Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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BMS Particles in Three Dimensions

Blagoje Oblak

ETH Zurich

March 2017

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Based on work with G. Barnich, A. Campoleoni, H. González, A. Maloney, M. Riegler

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Flat space-time

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Flat space-time

► Symmetries : Poincaré ~ Lorentz × Translations

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[Bondi et al. 1962, Barnich et al. 2009]

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Flat space-time

► Symmetries : Poincaré ~ Lorentz × Translations

But space-time is not flat ...

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UIRREPs of Poincaré define the notion of "particle"

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► 3D toy model : **BMS**₃

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1. Unitary reps of semi-direct products

- 2. Representations of BMS₃
- 3. Relation to gravity

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A. Semi-direct products

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A. Semi-direct products

B. Orbits

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B. Orbits (+ Example : Poincaré)

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A. Semi-direct products

B. Orbits (+ Example : Poincaré)

C. Unitary representations

Conclusion

Semi-direct products

Poincaré = Lorentz × Translations

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Conclusion

Semi-direct products

P = Lorentz \ltimes Translations

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Semi-direct products

$P = SO(2, 1) \ltimes$ Translations

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Semi-direct products



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SEMI-DIRECT PRODUCTS

 $P = \mathrm{SO}(2,1) \ltimes \mathbb{R}^3$

• Elements of $P = \text{pairs}(f, \alpha)$

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Conclusion

Semi-direct products

- Elements of $P = \text{pairs}(f, \alpha)$
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Semi-direct products

 $P = \mathrm{SO}(2,1) \ltimes \mathbb{R}^3$

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Unitary reps of semi-direct products 0000 Unitary reps of BMS₃

Relation to gravity

Conclusion 00

Semi-direct products

UIRREPS of $P = SO(2, 1) \ltimes \mathbb{R}^3$?

[Wigner 1939]

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Semi-direct products

UIRREPS of $P = SO(2, 1) \ltimes \mathbb{R}^3$?

• Start with Abelian group \mathbb{R}^3

[Wigner 1939]

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[Wigner 1939]

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- Start with Abelian group \mathbb{R}^3
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Semi-direct products

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[Wigner 1939]

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Which momenta?

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- ► Orbits !

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Orbits			

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Then,
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 $\mathcal{T}[(e, \alpha)]\Big|_{\mathbb{V}'} = e^{i\langle p, \sigma_{f-1}\alpha \rangle}$

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• **Orbit** of
$$p : \mathcal{O}_p \equiv \{f \cdot p \mid f \in G\}$$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Orbits			

Orbits of momenta for Poincaré?

► Lorentz tsfs preserve the Minkowski metric

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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- ► Lorentz tsfs preserve the Minkowski metric
- $\eta^{\mu\nu}p_{\mu}p_{\nu} \equiv p^2$ is Lorentz-invariant

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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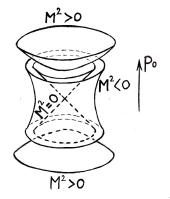
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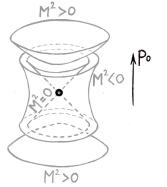
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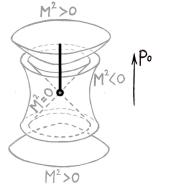
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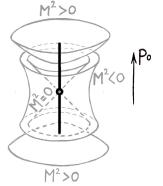
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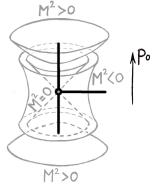
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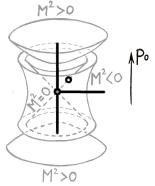
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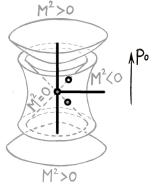
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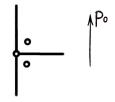


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ORBITS

Orbits of momenta for Poincaré?

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Let's build UIRREPS of *P* !

• Pick an orbit \mathcal{O}_p



Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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- Let $\mathcal{H} =$ space of **wavefunctions** Ψ

Unitary reps of semi-direct products ○○○○●	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion 00

Let's build UIRREPS of *P* !

