

Deformational Berry Phases of Quantum Hall Droplets

Blagoje Oblak

LPTHE (Sorbonne) & CPHT (Polytechnique)

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With B. Estienne, to appear.

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro

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Intro 0●00	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Diffeomorphisms (cts deformations) ubiquitous in physics

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Diffeomorphisms (cts deformations) ubiquitous in physics :

► General relativity

(gauge symmetry)

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Ouestion : **Observables** sensitive to geometry of groups of diffeos?

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(fluid flows) (robust to deformations)

Question : Observables sensitive to geometry of groups of diffeos ? ...i.e. infinite-dimensional geometry ?

Intro 00●0	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Diffeomorphisms are crucial for quantum Hall effect (QHE)

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Diffeomorphisms are crucial for **quantum Hall effect** (QHE) :

Non-commutative space

[Girvin et al. 85, Bellissard 86]

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Nonlinear deformations

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- "Bulk-edge correspondence" for Hall viscosity

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Plan

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plan

- 1. Adiabatic diffeos in 1D
- 2. One-body sdiffeos in 2D
- 3. Adiabatic sdiffeos in 2D
- 4. Generalized Hall viscosity

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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1. Adiabatic diffeos in 1D

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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1. Adiabatic diffeos in 1D

A. Reminder on Berry phases

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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1. Adiabatic diffeos in 1D

A. Reminder on Berry phases

B. Deformational Berry ϕ in 1D

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Quantum system depending on **parameters** *g*



Intro 0000	Berry ϕ in 1D $\circ \bullet \circ$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

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Quantum system depending on **parameters** *g*

► Parameter-dep. energy eigenstates $|\psi(g)\rangle$

Intro 0000	Berry ϕ in 1D $\odot \bullet \odot$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on **parameters** *g*

- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically

Intro 0000	Berry ϕ in 1D $\odot \bullet \odot$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on **parameters** g

- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow g_t$

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Quantum system depending on **parameters** g

- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow g_t$
- Wavefet picks phase $\int dt E i \int dt \langle \psi(g_t) | \frac{\partial}{\partial t} | \psi(g_t) \rangle$

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Intro 0000	Berry ϕ in 1D $\circ \bullet \circ$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on **parameters** *g*

- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically \Rightarrow g_t
- Wavefct picks phase

$$\underbrace{\int \mathrm{d}t \, E}_{\text{Dynamical } \phi} - i \int \mathrm{d}t \left\langle \psi(g_t) \right| \frac{\partial}{\partial t} \big| \psi(g_t) \right\rangle$$

Intro 0000	Berry ϕ in 1D $\circ \bullet \circ$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on **parameters** g

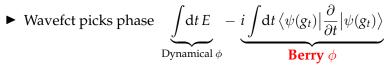
- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
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$$\underbrace{\int dt E}_{\text{Dynamical }\phi} - \underbrace{i \int dt \left\langle \psi(g_t) \right| \frac{\partial}{\partial t} |\psi(g_t) \right\rangle}_{\text{Berry }\phi}$$

Intro 0000	Berry ϕ in 1D $\circ \bullet \circ$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on parameters g

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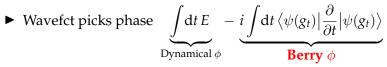
Example : time-dependent rotations of spin [Berry 84]

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Intro 0000	Berry ϕ in 1D $\circ \bullet \circ$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Quantum system depending on parameters g

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Example : time-dependent rotations of spin [Berry 84]

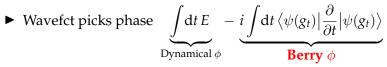
• General Berry ϕ due to unitary group actions

[Jordan 87]

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Quantum system depending on parameters g

- ► Parameter-dep. energy eigenstates $|\psi(g)\rangle$
- Vary parameters adiabatically and cyclically $\Rightarrow g_t$



Example : time-dependent rotations of spin [Berry 84]

