Deformational Berry Phases of Quantum Hall Droplets

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December 2021

With B. Estienne, to appear.

Intro

Motivation

Diffeomorphisms (cts deformations) ubiquitous in physics

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...i.e. **infinite-dimensional geometry** ?

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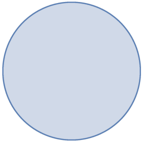

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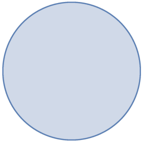

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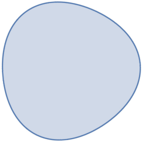

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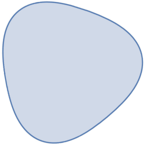

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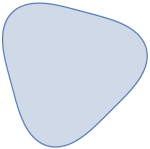

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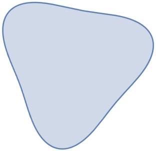

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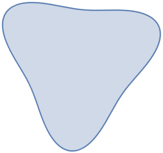

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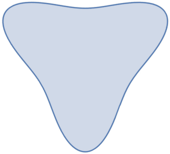

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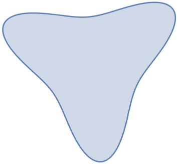

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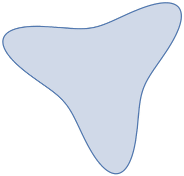

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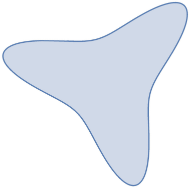

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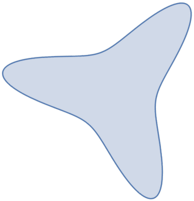

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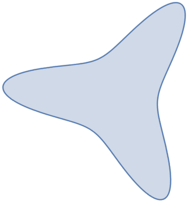

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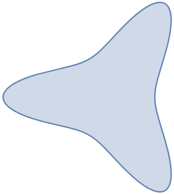

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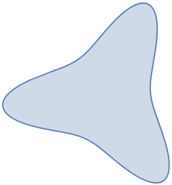

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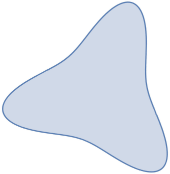

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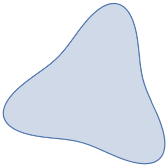

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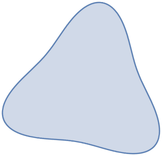

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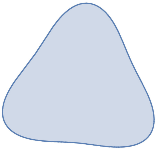

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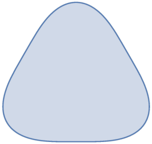

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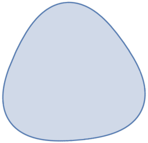

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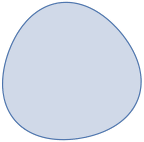

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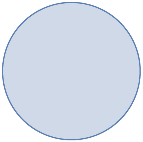

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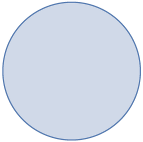

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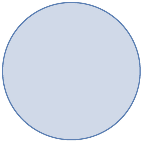

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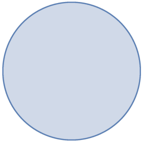

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- Finite droplets $\Rightarrow$ edge contributions
- "**Bulk-edge correspondence**" for Hall viscosity

PLAN

1. **Adiabatic diffeos in 1D**
2. One-body sdiffeos in 2D
3. Adiabatic sdiffeos in 2D
4. Generalized Hall viscosity

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A. Reminder on Berry phases

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A. Reminder on Berry phases

B. Deformational Berry ϕ in 1D

Berry phases

BERRY PHASES

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Berry phases

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- Parameter-dep. energy eigenstates $|\psi(g)\rangle$

Berry phases

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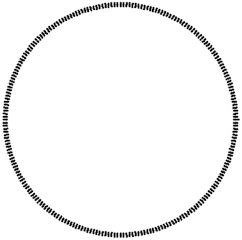