- Pick an orbit \mathcal{O}_p
- Let $\mathcal{H} =$ space of wavefunctions $\Psi : \mathcal{O}_p \to \mathbb{C} : q \mapsto \Psi(q)$

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Unitary reps of semi-direct products ○○○○●	Unitary reps of BMS ₃ 0000000	Relation to gravity	Conclusion 00

- Pick an orbit \mathcal{O}_p
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- Define $\mathcal{T}[(f, \alpha)]\Psi$

Unitary reps of semi-direct products ○○○○●	Unitary reps of BMS ₃ 0000000	Relation to gravity	Conclusion 00

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• Define $(\mathcal{T}[(f,\alpha)]\Psi)(q) = e^{i\langle q,\alpha\rangle}\Psi(f^{-1}\cdot q)$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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- Pick an orbit \mathcal{O}_p
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Let's build UIRREPS of *P* !

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How to make T unitary?

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How to make T unitary?

• Pick a *G*-invariant measure μ on \mathcal{O}_p

• Scalar product
$$\langle \Phi | \Psi \rangle \equiv \int_{\mathcal{O}_p} d\mu(q) \left(\Phi(q) \right)^* \Psi(q)$$

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All UIRREPS of *P* are of this form !

[Mackey ~1950]

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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2. Unitary representations of BMS₃

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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2. Unitary representations of BMS₃

A. Definition of the BMS₃ group

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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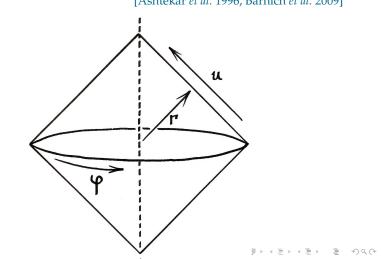
2. Unitary representations of BMS₃

A. Definition of the BMS₃ group

B. Orbits and unitary reps

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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BMS₃ = Aspt. symmetry group of 3D aspt. flat space-times [Ashtekar *et al.* 1996, Barnich *et al.* 2009]

► Aspt. flat metrics in 3D :

 $ds^2 \stackrel{r \to +\infty}{\sim} -du^2 - 2dudr + r^2 d\varphi^2$

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► Aspt. flat metrics in 3D :

 $ds^2 \stackrel{r \to +\infty}{\sim} - du^2 - 2dudr + r^2 d\varphi^2 +$ subleading terms

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DEFINITION OF BMS₃

- ► Aspt. flat metrics in 3D : $ds^2 \stackrel{r \to +\infty}{\sim} -du^2 - 2dudr + r^2 d\varphi^2 + \text{ subleading terms}$
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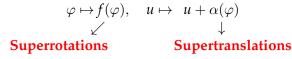
Conclusion

DEFINITION OF BMS₃

BMS₃ = Aspt. symmetry group of 3D aspt. flat space-times [Ashtekar *et al.* 1996, Barnich *et al.* 2009]

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Definition of BMS_3

 BMS_3 transformations :

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DEFINITION OF BMS₃

 BMS_3 transformations :

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• Elements of BMS₃ = pairs (f, α)

BMS₃ transformations :

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DEFINITION OF BMS₃

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Conclusion

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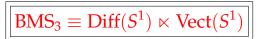
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[Barnich & BO 2014]

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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► What should we expect ?

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Poincaré : exact space-time symmetry

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BMS₃ : aspt. space-time symmetry

• UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons

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► UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons = **BMS**₃ particle

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► UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons = **BMS**₃ particle

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$$\bullet \ \mathcal{H}_{BMS} = \mathcal{H}_{Poinc} \otimes \mathcal{H}_{Soft \, grav}$$

• Can we classify BMS₃ particles ?

► What should we expect ?