- General Berry ϕ due to unitary group actions
- [Jordan 87]
- Berry φ produced by unitary sample diffeos ?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Particle on circle \Rightarrow position $\varphi \sim \varphi + 2\pi$

• Wavefct $\psi(\varphi)$ on S^1

Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

- Wavefet $\psi(\varphi)$ on S^1
- Let $\varphi \mapsto g(\varphi)$ be a **diffeo**

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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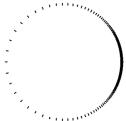
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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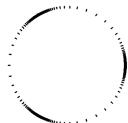
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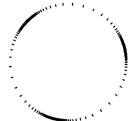
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Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

- Wavefet $\psi(\varphi)$ on S^1
- Let $\varphi \mapsto g(\varphi)$ be a **diffeo**



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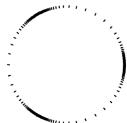
Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

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Deformational Berry ϕ

- Wavefct $\psi(\varphi)$ on S^1
- Let $\varphi \mapsto g(\varphi)$ be a **diffeo**
- Rotations $g(\varphi) = \varphi + \theta$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- Wavefct $\psi(\varphi)$ on S^1
- Let $\varphi \mapsto g(\varphi)$ be a **diffeo**
- Rotations $g(\varphi) = \varphi + \theta$
- Unitary Diff S^1 action : $(\mathcal{U}[g]\psi)(\varphi)$

Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

- Wavefct $\psi(\varphi)$ on S^1
- Let $\varphi \mapsto g(\varphi)$ be a **diffeo**
- Rotations $g(\varphi) = \varphi + \theta$
- Unitary Diff S¹ action : $(\mathcal{U}[g]\psi)(\varphi) = \sqrt{(g^{-1})'(\varphi)}\psi(g^{-1}(\varphi))$

Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

- Wavefct $\psi(\varphi)$ on S^1
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Particle on circle \Rightarrow position $\varphi \sim \varphi + 2\pi$

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Choose adiabatic, cyclic $g_t(\varphi)$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Choose adiabatic, cyclic $g_t(\varphi)$

► Berry phases ?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- Assume $\psi(\varphi) \propto e^{ij\varphi}$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- ► Berry phases ?
- Assume $\psi(\varphi) \propto e^{ij\varphi}$

• Berry =
$$i \int dt \langle \psi | \mathcal{U}[g_t]^{-1} \frac{\partial}{\partial t} \mathcal{U}[g_t] | \psi \rangle$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro 0000	Berry ϕ in 1D $\circ \circ \bullet$	Sdiffeos in 2D 0000	Berry <i>φ</i> in 2D 0000	Hall viscosity 0000	The End 00

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Analogue for 2D electrons in magnetic field ?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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2. One-body sdiffeos in 2D

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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2. One-body sdiffeos in 2D

A. Area-preserving diffeos

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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2. One-body sdiffeos in 2D

A. Area-preserving diffeos

B. Unitary action of sdiffeos

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

• Magnetic field $\mathbf{B} = \mathbf{dA}$



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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

- Magnetic field $\mathbf{B} = \mathbf{dA}$
- Diffeo $g : \mathbf{x} \mapsto g(\mathbf{x})$ preserves area if $g^* \mathbf{B} = \mathbf{B}$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

- Magnetic field $\mathbf{B} = \mathbf{dA}$
- ► Diffeo $g : \mathbf{x} \mapsto g(\mathbf{x})$ preserves area if $g^* \mathbf{B} = \mathbf{B}$
- ► $SDiff(\mathbb{R}^2) = \{area-preserving diffeos\}$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D ○●○○	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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Example :

 (r^2,φ)

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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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Example :

$$(r^2, \varphi) \longmapsto \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D ○●○○	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D ○●○○	Berry ϕ in 2D 0000	Hall viscosity 0000	The End 00

Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

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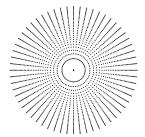
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