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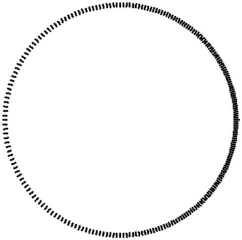

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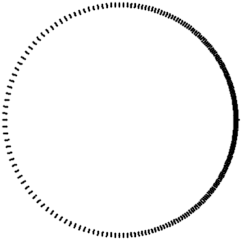

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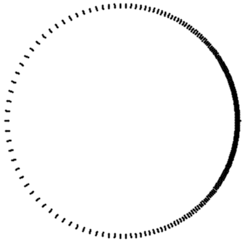

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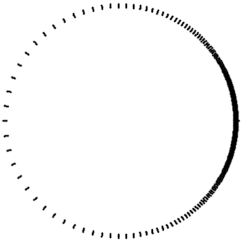

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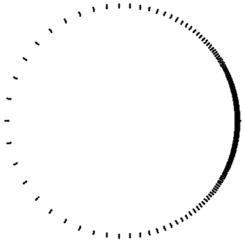

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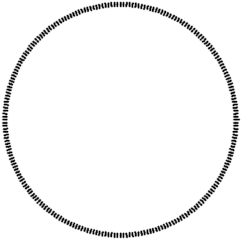

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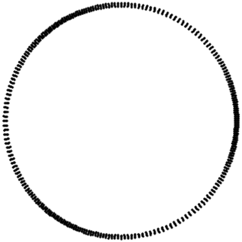

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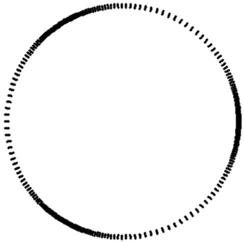

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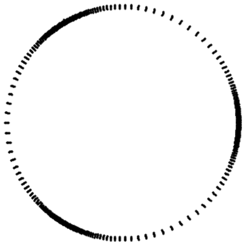

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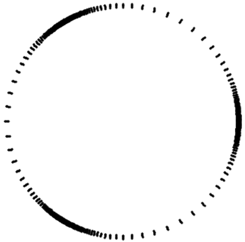

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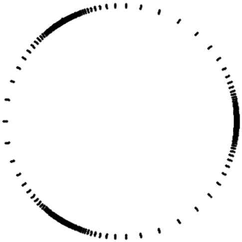

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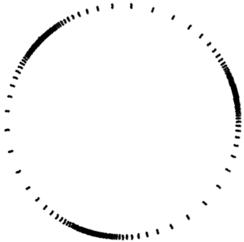

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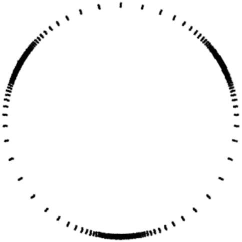

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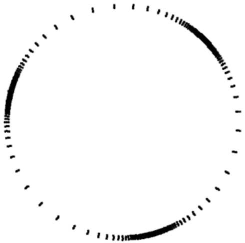

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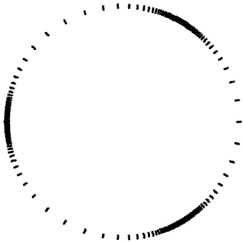

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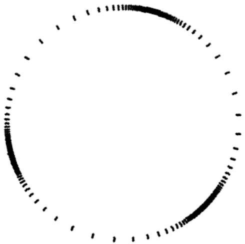

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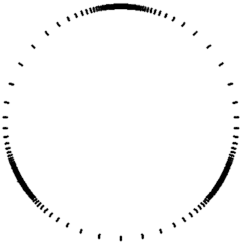

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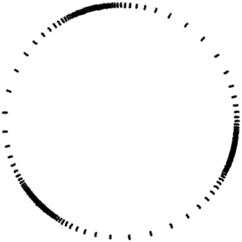

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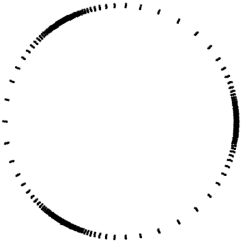

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Analogue for **2D electrons in magnetic field** ?