Poincaré : exact space-time symmetry

• UIRREP = \mathcal{H}_{Poinc} = Particle

BMS₃ : aspt. space-time symmetry

► UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons = **BMS**₃ particle

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- $\blacktriangleright \ \mathcal{H}_{BMS} = \mathcal{H}_{Poinc} \otimes \mathcal{H}_{Soft \ grav}$
- Can we classify BMS₃ particles ?
- Can we describe $\mathcal{H}_{\text{Soft grav}}$?

► What should we expect ?

Poincaré : exact space-time symmetry

• UIRREP = \mathcal{H}_{Poinc} = Particle

BMS₃ : aspt. space-time symmetry

- ► UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons = **BMS**₃ particle
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- Can we compute stuff with it ?

► What should we expect ?

Poincaré : exact space-time symmetry

• UIRREP = \mathcal{H}_{Poinc} = Particle

 $BMS_3:$ aspt. space-time symmetry

- ► UIRREP = \mathcal{H}_{BMS} = Particle dressed w/ soft gravitons = **BMS**₃ particle
- $\blacktriangleright \ \mathcal{H}_{BMS} = \mathcal{H}_{Poinc} \otimes \mathcal{H}_{Soft\,grav}$
- Can we classify BMS₃ particles ? \longrightarrow Yes !
- ► Can we describe $\mathcal{H}_{\text{Soft grav}}$? \longrightarrow Yes !
- Can we compute stuff with it ? \rightarrow Yes !

Unitary	reps of semi-direct products	
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Relation to gravity 0000 Conclusion 00

ORBITS AND UNITARY REPS

Unitary reps of $BMS_3 = Diff(S^1) \ltimes Vect(S^1)$?



Unitary reps of semi-direct products 00000

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ORBITS AND UNITARY REPS

Unitary reps of BMS₃ =
$$\underbrace{\text{Diff}(S^1) \ltimes \text{Vect}(S^1)}_{G \ltimes A}$$
?

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ORBITS AND UNITARY REPS

Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
?
• $A^* =$ space of momenta

 $A^* =$ space of ►

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ORBITS AND UNITARY REPS

Unitary reps of BMS₃ =
$$\underbrace{\text{Diff}(S^1) \ltimes \text{Vect}(S^1)}_{G \ltimes A}$$
?

► Vect(*S*¹)* = space of **supermomenta**

Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
?

• Vect $(S^1)^*$ = space of supermomenta $p(\varphi)$:



Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
?

• Vect $(S^1)^*$ = space of supermomenta $p(\varphi)$:

 $\langle p, \alpha \rangle$



Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
?

• Vect $(S^1)^*$ = space of supermomenta $p(\varphi)$:



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Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
?

► Vect(S¹)* = space of supermomenta $p(\varphi)$: $\langle p, \alpha \rangle = \frac{1}{2\pi} \int d\varphi \, p(\varphi) \alpha(\varphi)$

$$\underbrace{\langle \underline{p}, \alpha \rangle}_{\in \mathbb{R}} = \frac{1}{2\pi} \int d\varphi \, p(\varphi) \alpha(\varphi)$$

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Unitary reps of
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► Vect(S¹)* = space of supermomenta $p(\varphi)$: $\underbrace{\langle p, \alpha \rangle}_{\in \mathbb{R}} = \frac{1}{2\pi} \int d\varphi \, p(\varphi) \alpha(\varphi)$

$$p(\varphi) = \sum_{n \in \mathbb{Z}} p_n e^{-in\varphi}$$

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Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
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► Vect(S¹)* = space of supermomenta $p(\varphi)$: $\underbrace{\langle p, \alpha \rangle}_{\in \mathbb{R}} = \frac{1}{2\pi} \int d\varphi \, p(\varphi) \alpha(\varphi)$

$$p(\varphi) = \sum_{n \in \mathbb{Z}} p_n e^{-in\varphi}$$
$$\blacktriangleright p_0 = \text{energy}$$

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ORBITS AND UNITARY REPS

Unitary reps of BMS₃ =
$$\underbrace{\text{Diff}(S^1) \ltimes \text{Vect}(S^1)}_{G \ltimes A}$$
?

