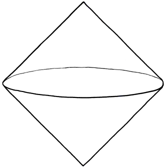

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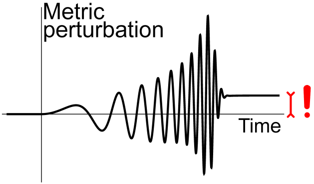

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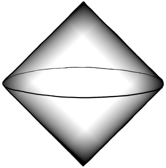

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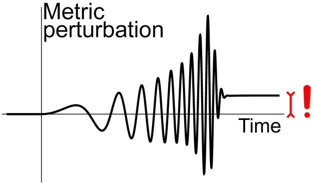

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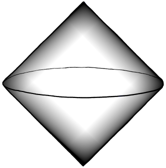

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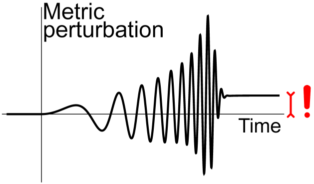

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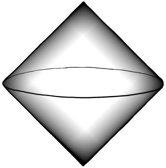
     (Noether currents of boundary theory)



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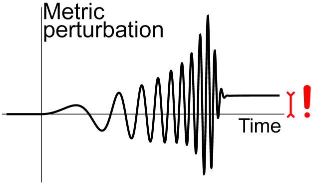

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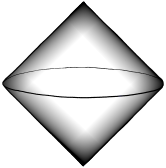

Memory = **observable** effect of gravitational **symmetries**

This Talk in a Nutshell

THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes**

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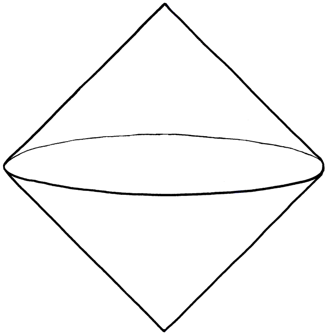

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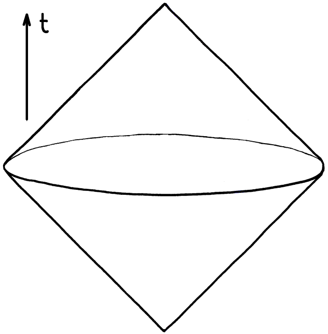

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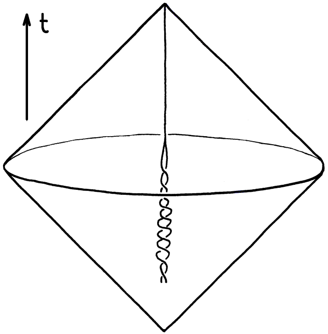

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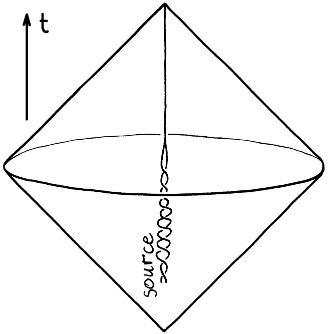

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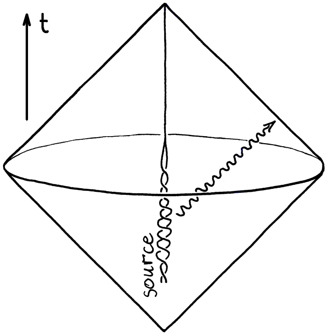

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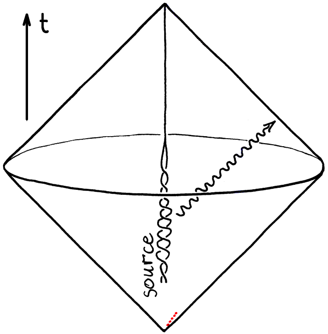

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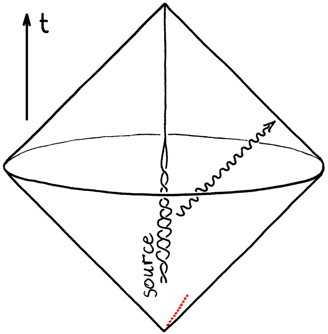

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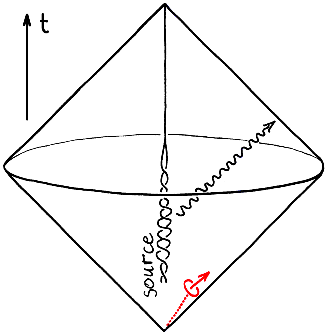

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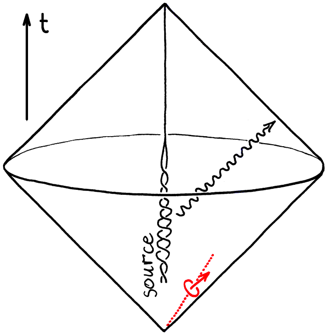