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A. Area-preserving diffeos

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A. Area-preserving diffeos

B. Unitary action of sdiffeos

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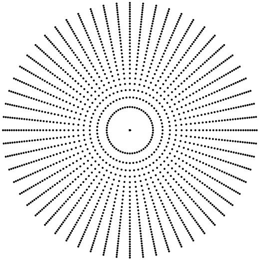

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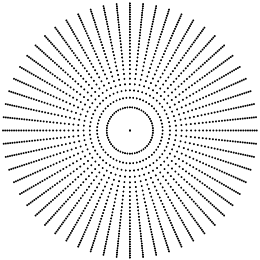

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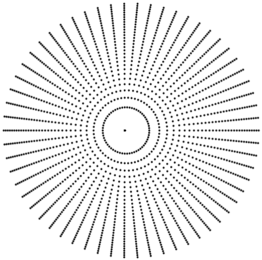

SDIFFEOS

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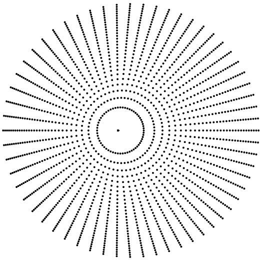

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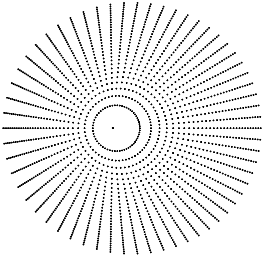

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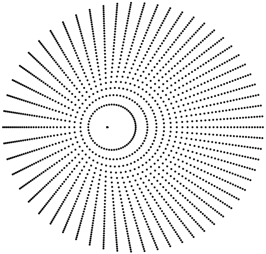

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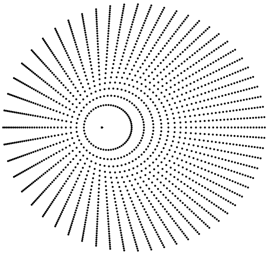

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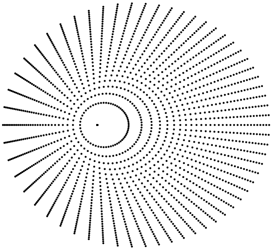

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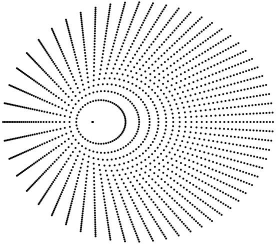

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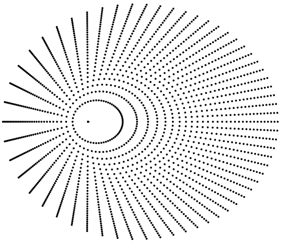

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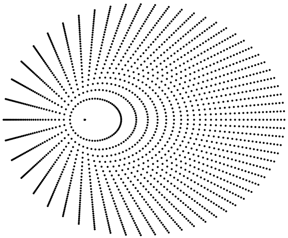

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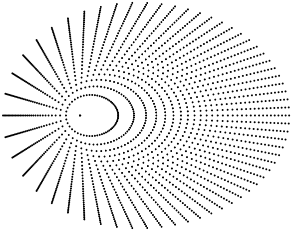

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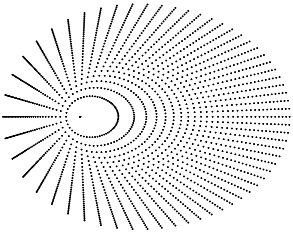

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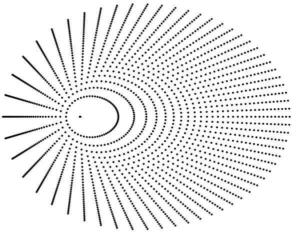

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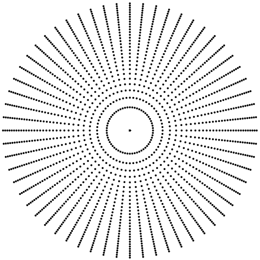

