BMS Particles in Three Dimensions

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Based on work with
G. Barnich, A. Campoleoni, H. González, A. Maloney, M. Riegler

MOTIVATION

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- 3D toy model : **BMS_3**

Plan of the talk

1. **Unitary reps of semi-direct products**

2. Representations of BMS_3

3. Relation to gravity

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C. Unitary representations

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- Orbits !

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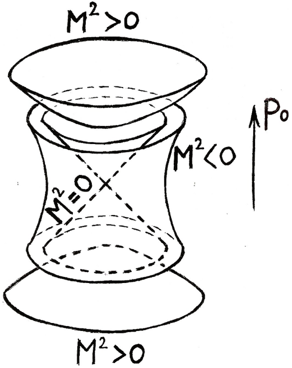

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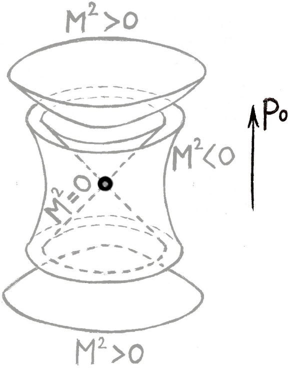

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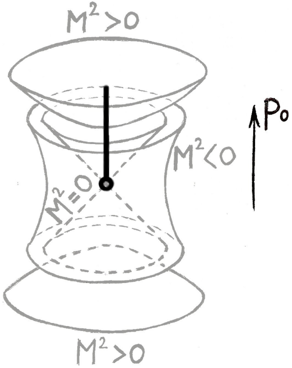

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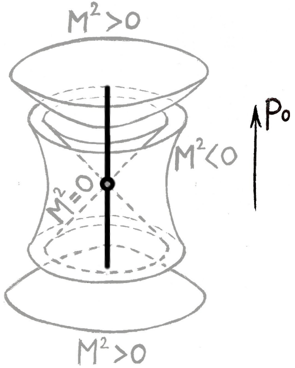

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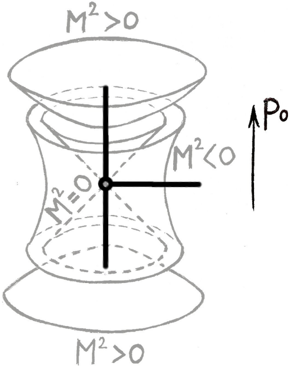

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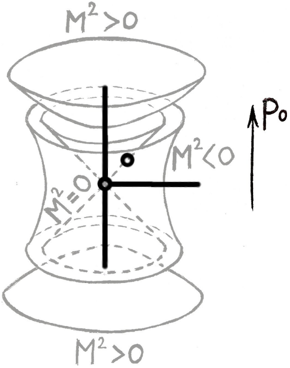

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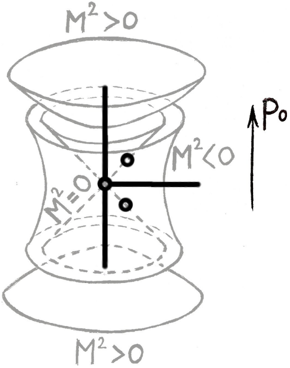

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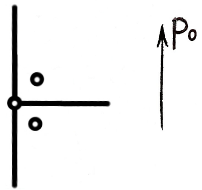

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- Let $\mathcal{H}$ = space of **wavefunctions** $\Psi : \mathcal{O}_p \to \mathbb{C} : q \mapsto \Psi(q)$
- Define $\left(\mathcal{T}[(f,\alpha)]\Psi\right)(q) = e^{i\langle q,\alpha\rangle}\Psi(f^{-1}\cdot q)$
- This is an IRREP of P !

How to make $\mathcal{T}$ unitary?

- Pick a G-invariant measure μ on $\mathcal{O}_p$
- Scalar product $\langle\Phi|\Psi\rangle \equiv \int_{\mathcal{O}_p} d\mu(q)\,(\Phi(q))^*\,\Psi(q)$

All UIRREPs of P are of this form ! [Mackey ∼1950]

2. Unitary representations of BMS_3

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A. Definition of the BMS_3 group