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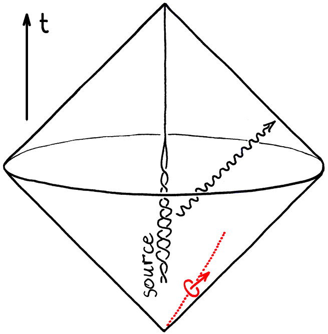

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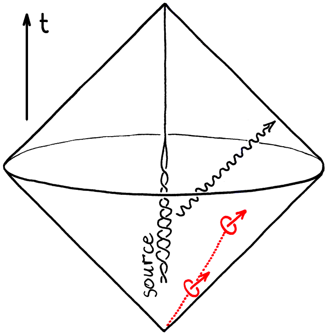

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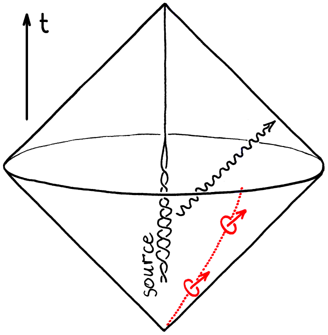

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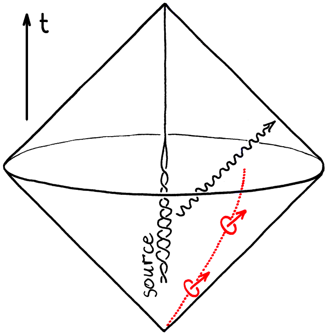

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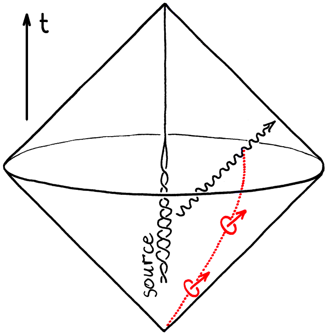

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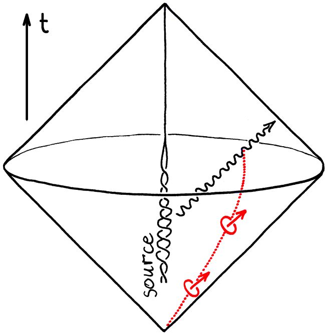
Waves cause precession



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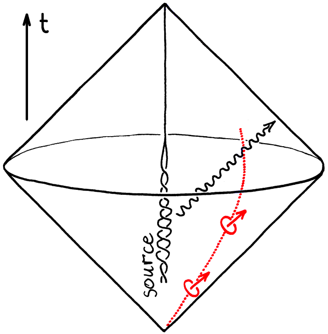

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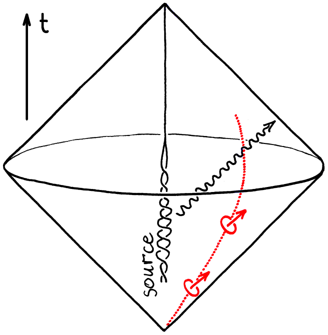

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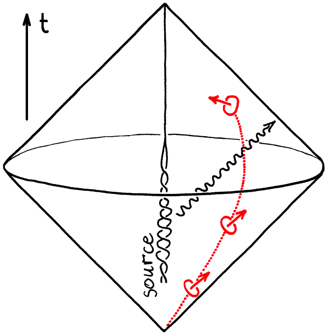

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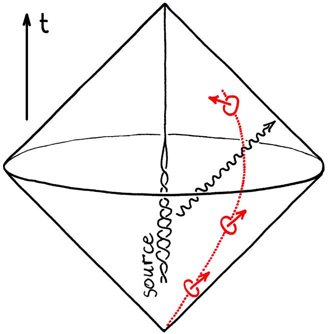

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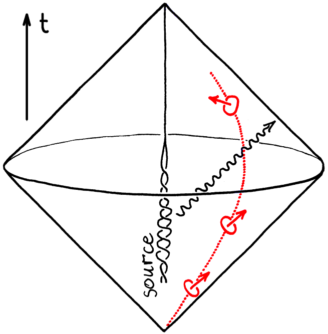

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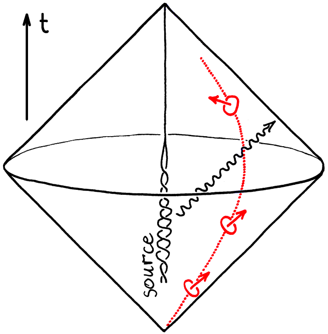

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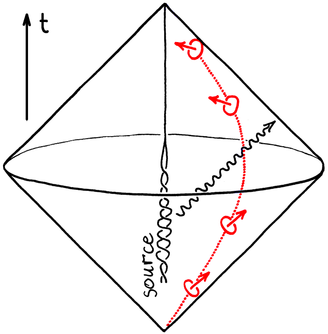