► Vect(S¹)* = space of supermomenta $p(\varphi)$: $\underbrace{\langle p, \alpha \rangle}_{\in \mathbb{R}} = \frac{1}{2\pi} \int d\varphi \, p(\varphi) \alpha(\varphi)$

$$p(\varphi) = \sum_{n \in \mathbb{Z}} p_n e^{-in\varphi}$$

$$p_0 = \text{energy}$$

$$p_{\pm 1} = \text{spatial momentum}$$

Unitary reps of
$$BMS_3 = \underbrace{Diff(S^1) \ltimes Vect(S^1)}_{G \ltimes A}$$
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► Vect(S¹)* = space of supermomenta
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$$p(\varphi) = \sum_{n \in \mathbb{Z}} p_n e^{-in\varphi}$$

$$p_0 = \text{energy}$$

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Orbits of supermomenta under superrotations?

Fix a supermomentum $p(\varphi)$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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ORBITS AND UNITARY REPS

Fix a supermomentum $p(\varphi)$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclus
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ORBITS AND UNITARY REPS

Fix a supermomentum $p(\varphi)$

• Find all $f \cdot p$, where $f \in \text{Diff}(S^1)$.

 $f \cdot p$

Unitary reps of semi-direct products	Unitary reps of BMS ₃
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Fix a supermomentum $p(\varphi)$

• Find all $f \cdot p$, where $f \in \text{Diff}(S^1)$.

 $\langle f \cdot p, \alpha \rangle$

Unitary reps of semi-direct products	Unitary
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ORBITS AND UNITARY REPS

Fix a supermomentum $p(\varphi)$

$$\langle f \cdot p, \alpha \rangle = \langle p, \sigma_{f^{-1}} \alpha \rangle$$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Fix a supermomentum $p(\varphi)$

$$\langle f \cdot p, \alpha \rangle = \langle p, \sigma_{f^{-1}} \alpha \rangle$$

 $\blacktriangleright f \cdot p$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Fix a supermomentum $p(\varphi)$

$$\langle f \cdot p, \alpha \rangle = \langle p, \sigma_{f^{-1}} \alpha \rangle$$

•
$$f \cdot p|_{f(\varphi)}$$

Unitary	reps	of	semi-dire	ct	products	
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Conclusion

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ORBITS AND UNITARY REPS

Fix a supermomentum $p(\varphi)$

Find all $f \cdot p$, where $f \in \text{Diff}(S^1)$.

$$\begin{split} \langle f \cdot p, \alpha \rangle &= \langle p, \sigma_{f^{-1}} \alpha \rangle \\ \bullet & f \cdot p|_{f(\varphi)} = \frac{1}{(f'(\varphi))^2} \ p(\varphi) \end{split}$$

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► BMS₃ orbits = orbits of stress tensors under conf. tsfs !

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▶ p(φ) ~ CFT stress tensor on S¹

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Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃	Relation to gravity 0000	Conclusion 00

$$f\cdot p|_{f(\varphi)} = \frac{1}{(f'(\varphi))^2} \left[p(\varphi) + \frac{c}{12} \left\{ f;\varphi \right\} \right]$$

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Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 000000	Relation to gravity 0000	Conclusion 00
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Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 000000	Relation to gravity 0000	Conclusion 00

$$f \cdot p|_{f(\varphi)} = \frac{1}{(f'(\varphi))^2} \left[p(\varphi) + \frac{c}{12} \left\{ f; \varphi \right\} \right]$$

- $\mathcal{O}_p = \left\{ f \cdot p | f \in \text{Diff}(S^1) \right\} \rightarrow \text{complicated } !$
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1. Constant supermomentum $p(\varphi) = p_0$ (~ particle at rest)

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$$f \cdot p|_{f(\varphi)} = \frac{1}{(f'(\varphi))^2} \left[p_0 + \frac{c}{12} \{f; \varphi\} \right] = p_0$$

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•
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Generic p_0 :

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 $p_0 = -n^2 c/24$:

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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$$\frac{1}{(f'(\varphi))^2} \left[p_0 + \frac{c}{12} \left\{ f; \varphi \right\} \right] = p_0$$