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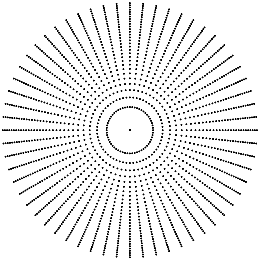

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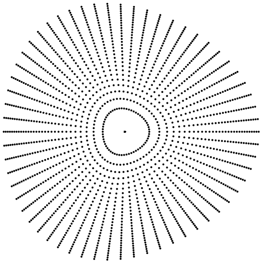

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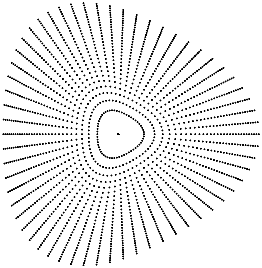

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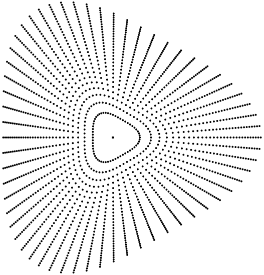

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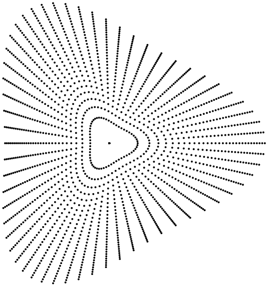

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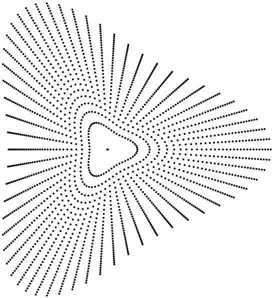

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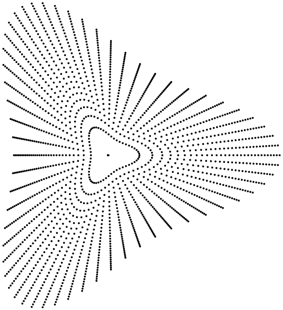

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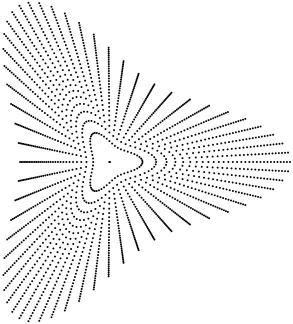

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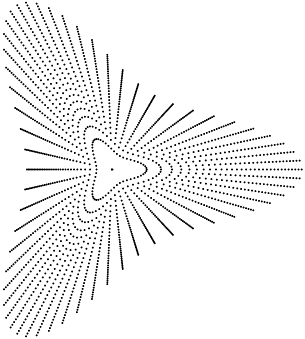

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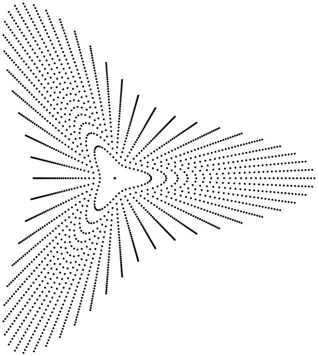

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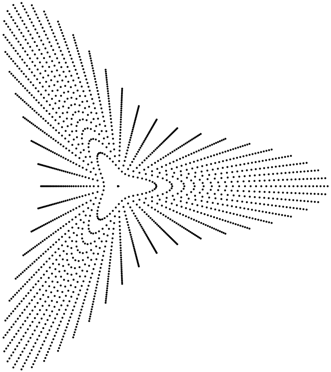

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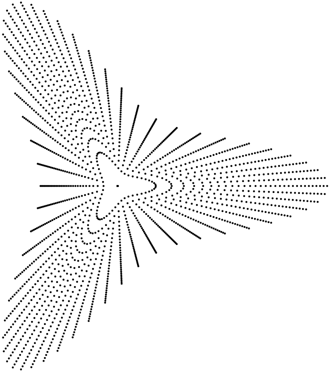