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B. Orbits and unitary reps

DEFINITION OF BMS_3

BMS_3 = Aspt. symmetry group of 3D aspt. flat space-times

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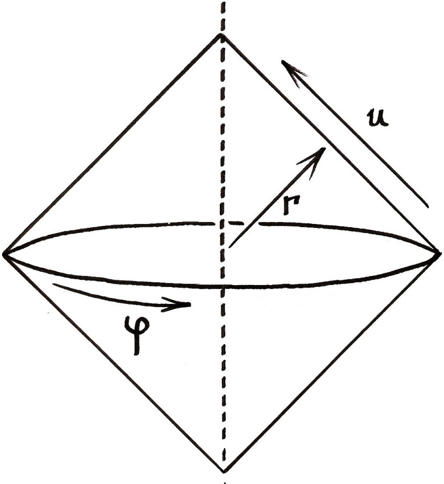

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$$\boxed{\mathrm{BMS}_3 \equiv \mathrm{Diff}(S^1) \ltimes \mathrm{Vect}(S^1)}$$

[Barnich & BO 2014]

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Orbits of supermomenta under superrotations?

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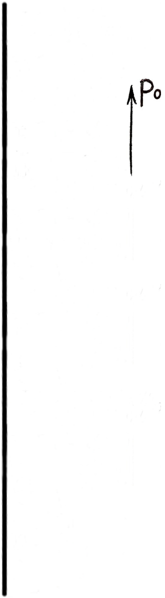

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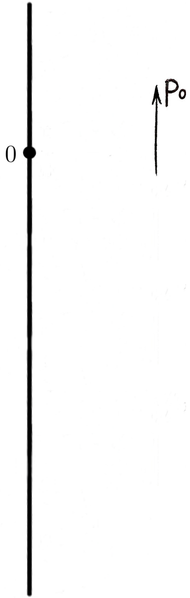

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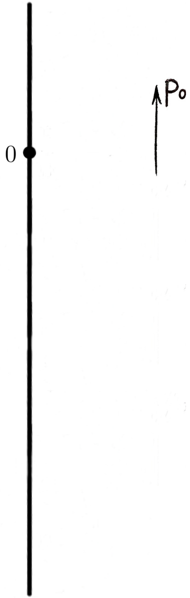

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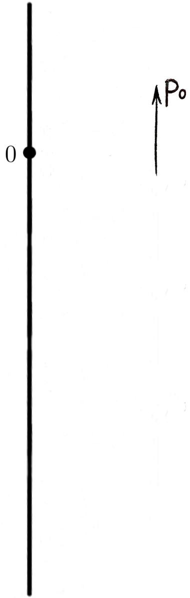

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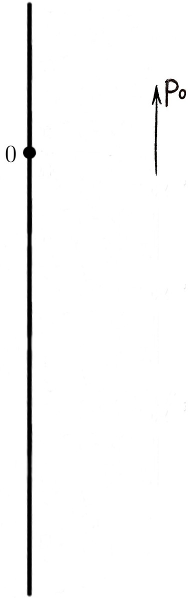

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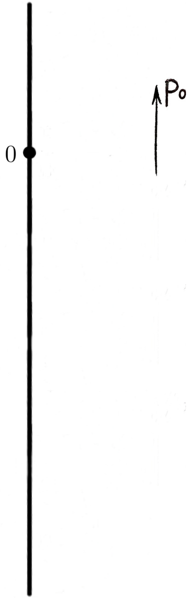

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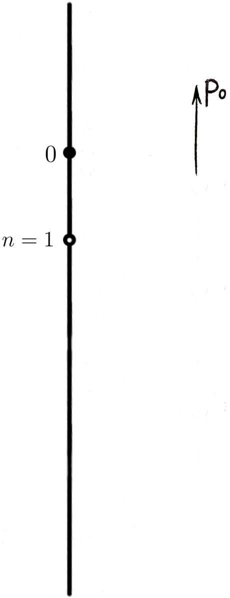

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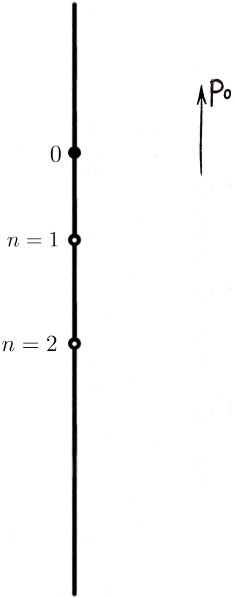

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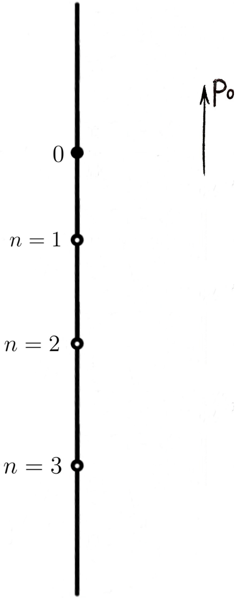