- ► $\mathcal{O}_p = \{f \cdot p | f \in \text{Diff}(S^1)\} \rightarrow \text{complicated } !$
- Let's make it simple :
 - 1. Constant supermomentum $p(\varphi) = p_0$ (~ particle at rest) 2. Look for stabilizer G_p
 - $\mathcal{O}_p \cong \operatorname{Diff}(S^1)/G_p$

Generic p_0 : $f(\varphi) = \varphi + \theta \quad \rightsquigarrow G_p \cong U(1)$

$$p_0 = -n^2 c/24$$
: $f(\varphi) = \varphi + \theta + \cdots$

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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$$\frac{1}{(f'(\varphi))^2} \left[p_0 + \frac{c}{12} \left\{ f; \varphi \right\} \right] = p_0$$

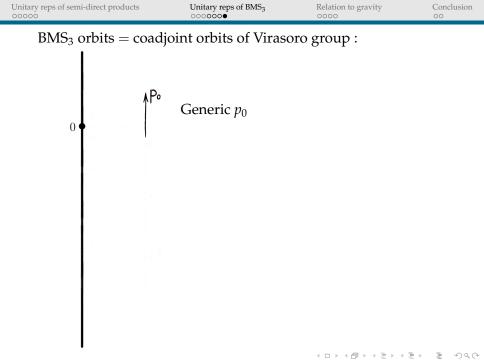
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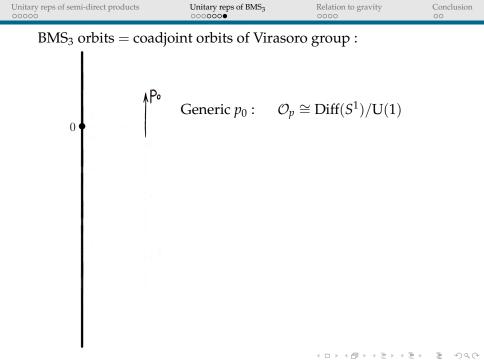
$$p_0 = -n^2 c/24$$
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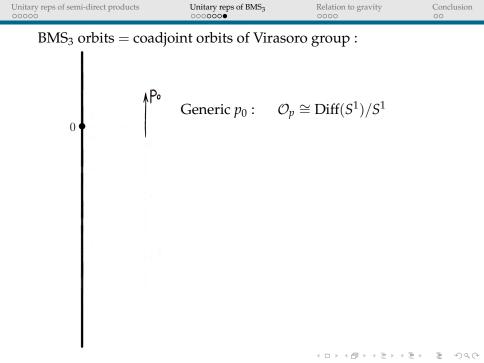
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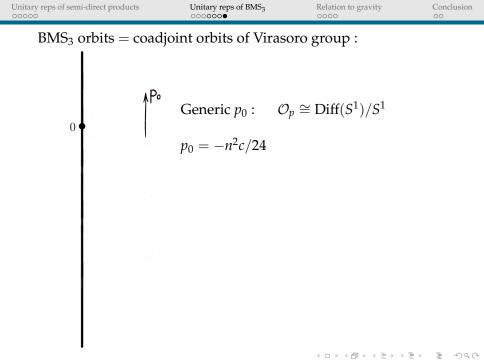
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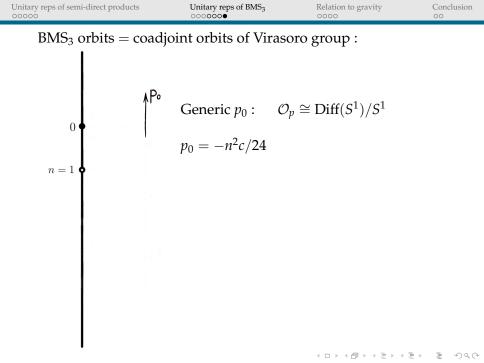
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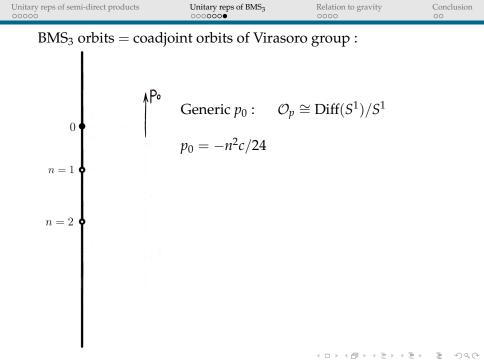