- Magnetic field $\mathbf{B} = d\mathbf{A}$
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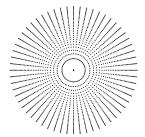
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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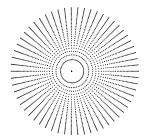
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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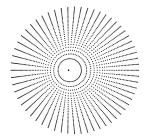
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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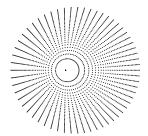
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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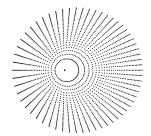
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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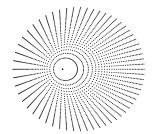
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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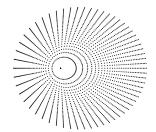
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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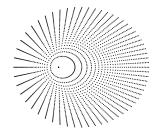
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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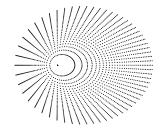
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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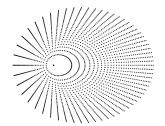
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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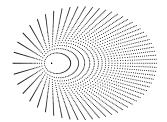
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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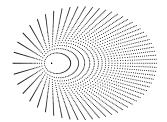
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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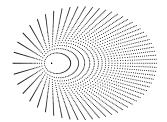
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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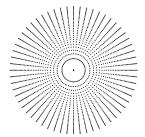
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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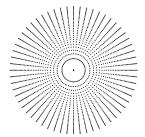
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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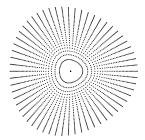
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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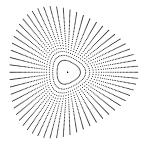
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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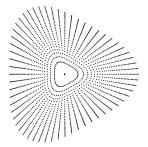
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- ▶ SDiff(\mathbb{R}^2) = {area-preserving diffeos} ≡ {sdiffeos}

Example :

$$(r^2, \varphi) \longmapsto \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$



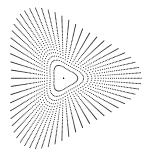
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Plane \mathbb{R}^2 , potential $\mathbf{A} = A_i(\mathbf{x}) dx^i$

- Magnetic field $\mathbf{B} = d\mathbf{A}$
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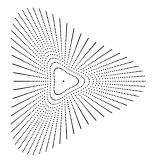
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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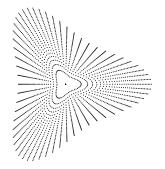
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$$(r^2, \varphi) \longmapsto \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

"edge diffeos"



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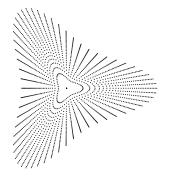
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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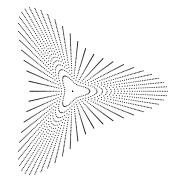
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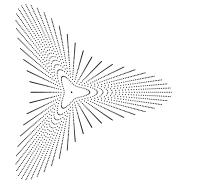
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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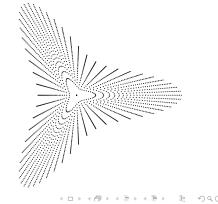
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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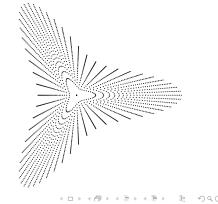
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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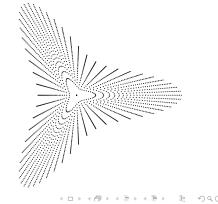
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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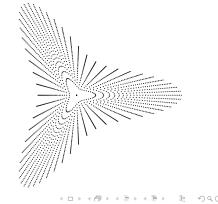
Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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UNITARY SDIFFEOS

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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UNITARY SDIFFEOS

Electron in \mathbb{R}^2

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

• Hilbert space $L^2(\mathbb{R}^2)$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

- Hilbert space $L^2(\mathbb{R}^2)$
- Unitary action of sdiffeos ?