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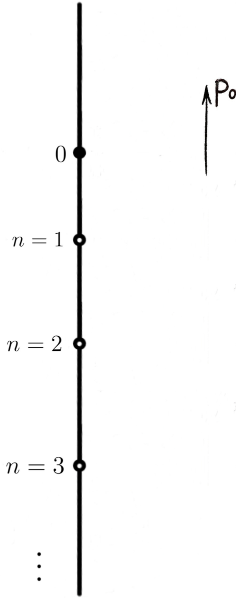

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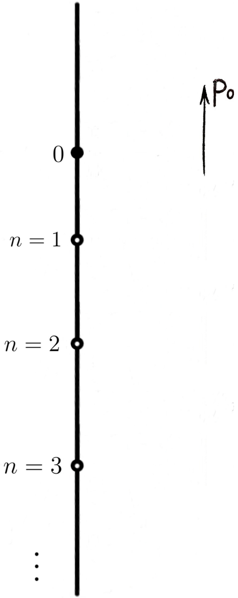

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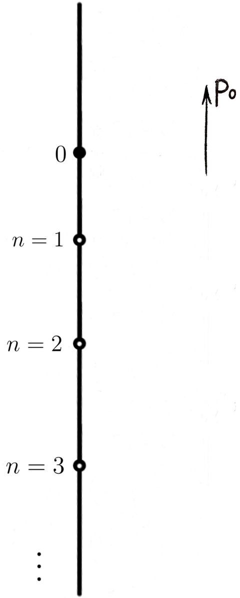

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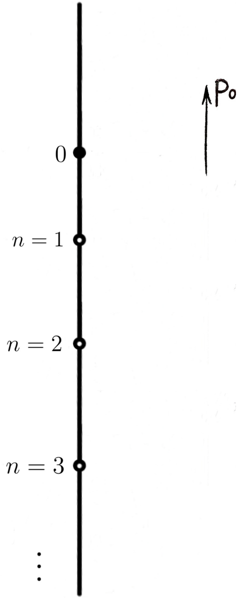

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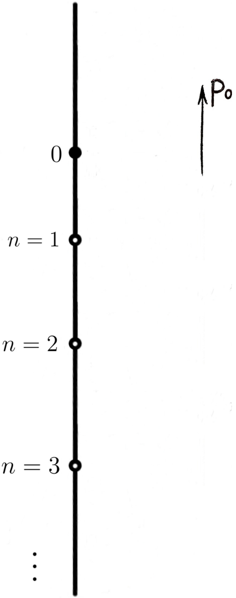

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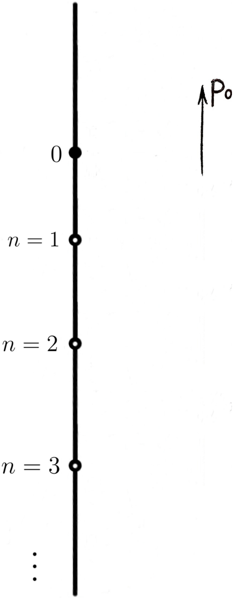

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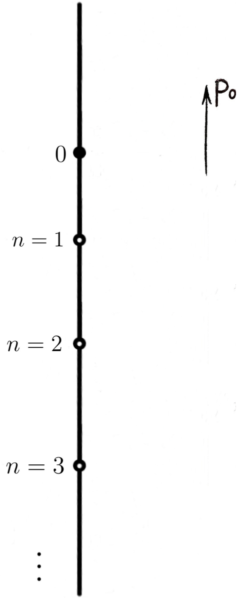

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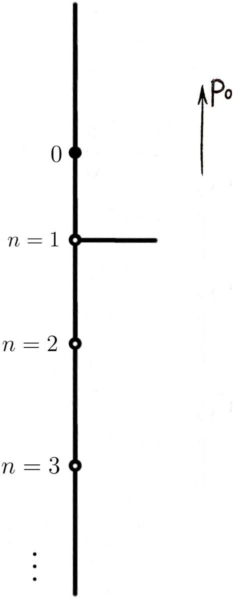