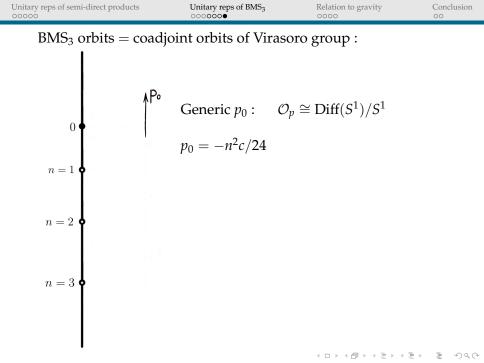


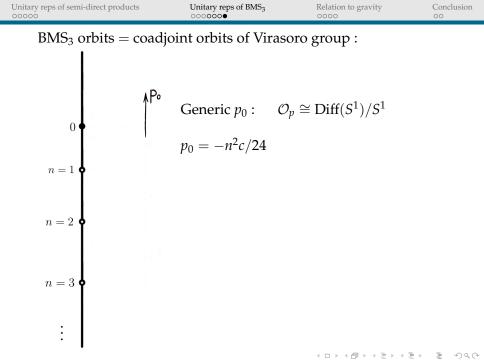


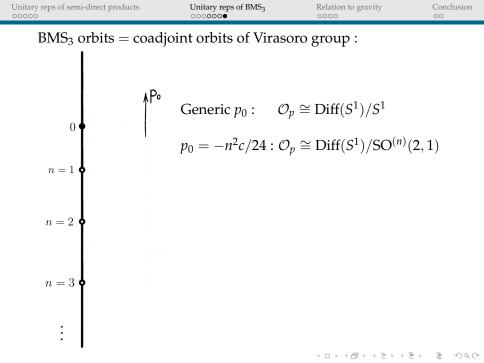


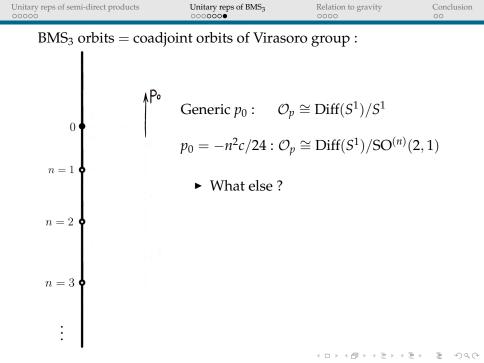












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Unitary reps of semi-direct products Unitary reps of BMS₃ Relation to gravity Conclusion 000000 BMS_3 orbits = coadjoint orbits of Virasoro group : Generic p_0 : $\mathcal{O}_p \cong \operatorname{Diff}(S^1)/S^1$ $p_0 = -n^2 c/24 : \mathcal{O}_p \cong \operatorname{Diff}(S^1)/\operatorname{SO}^{(n)}(2,1)$ n = 1What else ? • Perturbations $-\frac{n^2c}{24} + \delta p(\varphi)$ n = 2• $\delta p = 3$ -momentum under SO⁽ⁿ⁾(2, 1) "Poincaré orbits for each *n*" ! n=3New orbits ! :

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Unitary reps of semi-direct products Unitary reps of BMS₃ Relation to gravity Conclusion 000000 BMS_3 orbits = coadjoint orbits of Virasoro group : Generic p_0 : $\mathcal{O}_p \cong \text{Diff}(S^1)/S^1$ $p_0 = -n^2 c/24 : \mathcal{O}_p \cong \operatorname{Diff}(S^1)/\operatorname{SO}^{(n)}(2,1)$ n = 1What else ? • Perturbations $-\frac{n^2c}{24} + \delta p(\varphi)$ n=2 s • $\delta p = 3$ -momentum under SO⁽ⁿ⁾(2, 1) "Poincaré orbits for each n" ! n=3New orbits ! :