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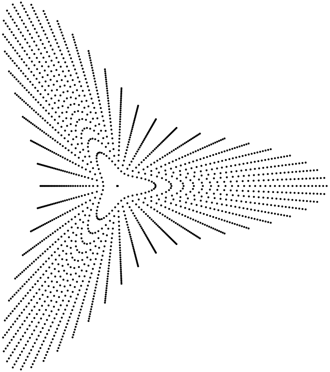

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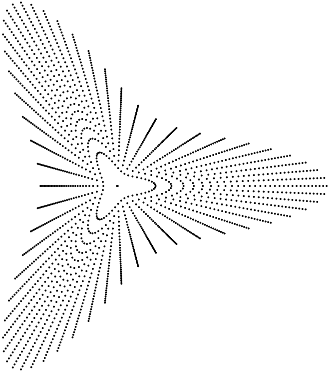

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Electron in $\mathbb{R}^2$

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Unitary sdiffeos

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- **Unitary action of sdiffeos** ?

$$\left(\mathcal{U}[g]\psi\right)(\mathbf{x})$$

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  (we wish to compare wavefcts)

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$$(\mathcal{U}[g]\psi)\,(\mathbf{x}) \equiv \underbrace{e^{iq\,\mathrm{d}^{-1}(\mathbf{A}-\bar{g}^*\mathbf{A})(\mathbf{x})}}_{\text{compensating gauge tsf}}\,\psi\big(\bar{g}(\mathbf{x})\big)$$

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3. Adiabatic sdiffeos

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A. One-body Berry ϕ

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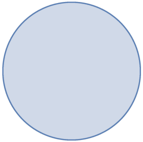

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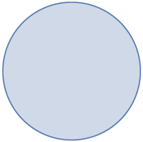

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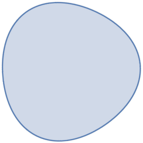

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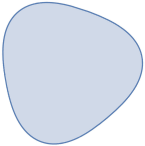

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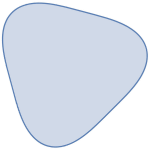

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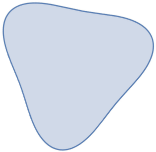

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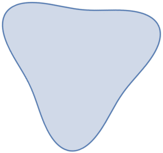

Many-body Berry ϕ

Droplet of $N \gg 1$ electrons

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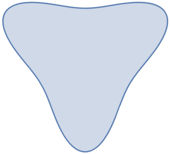

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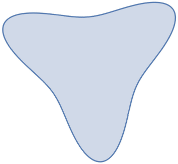

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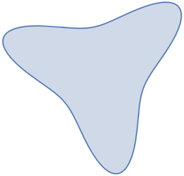

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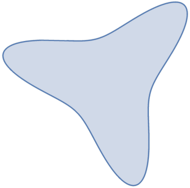

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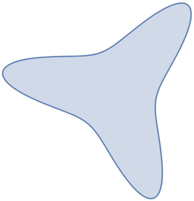

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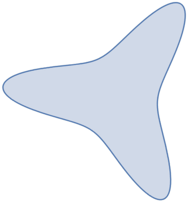

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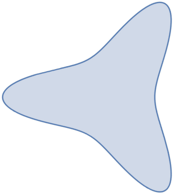

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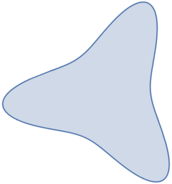

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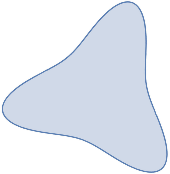

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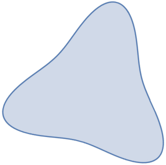

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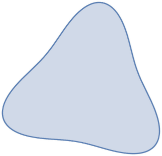

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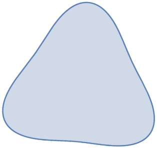

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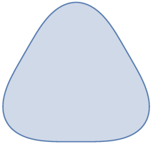

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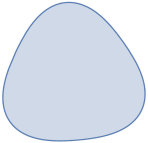

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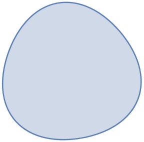

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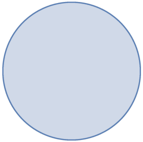