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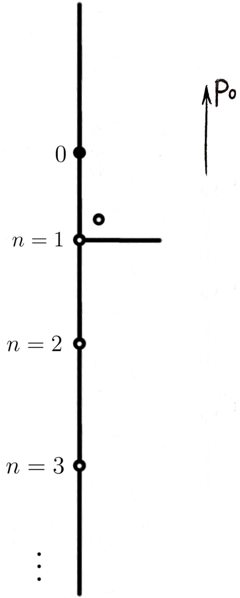

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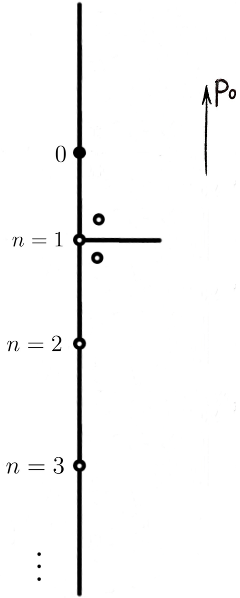

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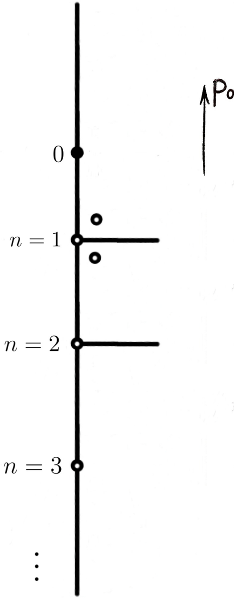

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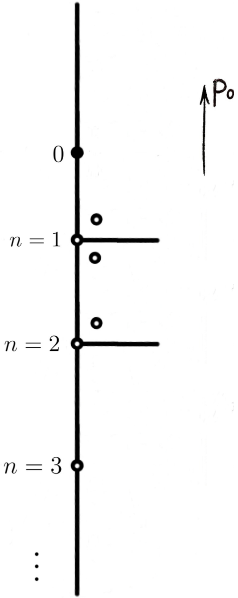

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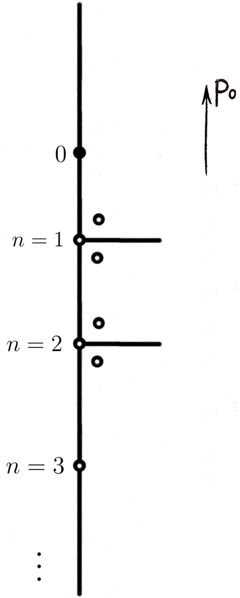

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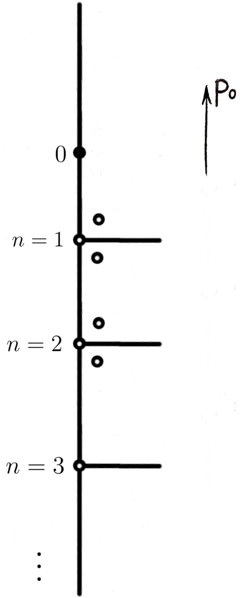

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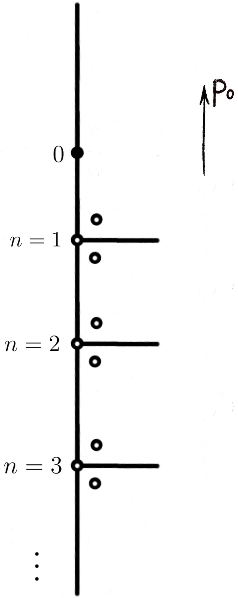

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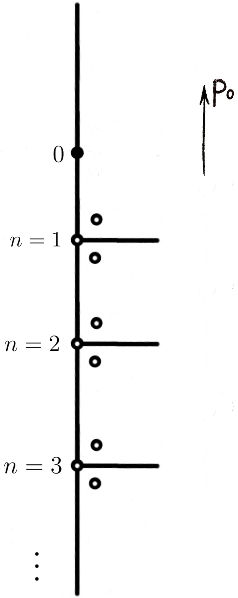

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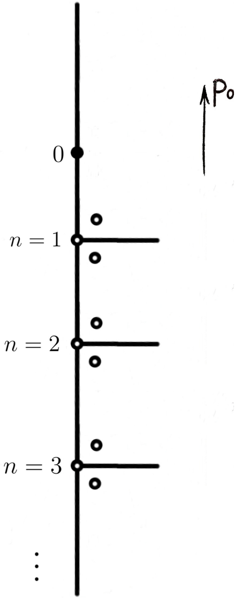