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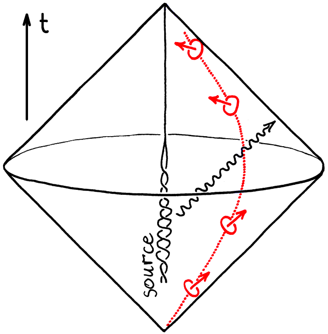

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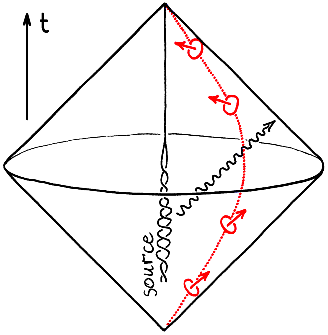

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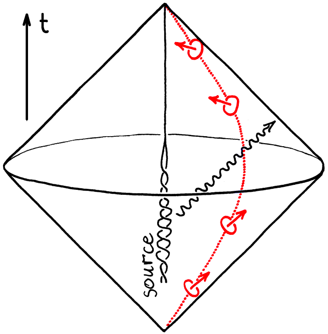

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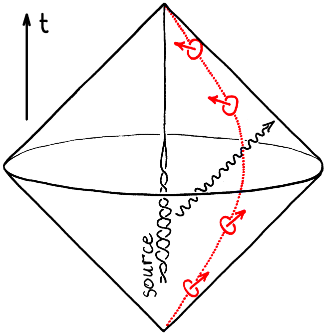

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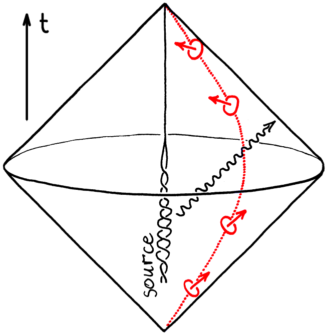
Underlying symmetries :

- Precession rate = current of **dual aspt symmetries**



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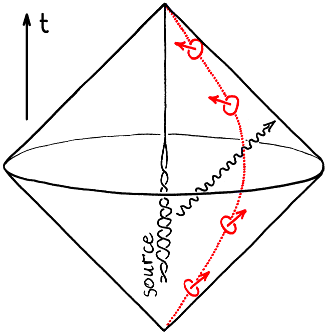
- Memory involves **electric-magnetic duality**



PLAN

1. Orientation memory in electrodynamics
2. Frames tied to distant stars
3. Orientation memory in gravity

Plan

1. **Orientation memory in electrodynamics**

2. Frames tied to distant stars

3. Orientation memory in gravity

Plan

1. Orientation memory in electrodynamics

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PLAN

1. Orientation memory in electrodynamics
2. **Frames tied to distant stars**... in radiative metric
3. Orientation memory in gravity

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1. Orientation Memory in Electrodynamics

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A. Bondi coordinates and null infinity

1. Orientation Memory in Electrodynamics

A. Bondi coordinates and null infinity

B. Radiation causes precession

Electrodynamics at null infinity

ELECTRODYNAMICS AT NULL INFINITY

Minkowski in polar coordinates :

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \quad \mathrm{d}r^2 \quad + r^2\, h_{ab}\mathrm{d}\theta^a \mathrm{d}\theta^b$$

Electrodynamics at null infinity

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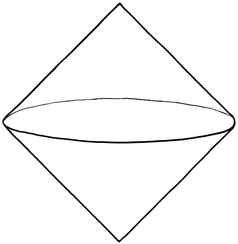

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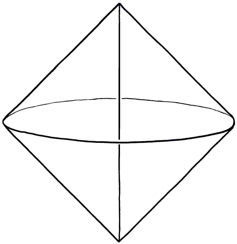

Electrodynamics at null infinity

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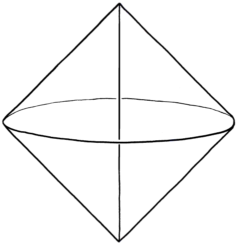

ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

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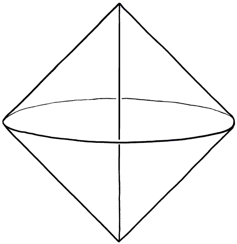

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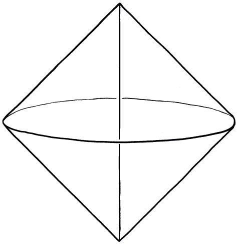

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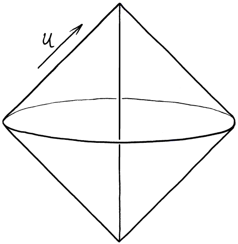

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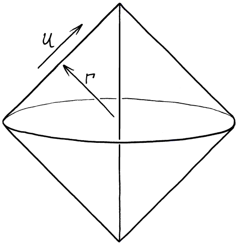