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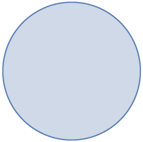

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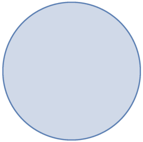

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(Adiabatic thm holds despite haplessness [Avron-Elgart 98])

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Hall droplet :

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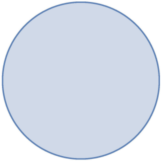

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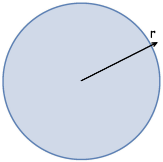

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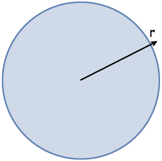

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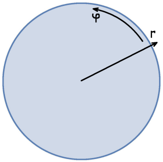

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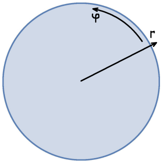

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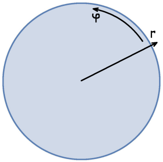

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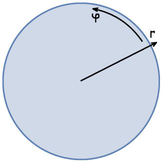

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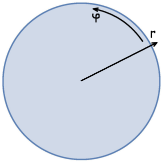

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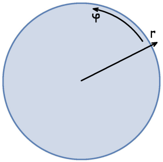

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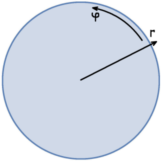

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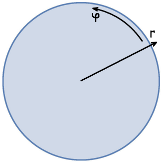

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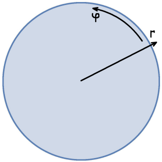

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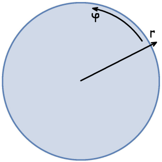

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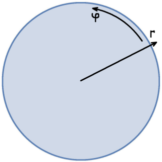
- Edge diffeos $g(r^2,\varphi)=\left(\frac{r^2}{g'(\varphi)},\,g(\varphi)\right)$
- Linear sdiffeos take this form

$$g(\mathbf{x})=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

MANY-BODY BERRY ϕ

Hall droplet :

- Assume isotropic potential
- Density $\rho(r)$ (bulk)
- Current $J(r)\mathrm{d}\varphi$ (edge)
- Symmetric gauge



$$\text{Berry} = \int r\,\mathrm{d}r \left[J + \rho\, r^2/(2\ell^2)\right] \int \mathbf{d}t\,\mathbf{d}\varphi \frac{\dot{g}}{g'}$$ as in 1D !

- Edge diffeos $g(r^2, \varphi) = \left(\frac{r^2}{g'(\varphi)}, g(\varphi)\right)$
- Linear sdiffeos take this form
- Relation to Hall viscosity ?

$$g(\mathbf{x}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

4. Hall viscosity

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A. Reminder on Hall viscosity

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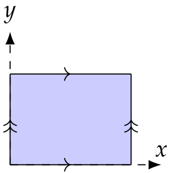
B. Comparison with SDiff Berry ϕ

Hall viscosity

QHE on torus

HALL VISCOSITY

QHE on torus



Hall viscosity

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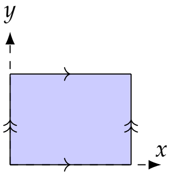
- **Linear sdiffeos** deform torus



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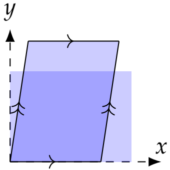

Hall viscosity

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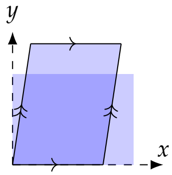

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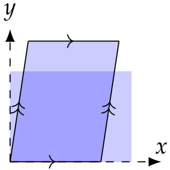
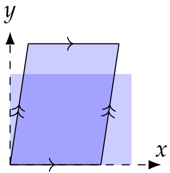


HALL VISCOSITY



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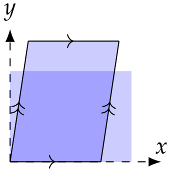
$$H \sim (\mathbf{p} - q\mathbf{A})^2 \quad \longrightarrow \quad \mathcal{U} H \mathcal{U}^{-1} \sim (p_j - qA_j) G^{ij} (p_k - qA_k)$$