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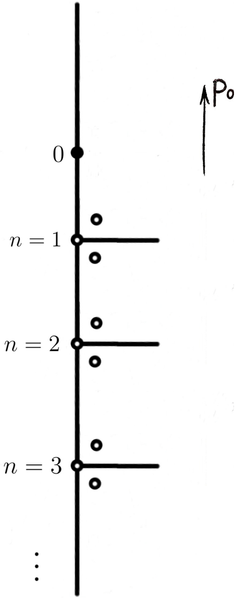

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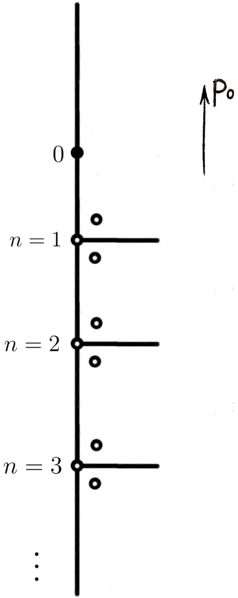

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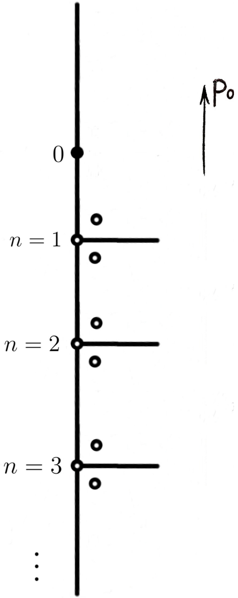

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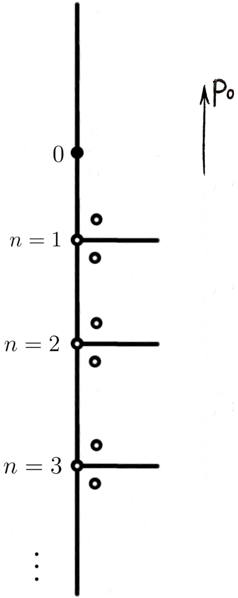

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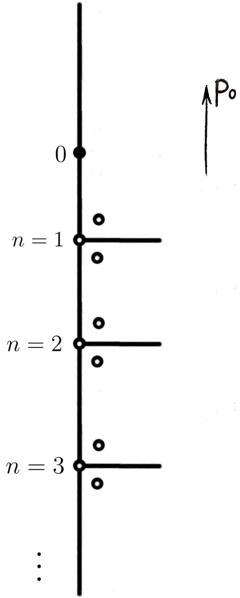

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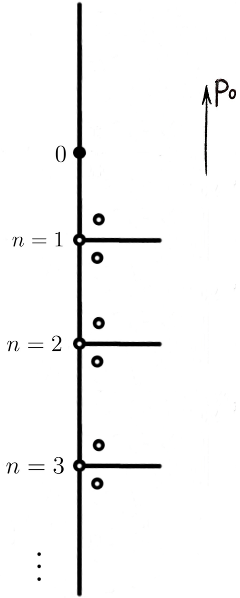

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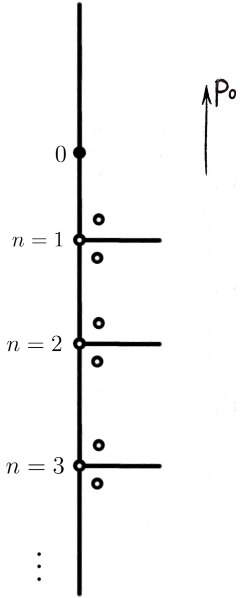

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3. Relation to gravity

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A. Supermomentum and Bondi mass

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A. Supermomentum and Bondi mass

B. Characters and partition functions

SUPERMOMENTUM & BONDI MASS

On-shell aspt. flat metrics

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On-shell aspt. flat metrics : [Barnich & Troessaert 2010]

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[BO 2015]

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- Flat analogue of $\mathrm{AdS}_3/\mathrm{CFT}_2$ results

  [Giombi, Maloney, Yin 2008]

  [Gaberdiel, Gopakumar 2010]

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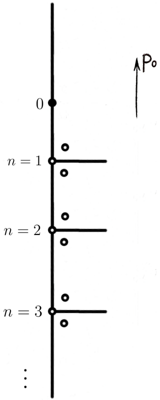
BMS$_3$ characters	$\leftrightarrow$	One-loop partition fcts around flat space



Thank you !