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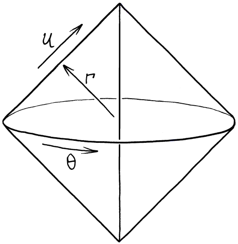

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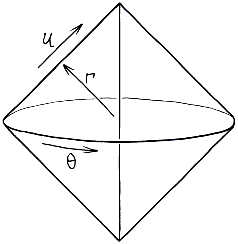
$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab} d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



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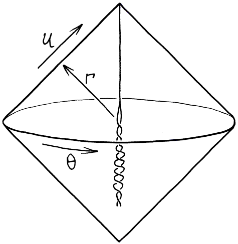

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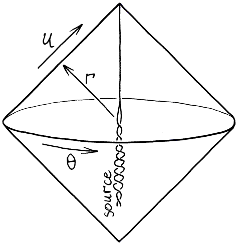

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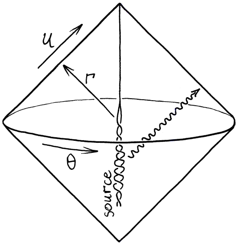

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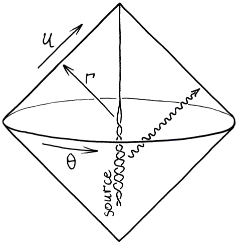

Electromagnetic field A

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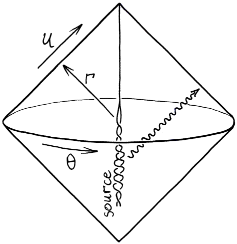

Electromagnetic field A

[Radial gauge $A_r \equiv 0$]

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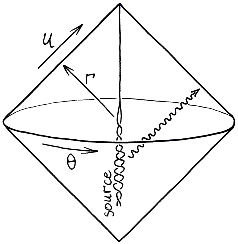

Electromagnetic field A [Radial gauge $A_r \equiv 0$]

- $A \sim C_a\,d\theta^a + \frac{1}{r} D_a C^a\,du + \cdots$

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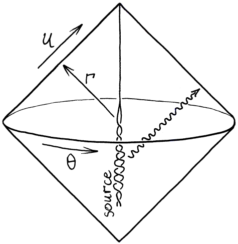

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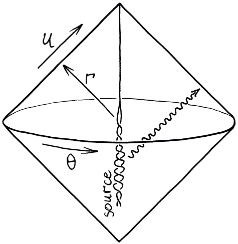

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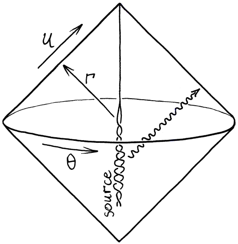

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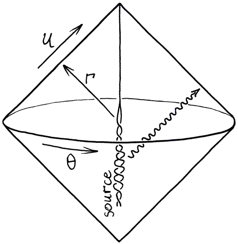

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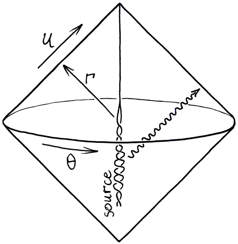

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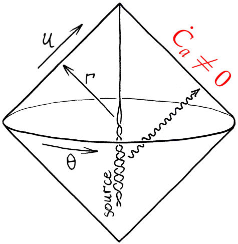

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RADIATION CAUSES PRECESSION

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M}$

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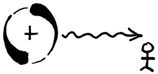

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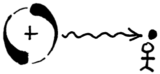

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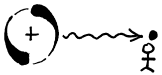

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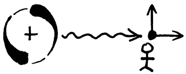

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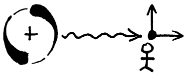

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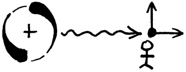

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Use **source-oriented frame**

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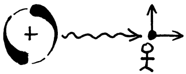

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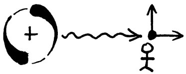

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with $\widetilde{C}_a = \epsilon_a{}^b\, C_b$ dual boundary data

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Radiation causes precession

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RADIATION CAUSES PRECESSION

Orientation memory $= \int \mathrm{d}u\, \Omega_{\parallel}$

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$ involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a$

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  of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$

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- Canonical generator
  of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$
- Coincides with **helicity** of wave burst
  $=$ # left-hand photons $-$ # right-handed photons

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\left(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\right)$$

Radiation causes precession

Precession caused by specific sources ?

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Radiation causes precession

Precession caused by specific sources ?

- Point charge moving up and down

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Radiation causes precession

Precession caused by specific sources ?

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  $D_a \widetilde{C}^a = 0$ but $\dot{C}_a \widetilde{C}^a \sim \omega \frac{v^2}{c^2} \cos\theta$

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2} \left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a \right)$$

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$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\left(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\right)$$

Radiation causes precession

Precession caused by specific sources ?