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Unitary reps of semi-direct products Unitary reps of BMS₃ Relation to gravity Conclusion 000000 BMS_3 orbits = coadjoint orbits of Virasoro group : Generic p_0 : $\mathcal{O}_p \cong \text{Diff}(S^1)/S^1$ $p_0 = -n^2 c/24 : \mathcal{O}_p \cong \operatorname{Diff}(S^1)/\operatorname{SO}^{(n)}(2,1)$ ▶ What else ? • Perturbations $-\frac{n^2c}{24} + \delta p(\varphi)$ n=2• $\delta p = 3$ -momentum under SO⁽ⁿ⁾(2, 1) "Poincaré orbits for each n" ! n=3New orbits ! :

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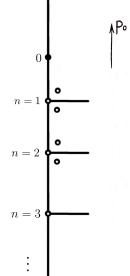
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Unitary reps of semi-direct products

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 BMS_3 orbits = coadjoint orbits of Virasoro group :

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Generic
$$p_0$$
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$$p_0 = -n^2 c/24 : \mathcal{O}_p \cong \mathrm{Diff}(S^1)/\mathrm{SO}^{(n)}(2,1)$$

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Unitary reps of semi-direct products Unitary reps of BMS₃ Relation to gravity Conclusion 000000 BMS_3 orbits = coadjoint orbits of Virasoro group : Generic p_0 : $\mathcal{O}_p \cong \text{Diff}(S^1)/S^1$ $p_0 = -n^2 c/24 : \mathcal{O}_p \cong \text{Diff}(S^1)/\text{SO}^{(n)}(2,1)$ ▶ What else ? • Perturbations $-\frac{n^2c}{24} + \delta p(\varphi)$

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- ▶ "Poincaré orbits for each *n*" !
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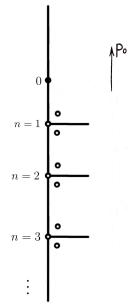
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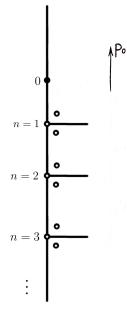
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BMS₃ orbits



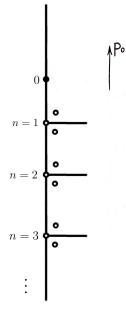
These are orbits of supermomenta in unitary reps of BMS₃.



These are orbits of supermomenta in unitary reps of BMS₃.

Each orbit determines UIRREPS of BMS₃.

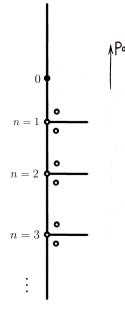
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These are orbits of supermomenta in unitary reps of BMS₃.

Each orbit determines UIRREPS of BMS₃. ► BMS₃ particles !

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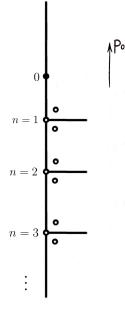
- ► **BMS**₃ particles !
- $\mathcal{H}_{BMS} = \{ wavefcts on \mathcal{O}_p \}$

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BMS₃ orbits

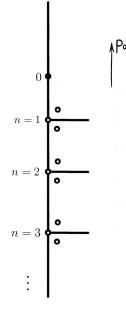


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- $\mathcal{H}_{BMS} = \{ wavefcts on \mathcal{O}_p \}$

► Vacuum :



These are orbits of supermomenta in unitary reps of BMS_3 .

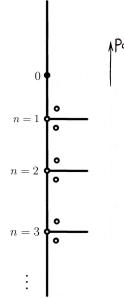
Each orbit determines UIRREPS of BMS₃.

- ► **BMS**₃ particles !
- $\mathcal{H}_{BMS} = \{ wavefcts on \mathcal{O}_p \}$
- **Vacuum** : $p_0 = -c/24$

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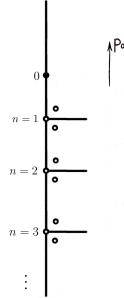


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Each orbit determines UIRREPS of BMS₃.