 $(\mathcal{U}[g]\psi)(\mathbf{x})$



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

- Hilbert space $L^2(\mathbb{R}^2)$
- Unitary action of sdiffeos :

$$(\mathcal{U}[g]\psi)(\mathbf{x}) \equiv \psi(\bar{g}(\mathbf{x}))$$

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

- Hilbert space $L^2(\mathbb{R}^2)$
- Unitary action of sdiffeos preserving A : (we wish to compare wavefcts)

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

- Hilbert space $L^2(\mathbb{R}^2)$
- Unitary action of sdiffeos preserving A : (we wish to compare wavefcts)

$$(\mathcal{U}[g]\psi)(\mathbf{x}) \equiv \underbrace{e^{iq\,\mathbf{d}^{-1}(\mathbf{A}-\bar{g}^*\mathbf{A})(\mathbf{x})}}_{\text{compensating gauge tsf}}\psi\big(\bar{g}(\mathbf{x})\big)$$

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Action on Hamiltonian?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Action on $H \sim (\mathbf{p} - q\mathbf{A})^2 + V(\mathbf{x})$?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Action on $H \sim (\mathbf{p} - q\mathbf{A})^2 + V(\mathbf{x})$: $\blacktriangleright \mathcal{U}[g]H\mathcal{U}[g]^{-1} \sim (p_j - qA_j) G^{jk}(\mathbf{x}) (p_k - qA_k) + V(\bar{g}(\mathbf{x}))$

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

- ► Hilbert space *L*²(ℝ²)
- Unitary action of sdiffeos preserving A : (we wish to compare wavefcts)

$$\left(\mathcal{U}[g]\psi\right)(\mathbf{x}) \equiv \underbrace{e^{iq\,\mathrm{d}^{-1}(\mathbf{A}-\bar{g}^*\mathbf{A})(\mathbf{x})}}_{\text{compensating gauge tsf}}\psi\big(\bar{g}(\mathbf{x})\big)$$

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Action on $H \sim (\mathbf{p} - q\mathbf{A})^2 + V(\mathbf{x})$:

• $\mathcal{U}[g]H\mathcal{U}[g]^{-1} \sim (p_j - qA_j) G^{jk}(\mathbf{x})(p_k - qA_k) + V(\overline{g}(\mathbf{x}))$ with G^{jk} = metric induced by sdiffeo

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- Sdiffeos change metric and deform potential

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Electron in \mathbb{R}^2

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- Sdiffeos change metric and deform potential
- ∞ parameters !

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Composition of sdiffeos ?



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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UNITARY SDIFFEOS

Composition of sdiffeos ?

$$\blacktriangleright \ \mathcal{U}[f] \circ \mathcal{U}[g] = \qquad \qquad \mathcal{U}[f \circ g]$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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UNITARY SDIFFEOS

Composition of sdiffeos ?

$$\blacktriangleright \mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g]$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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UNITARY SDIFFEOS

Composition of sdiffeos ?

• $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g]$ with $\mathbf{C} \neq 0$!

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

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$$\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g]$$
 with $\mathbf{C} \neq 0$!

 $\mathbf{C}(f,g) = \mathbf{d}^{-1}(f^*(g^*\mathbf{A} - \mathbf{A})) - f^*(\mathbf{d}^{-1}(g^*\mathbf{A} - \mathbf{A}))$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

• $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g]$ with $\mathbf{C} \neq 0$!

 $\mathsf{C}(f,g) = \mathsf{d}^{-1}(f^*(g^*\mathbf{A} - \mathbf{A})) - f^*(\mathsf{d}^{-1}(g^*\mathbf{A} - \mathbf{A}))$

• Projective representation of $SDiff(\mathbb{R}^2)$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

• $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g]$ with $\mathbf{C} \neq 0$!