- Point charge moving up and down : $\Omega_{||} = 0$
- **Rotating point charge** :
  $D_a \widetilde{C}^a = 0$ but $\dot{C}_a \widetilde{C}^a \sim \omega \frac{v^2}{c^2} \cos\theta \;\; \Rightarrow \;\; \boldsymbol{\Omega_{||} \neq 0}$!
- Analogue for binary black holes ?

$$\Omega_{||} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

2. Frames Tied to Distant Stars

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A. Asymptotically flat metrics

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A. Asymptotically flat metrics

B. Source-oriented tetrad

2. Frames Tied to Distant Stars

A. Asymptotically flat metrics

B. Source-oriented tetrad

C. Star-oriented tetrad

METRIC AT INFINITY

Metric at infinity

Minkowski in Bondi coord $(u = t - r)$:

$$ds^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$

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Minkowski in Bondi coord ($u = t - r$) :

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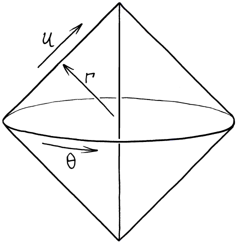

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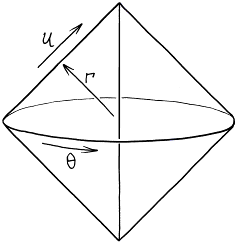

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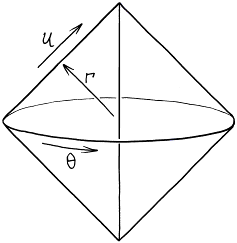

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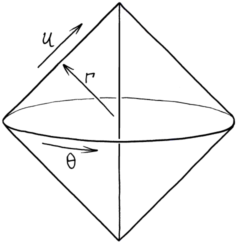

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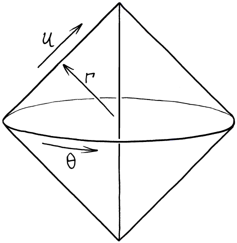

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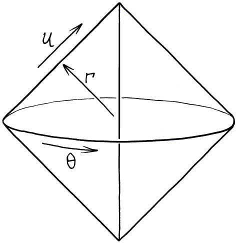

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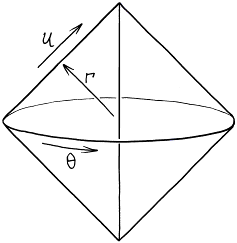

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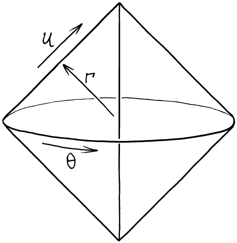

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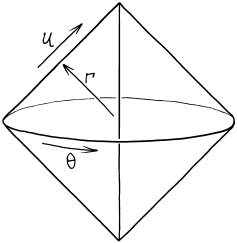

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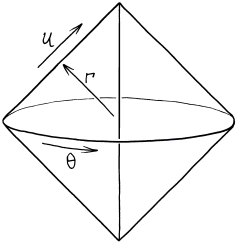

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- $C_{ab}(u, \theta)$

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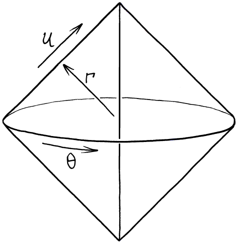

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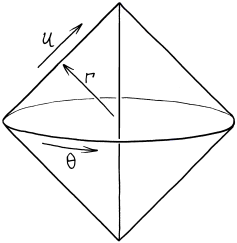

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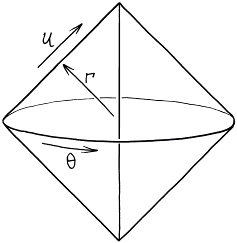

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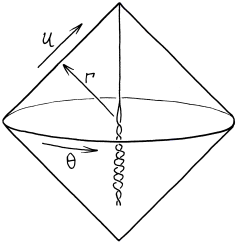

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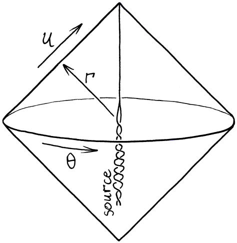

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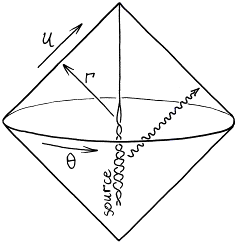

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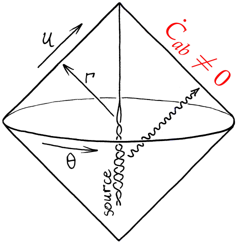

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Metric dynamics

METRIC DYNAMICS

Asymptotic solution of Einstein eqns ?