Hall viscosity



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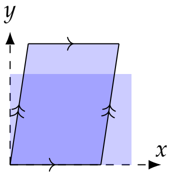

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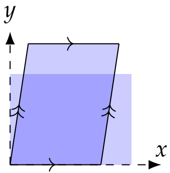

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$$\downarrow$$

$$a^\dagger a$$

Hall viscosity

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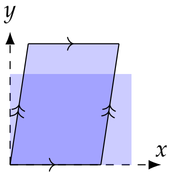

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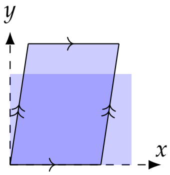
$$\downarrow \qquad\qquad \downarrow$$

$$a^\dagger a \qquad\qquad \exp[\# a^2 + \cdots]$$

HALL VISCOSITY



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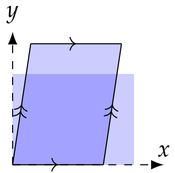
- Unitary SL(2,$\mathbb{R}$) action by mechanical momenta

Hall viscosity



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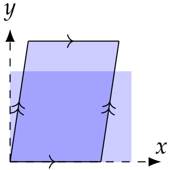

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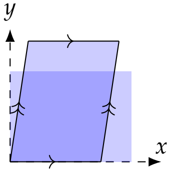

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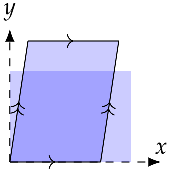

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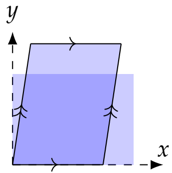

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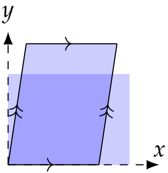

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Fill ν Landau levels

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- Reproduce with planar sdiffeos ?

Hall viscosity revisited

Hall viscosity revisited

$$\text{Berry} = \left[\int r\,\mathrm{d}r\,J + \int r^3\,\mathrm{d}r\,\rho/(2\ell^2)\right]\int \mathrm{d}t\,\mathrm{d}\varphi\,\frac{\dot{g}}{g'}$$

[edge diffeos]

HALL VISCOSITY REVISITED

$$\text{Berry} = \left[\int r\,\mathrm{d}r\,J + \int r^3\,\mathrm{d}r\,\rho/(2\ell^2)\right]\int \mathrm{d}t\,\mathrm{d}\varphi\,\frac{\dot{g}}{g'}$$ [edge diffeos]

- Fill ν Landau levels

Hall viscosity revisited

$$\text{Berry} = \Big[\underbrace{\int r\,\mathrm{d}r\,J}_{-N\nu} + \int r^3\,\mathrm{d}r\,\rho/(2\ell^2)\Big]\int \mathrm{d}t\,\mathrm{d}\varphi\,\frac{\dot{g}}{g'}$$

[edge diffeos]

- Fill ν Landau levels

Hall viscosity revisited

$$\text{Berry} = \Big[\underbrace{\int r\, \mathrm{d}r\, J}_{\substack{-N\nu \\ \text{(extensive)}}} + \int r^3\, \mathrm{d}r\, \rho/(2\ell^2) \Big] \int \mathrm{d}t\, \mathrm{d}\varphi \frac{\dot{g}}{g'}$$

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[edge diffeos]

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- **Hall viscosity** ! ... but why ? [Avron *et al.* 95]

Hall viscosity revisited

Hall viscosity revisited

Small **linear sdiffeo**

Hall viscosity revisited

Small **linear sdiffeo** $\mathbf{x} \mapsto \mathbf{x} + \begin{pmatrix} -\mathrm{Im}(\epsilon) & \omega - \mathrm{Re}(\epsilon) \\ -\omega - \mathrm{Re}(\epsilon) & \mathrm{Im}(\epsilon) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Hall viscosity revisited

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$[\mathcal{U}[\text{lin.sdiff}] \sim 1 + i\epsilon a^2 + \cdots]$

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Thanks for listening !