 $C(f,g) = d^{-1}(f^*(g^*A - A)) - f^*(d^{-1}(g^*A - A))$

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- Projective representation of $SDiff(\mathbb{R}^2)$
- ► Central charge = electric charge × magnetic field

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

- $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g] \text{ with } \mathbf{C} \neq 0!$ $\mathbf{C}(f,g) = d^{-1}(f^*(g^*\mathbf{A} \mathbf{A})) f^*(d^{-1}(g^*\mathbf{A} \mathbf{A}))$
- Projective representation of $SDiff(\mathbb{R}^2)$
- ► Central charge = electric charge × magnetic field

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For translations, $C(f,g) = B \times (\text{area in } f g \overline{f} \overline{g})$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

- $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g] \text{ with } \mathbf{C} \neq 0!$ $\mathbf{C}(f,g) = d^{-1}(f^*(g^*\mathbf{A} \mathbf{A})) f^*(d^{-1}(g^*\mathbf{A} \mathbf{A}))$
- Projective representation of $SDiff(\mathbb{R}^2)$
- ► Central charge = electric charge × magnetic field

For translations, $C(f,g) = B \times (\text{area in } f g \overline{f} \overline{g})$

Magnetic translations

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

 $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g] \text{ with } \mathbf{C} \neq 0!$ $\mathbf{C}(f,g) = d^{-1}(f^*(g^*\mathbf{A} - \mathbf{A})) - f^*(d^{-1}(g^*\mathbf{A} - \mathbf{A}))$

• Projective representation of $SDiff(\mathbb{R}^2)$

► Central charge = electric charge × magnetic field

For translations, $C(f,g) = B \times (\text{area in } f g \overline{f} \overline{g})$

- Magnetic translations
- C cannot be absorbed by redefinition (non-trivial cocycle)

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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Composition of sdiffeos ?

 $\mathcal{U}[f] \circ \mathcal{U}[g] = e^{iq\mathbf{C}(f,g)} \mathcal{U}[f \circ g] \text{ with } \mathbf{C} \neq 0!$ $\mathbf{C}(f,g) = d^{-1}(f^*(g^*\mathbf{A} - \mathbf{A})) - f^*(d^{-1}(g^*\mathbf{A} - \mathbf{A}))$

• Projective representation of $SDiff(\mathbb{R}^2)$

► Central charge = electric charge × magnetic field

For translations, $C(f,g) = B \times (\text{area in } f g \overline{f} \overline{g})$

- Magnetic translations
- C cannot be absorbed by redefinition (non-trivial cocycle)
- Will add Aharonov-Bohm to Berry ϕ

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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3. Adiabatic sdiffeos

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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3. Adiabatic sdiffeos

A. One-body Berry ϕ



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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3. Adiabatic sdiffeos

A. One-body Berry ϕ

B. Many-body Berry ϕ

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $0 \bullet 00$	Hall viscosity 0000	The End 00

Neutral preliminary

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $0 \bullet 00$	Hall viscosity 0000	The End 00

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $0 \bullet 00$	Hall viscosity 0000	The End 00

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One-body Berry ϕ

Neutral preliminary : $(\mathcal{U}[g]\psi)(\mathbf{x}) \equiv \psi(\bar{g}(\mathbf{x}))$

► Energy eigenstate ψ

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \bullet \circ \circ$	Hall viscosity 0000	The End 00

- ► Energy eigenstate ψ
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Berry =
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This is really
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$$\int dt \Big(\langle \psi | \mathcal{U}[g]^{-1} \partial_t \mathcal{U}[g] | \psi \rangle + q \partial_\tau \mathbf{C}(g_t^{-1}, g_\tau) \Big)$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Droplet of $N \gg 1$ electrons

Assume non-degenerate one-body energies

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Many-body Berry ϕ

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$$\int dt d^2 \mathbf{x} \langle \mathbf{J}(\mathbf{x}), \, \bar{g} \, \dot{g} \rangle + q \int d^2 \mathbf{x} \, \boldsymbol{\rho}(\mathbf{x}) \oint_{g_t(\mathbf{x})} \mathbf{A}$$



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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

Droplet of $N \gg 1$ electrons

- Assume non-degenerate one-body energies
- Ground state = Slater determinant of ψ_n 's

Berry =
$$\int dt d^2 \mathbf{x} \langle \mathbf{J}(\mathbf{x}), \, \bar{g} \, \dot{g} \rangle + q \int d^2 \mathbf{x} \, \boldsymbol{\rho}(\mathbf{x}) \oint_{g_t(\mathbf{x})} \mathbf{A}$$