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$$\mathrm{d}s^2 \sim -\big(1+\cdots\big)\mathrm{d}u^2 - (2+\cdots)\mathrm{d}u\,\mathrm{d}r$$
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METRIC DYNAMICS

Asymptotic solution of Einstein eqns :

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Dynamics ?

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$\dot{m} \sim -\dot{C}^2$ (radiation carries energy)

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Dynamics = **balance equations**

$$\dot{m} \sim -\dot{C}^2 \qquad \text{(radiation carries energy)}$$
$$\dot{L}_a \sim 3D_b\dot{C}^{bc}C_{ac} + \dot{C}^{bc}D_bC_{ac}$$

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$\dot{m} \sim -\dot{C}^2$ (radiation carries energy)

$\dot{L}_a \sim C\dot{C}$ (useful later)

METRIC DYNAMICS

Asymptotic solution of Einstein eqns :

$$ds^2 \sim -(1 - \tfrac{2}{r}\boldsymbol{m})du^2 - (2 + \cdots)du\,dr + (\boldsymbol{D^b C_{ab}} + \tfrac{1}{r}\boldsymbol{L_a})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab})d\theta^a d\theta^b$$

- $m(u,\theta)$ = Bondi mass = celestial energy density
- $L_a(u,\theta)$ = angular momentum $\ll$ shear

Dynamics = **balance equations**

$\dot{m} \sim -\dot{C}^2$ (radiation carries energy)
$\dot{L}_a \sim C\dot{C}$ (useful later)

- **Shear** C_{ab} fixes everything

Source-oriented frame

Source-oriented frame

Source-oriented frame

SOURCE-ORIENTED FRAME

Source-oriented frame

Freely falling observer at large r

SOURCE-ORIENTED FRAME

Freely falling observer at large r

- Geodesic eqn yields **velocity** $\mathbf{v}$

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Observer carries gyroscope

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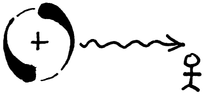

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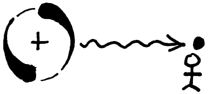

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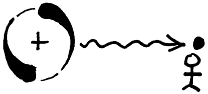

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- Measure orientation ?

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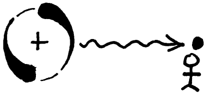

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- Measure orientation :
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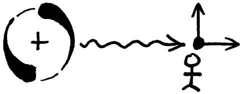

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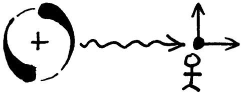

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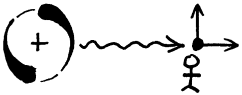

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$\swarrow$

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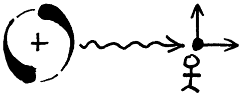

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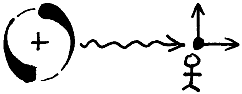

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$$\mathbf{v} \qquad \sim \partial_r - \partial_u$$

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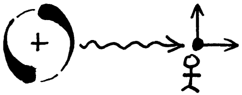

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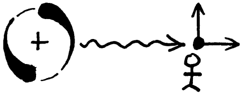

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$$\boldsymbol{e}_{\hat{0}} \swarrow \mathbf{v} \qquad \boldsymbol{e}_{\hat{r}} \downarrow \sim \partial_r - \partial_u \qquad \boldsymbol{e}_{\hat{a}} \searrow \sim \frac{1}{r}\boldsymbol{E}_{\hat{a}}$$

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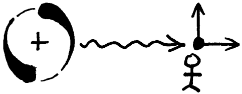

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- Expand in $1/r$

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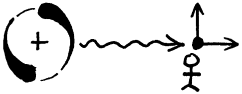

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- Expand in $1/r$

Note : Angular velocity $v^a \sim \frac{1}{r^2} D_b C^{ab}$

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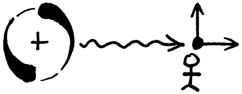

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- **Radiation** causes **angular motion**

Star-oriented frame

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Problem : Source-oriented frame **rotates without radiation**

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STAR-ORIENTED FRAME

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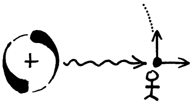

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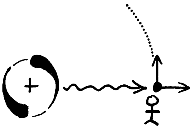

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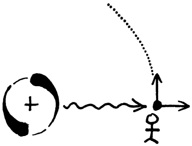

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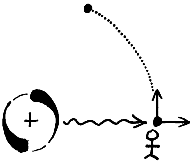

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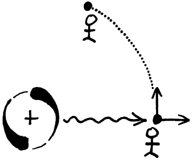

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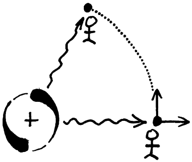

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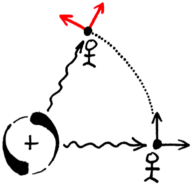