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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

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Many-body Berry ϕ

Droplet of $N \gg 1$ electrons

- Assume non-degenerate one-body energies
- Ground state = Slater determinant of ψ_n 's

Berry =
$$\int dt d^2 \mathbf{x} \langle \mathbf{J}(\mathbf{x}), \, \bar{g} \, \dot{g} \rangle + q \int d^2 \mathbf{x} \, \boldsymbol{\rho}(\mathbf{x}) \oint_{g_t(\mathbf{x})} \mathbf{A}$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

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Many-body Berry ϕ

Droplet of $N \gg 1$ electrons

- Assume non-degenerate one-body energies
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• Example : Translations $g_t(\mathbf{x}) = \mathbf{x} + \mathbf{a}(t)$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

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Many-body Berry ϕ

Droplet of $N \gg 1$ electrons

- Assume non-degenerate one-body energies
- Ground state = Slater determinant of ψ_n 's

Berry =
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• Berry =
$$qN \oint_a \mathbf{A}$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \bullet \circ$	Hall viscosity 0000	The End 00

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(Adiabatic thm holds despite gaplessness [Avron-Elgart 98])

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Hall droplet :

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
0000	000	0000	0000	0000	00

Hall droplet :

Assume isotropic potential

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

► Assume isotropic potential





Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

► Assume isotropic potential



Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

- Assume isotropic potential
- Density $\rho(r)$ (bulk)



Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

- Assume isotropic potential
- Density $\rho(r)$ (bulk)
- Current $J(r)d\varphi$ (edge)



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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

- Assume isotropic potential
- Density $\rho(r)$ (bulk)
- Current $J(r)d\varphi$ (edge)
- Symmetric gauge



Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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- Assume isotropic potential
- Density $\rho(r)$ (bulk)
- Current $J(r)d\varphi$ (edge)
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Berry =
$$\int dt \, d^2 \mathbf{x} \left[J(r) (\bar{g} \, \dot{g})^{\varphi} + \rho(r) \frac{(g^r(\mathbf{x}))^2}{2\ell^2} \dot{g}^{\varphi}(\mathbf{x}) \right]$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00
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Hall droplet :

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• Edge diffeos
$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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Berry =
$$\int r dr [J + \rho r^2 / (2\ell^2)] \int dt d\varphi \frac{\dot{g}}{g'}$$

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$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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Berry =
$$\int r dr [J + \rho r^2 / (2\ell^2)] \int dt d\varphi \frac{\dot{g}}{g'}$$

• Edge diffeos
$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

Hall droplet :

- Assume isotropic potential
- Density $\rho(r)$ (bulk)
- Current $J(r)d\varphi$ (edge)
- Symmetric gauge



Berry =
$$\int r dr [J + \rho r^2 / (2\ell^2)] \int dt d\varphi \frac{\dot{g}}{g'}$$
 as in 1D !

• Edge diffeos
$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

Linear sdiffeos take this form

$$g(\mathbf{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D $\circ \circ \circ \bullet$	Hall viscosity 0000	The End 00

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$$g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$$

- Linear sdiffeos take this form
- ► Relation to Hall viscosity ?

$$g(\mathbf{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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4. Hall viscosity

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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4. Hall viscosity

A. Reminder on Hall viscosity

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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4. Hall viscosity

A. Reminder on Hall viscosity

B. Comparison with SDiff Berry ϕ

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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QHE on torus



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
0000	000	0000	0000	0000	00

QHE on torus



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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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QHE on torus

► Linear sdiffeos deform torus



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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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QHE on torus

Linear sdiffeos deform torus



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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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QHE on torus

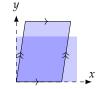
- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian



ntro	Berry φ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- ► Deform Hamiltonian :



$$H \sim (\mathbf{p} - q\mathbf{A})^2 \longrightarrow \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)G^{ij}(p_k - qA_k)$$

ntro	Berry φ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- ► Deform Hamiltonian :

$$H \sim (\mathbf{p} - q\mathbf{A})^2 \longrightarrow \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k)$$



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ntro	Berry φ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- ► Deform Hamiltonian :

$$H \sim (\mathbf{p} - q\mathbf{A})^2 \longrightarrow \mathcal{U} H \mathcal{U}^{-1} \sim (p_j - qA_j) \mathbf{G}^{ij}(p_k - qA_k)$$



ntro	Berry φ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- ► Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U} H \mathcal{U}^{-1} \sim (p_j - qA_j) \mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow \\ a^{\dagger}a \end{array}$$



ntro	Berry φ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- ► Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

► Unitary SL(2, ℝ) action by mechanical momenta



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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OHE on torus

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- Linear sdiffeos deform torus
- Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U} H \mathcal{U}^{-1} \sim (p_j - qA_j) \mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

• Unitary SL(2, \mathbb{R}) action by mechanical momenta

Fill ν Landau levels



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

► Unitary SL(2, ℝ) action by mechanical momenta

Fill ν Landau levels

• Berry =
$$N\nu \times$$
 (hyperbolic area) [Avron *et al.* 95]



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

► Unitary SL(2, ℝ) action by mechanical momenta

Fill ν Landau levels

• Berry =
$$N\nu \times$$
 (hyperbolic area) [Avron *et al.* 95]



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

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Fill ν Landau levels

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

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$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

► Unitary SL(2, ℝ) action by mechanical momenta

Fill ν Landau levels

• Berry = $N\nu \times (hyperbolic area)$ [Avron *et al.* 95]



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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry <i>φ</i> in 2D	Hall viscosity	The End
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QHE on torus

- Linear sdiffeos deform torus
- ► Equivalently, deform metric
- Deform Hamiltonian :

$$\begin{array}{ccc} H \sim (\mathbf{p} - q\mathbf{A})^2 & \longrightarrow & \mathcal{U}H\mathcal{U}^{-1} \sim (p_j - qA_j)\mathbf{G}^{ij}(p_k - qA_k) \\ \downarrow & & \downarrow \\ a^{\dagger}a & & \exp[\#a^2 + \cdots] \end{array}$$

► Unitary SL(2, ℝ) action by mechanical momenta

Fill ν Landau levels

- Berry = $N\nu \times (hyperbolic area)$
- Reproduce with planar sdiffeos ?



[Avron et al. 95]

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Berry =
$$\left[\int r \, \mathrm{d}r J + \int r^3 \, \mathrm{d}r \, \rho / (2\ell^2) \right] \int \mathrm{d}t \, \mathrm{d}\varphi \frac{\dot{g}}{g'}$$

[edge diffeos]

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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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[edge diffeos]

Fill ν Landau levels

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Berry =
$$\left[\underbrace{\int r \, \mathrm{d}r J}_{-N\nu} + \int r^3 \, \mathrm{d}r \, \rho / (2\ell^2) \right] \int \mathrm{d}t \, \mathrm{d}\varphi \frac{\dot{g}}{g'}$$

Fill ν Landau levels



[edge diffeos]

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(extensive)

[edge diffeos]

Fill ν Landau levels



Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Berry =
$$\left[\underbrace{\int r \, dr J}_{-N\nu} + \underbrace{\int r^3 \, dr \, \rho/(2\ell^2)}_{N^2/(2\nu)}\right] \int dt \, d\varphi \frac{\dot{g}}{g'}$$
(extensive)

[edge diffeos]

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Fill ν Landau levels

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$$\underbrace{\int dt \, d\varphi \frac{\dot{g}}{g'}}_{\text{(superextensive)}}$$

[edge diffeos]

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- Fill ν Landau levels
- **Superextensive** Berry ϕ

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[gauge-invariant]

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Linear sdiffeos $e^{2ig(\varphi)} = e^{2i\theta} \frac{e^{2i\varphi}\cosh\lambda + \sinh\lambda}{e^{2i\varphi}\sinh\lambda + \cosh\lambda}$