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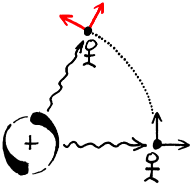

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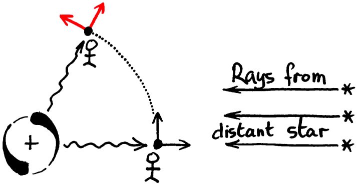

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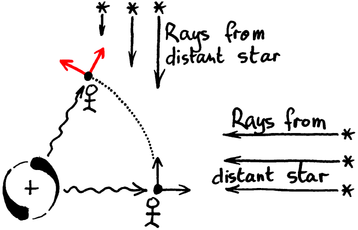

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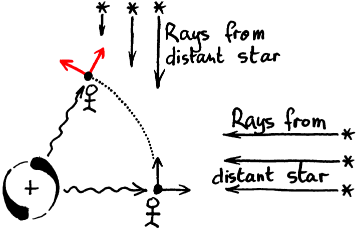

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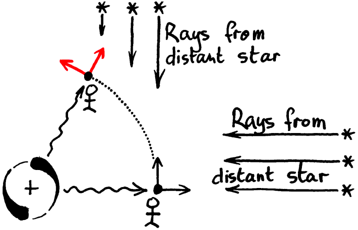

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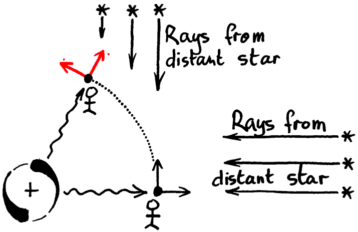

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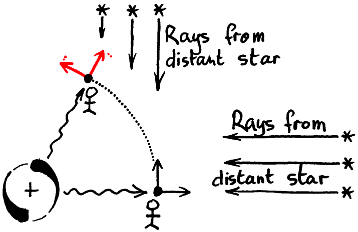

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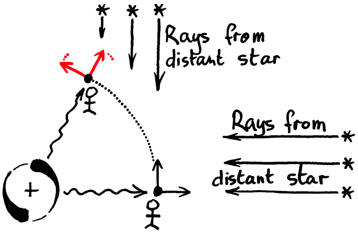

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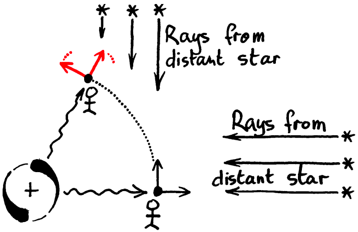

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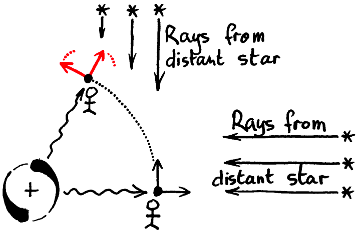

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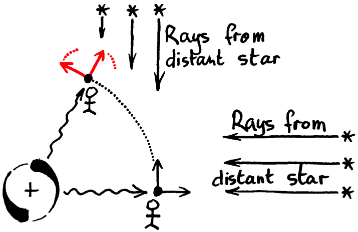

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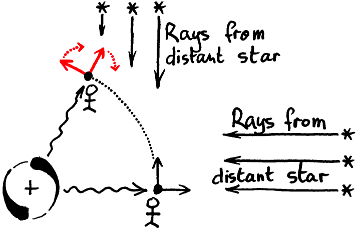

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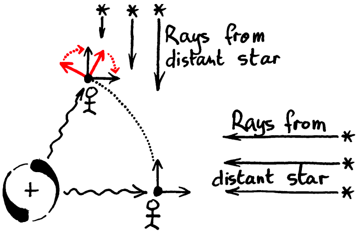

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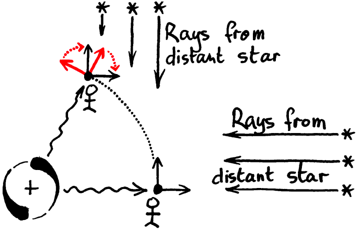

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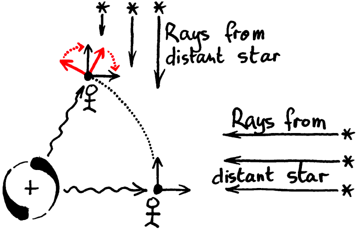

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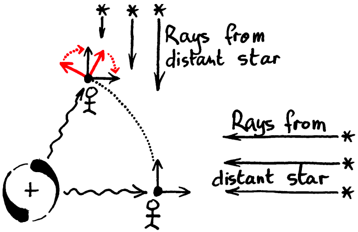

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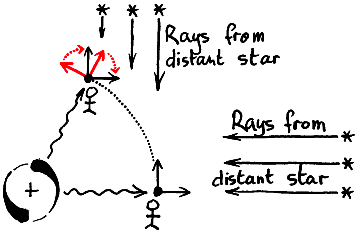