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• Berry-AB curvature = $N\nu \times \delta(\cosh \lambda) \wedge \delta\theta$

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[Avron et al. 95]

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- ► Hall viscosity ! ... but why ? [Avron *et al.* 95]

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Small linear sdiffeo

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Small linear sdiffeo
$$\mathbf{x} \mapsto \mathbf{x} + \begin{pmatrix} -\mathrm{Im}(\epsilon) & \omega - \mathrm{Re}(\epsilon) \\ -\omega - \mathrm{Re}(\epsilon) & \mathrm{Im}(\epsilon) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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HALL VISCOSITY REVISITED

Small linear sdiffeo $\mathbf{x} \mapsto \mathbf{x} + \begin{pmatrix} -\operatorname{Im}(\epsilon) & \omega - \operatorname{Re}(\epsilon) \\ -\omega - \operatorname{Re}(\epsilon) & \operatorname{Im}(\epsilon) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

► Action on Hall droplet ?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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- ► Action on Hall droplet ?
- Mechanical momenta a,a^{\dagger} change Landau levels

Intro 0000	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity ○○○●	The End 00
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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 $[\mathcal{U}[\text{lin.sdiff}] \sim 1 + i\epsilon a^2 + \cdots]$

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Hall viscosity only takes a, a^{\dagger}

[Lévay 95]

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Hall viscosity only takes a, a^{\dagger}

[Lévay 95]

• To recover it, **mod out** b, b^{\dagger}

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Small linear sdiffeo $\mathbf{x} \mapsto \mathbf{x} + \begin{pmatrix} -\operatorname{Im}(\epsilon) & \omega - \operatorname{Re}(\epsilon) \\ -\omega - \operatorname{Re}(\epsilon) & \operatorname{Im}(\epsilon) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

- Action on Hall droplet ?
- Mechanical momenta *a*,*a*[†] change Landau levels, Magnetic translations *b*,*b*[†] change angular mom
- Linear sdiffeo generated by $\underbrace{\epsilon a^2 + \omega a^{\dagger} a}_{\text{changes metric}} \underbrace{\overline{\epsilon} b^2 \omega b^{\dagger} b}_{\text{LLL projection}} + h.c.$

Hall viscosity only takes a, a^{\dagger}

[Lévay 95]

- To recover it, **mod out** b, b^{\dagger}
- Removes Aharonov-Bohm

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- To recover it, **mod out** b, b^{\dagger}
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- ▶ "Hall viscosity from the edge"

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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This talk in one sentence :

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End ●0

This talk in one sentence :

Arbitrary deformations of quantum Hall droplets

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End ●0

This talk in one sentence :

Arbitrary deformations of quantum Hall droplets yield Berry phases that **generalize Hall viscosity**

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End ●0

This talk in one sentence :

Arbitrary deformations of quantum Hall droplets yield Berry phases that generalize Hall viscosity and involve the edge current.

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Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End ●0

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Follow-ups :

Intro 0000	Berry ϕ in 1D 000	Sdiffeos in 2D 0000	Berry ϕ in 2D 0000	Hall viscosity 0000	The End ●0

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Follow-ups :

Independence of potential/disorder ?

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Follow-ups :

- Independence of potential/disorder ?
- Action of sdiffeos projected in LLL ?

[Cappelli et al. 94]

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- ► Generalization to FQHE ?

[Read 08, Bradlyn-Read 15]

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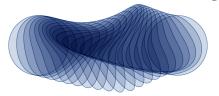
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Follow-ups :

- ► Independence of potential/disorder ?
- ► Action of sdiffeos projected in LLL ? [Cappelli *et al.* 94]
- ► Generalization to FQHE ? [Read 08, Bradlyn-Read 15]
- Sdiffeos produced by (nonlinear) edge modes ?

Intro	Berry ϕ in 1D	Sdiffeos in 2D	Berry ϕ in 2D	Hall viscosity	The End
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Thanks for listening !



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