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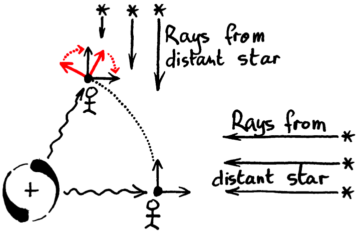

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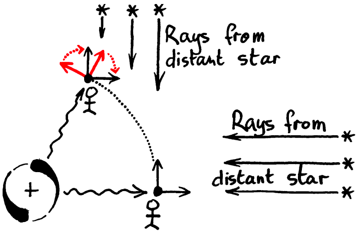

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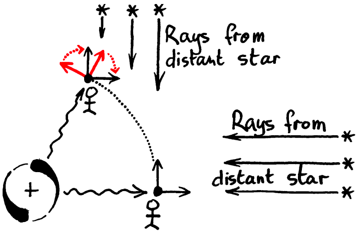

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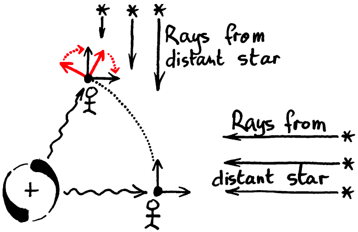

Precession with respect to this frame ?

3. Precession and Memory

3. Precession and Memory

A. Precession rate as dual mass

3. Precession and Memory

A. Precession rate as dual mass

B. Orientation memory

PRECESSION

Freely falling gyroscope

Precession

Freely falling gyroscope

- Parallel-transported spin $\nabla_{\mathbf{v}}\mathbf{S} = 0$

PRECESSION

Freely falling gyroscope

- Parallel-transported spin $\nabla_{\mathbf{v}}\mathbf{S} = 0$

$\mathbf{S} = S^{\hat{i}}\boldsymbol{f}_{\hat{i}}$ in local frame

PRECESSION

Freely falling gyroscope

- Parallel-transported spin $\nabla_{\mathbf{v}}\mathbf{S} = 0$

$\mathbf{S} = S^{\hat{i}}\boldsymbol{f}_{\hat{i}}$ in local frame

- Parallel transport reads $\dot{S}^{\hat{i}} = -f^{\hat{i}}_{\mu}\nabla_{\mathbf{v}}f^{\hat{j}\mu}\ S_{\hat{j}}$

Precession

Freely falling gyroscope

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Freely falling gyroscope

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Hence the core computation :

(i) Find **source**-oriented spin connection ω at large r

(ii) Rotate $\omega \to R\omega R^{-1} + R\mathrm{d}R^{-1}$ for **star**-oriented frame

Radiation causes precession

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  (No surprise: electric = distances, magnetic = orientations)

Orientation memory

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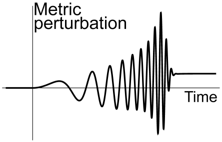

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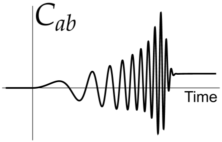

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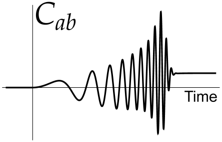

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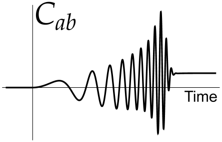

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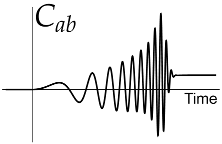

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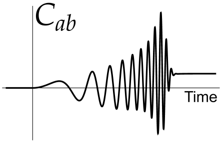

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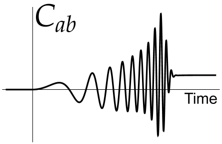

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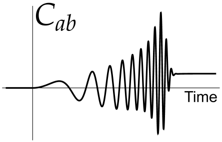

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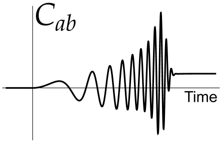

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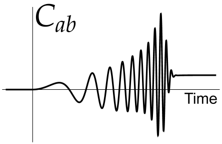

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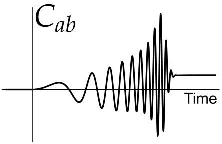

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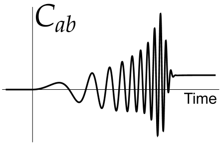

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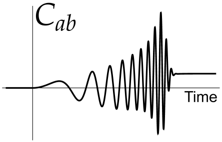

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Orientation Mem = Spin Mem + Duality

Conclusion

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by changing **orientation** wrt faraway stars

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  Estimate $\Phi \sim 10^{-39} \left(\frac{M/M_\odot}{r/1\,\text{Mpc}}\right)^2$

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- Use **pulsars** as distant gyroscopes !?

Ευχαριστό !