- ► **BMS**₃ particles !
- $\mathcal{H}_{BMS} = \{ wavefcts on \mathcal{O}_p \}$
- **Vacuum** : $p_0 = -c/24$
- ► Massive BMS₃ particles :

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These are orbits of supermomenta in unitary reps of BMS₃.

Each orbit determines UIRREPS of BMS₃.

- ► **BMS**₃ particles !
- $\mathcal{H}_{BMS} = \{ wavefcts on \mathcal{O}_p \}$
- Vacuum :
- ▶ Vacuum : p₀ = -c/24
 ▶ Massive BMS₃ particles : p₀ > -c/24

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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3. Relation to gravity

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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3. Relation to gravity

A. Supermomentum and Bondi mass

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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3. Relation to gravity

A. Supermomentum and Bondi mass

B. Characters and partition functions

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Unitary	reps of semi-direct products	
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Unitary reps of BMS₃ 0000000 Relation to gravity

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SUPERMOMENTUM & BONDI MASS

On-shell aspt. flat metrics

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SUPERMOMENTUM & BONDI MASS

On-shell aspt. flat metrics :

[Barnich & Troessaert 2010]

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 $ds^2 = 8G p(\varphi) du^2 - 2dudr + r^2 d\varphi^2$

Unitary reps of semi-direct products	Unitary re
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Conclusion

ps of BMS₃

On-shell aspt. flat metrics :

[Barnich & Troessaert 2010]

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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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On-shell aspt. flat metrics :

[Barnich & Troessaert 2010]

 $ds^2 = 8G p(\varphi) du^2 - 2du dr + r^2 d\varphi^2 + \cdots$

 $p(\varphi) =$ Bondi mass aspect

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• Action of BMS₃ on $p(\varphi)$:

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Ex. :

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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SUPERMOMENTUM & BONDI MASS

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Ex. :
$$p_0 = -c/24 \iff$$
 Minkowski space

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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 BMS_3 particle = Particle \otimes Soft gravitons

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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 BMS_3 particle = Particle \otimes Soft gravitons

• Vacuum BMS₃ character \leftrightarrow graviton partition function ?

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Characters of unitary reps of semi-direct products :

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Characters of unitary reps of semi-direct products :

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$$\chi[(f,\alpha)] = \operatorname{Tr}\left(\mathcal{T}[(f,\alpha)]\right)$$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Characters of unitary reps of semi-direct products :

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Conclusion 00

CHARACTERS & PARTITION FUNCTIONS

Massive BMS₃ particle

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Massive BMS₃ particle

► $p = p_0$



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Massive BMS₃ particle

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$$p = p_0 \rightarrow \mathcal{O}_p = \text{Diff}(S^1)/S^1$$

Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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Take $f(\varphi) = \varphi + \theta$

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	Relation to gravity	
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CHARACTERS & PARTITION FUNCTIONS

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CHARACTERS & PARTITION FUNCTIONS

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CHARACTERS & PARTITION FUNCTIONS

Unitary reps of BMS₃

Massive BMS₃ particle

• $p = p_0 \rightarrow \mathcal{O}_p = \operatorname{Diff}(S^1)/S^1$

Take $f(\varphi) = \varphi + \theta$ (rotation by θ)

• Character :

$$\chi[(\operatorname{rot}_{\theta},\alpha)] = \int_{\mathcal{O}_p} d\mu(q) \,\delta(q,\operatorname{rot}_{\theta}\cdot q) \, e^{i \, p_0 \alpha^0}$$

- ► The integral "localizes" to a point !
- Fourier modes = coordinates on \mathcal{O}_p

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$$[\operatorname{rot}_{\theta} \cdot q]_n = e^{in\theta}q_n$$

Massive BMS3 particle

• $p = p_0 \rightarrow \mathcal{O}_p = \operatorname{Diff}(S^1)/S^1$

Take $f(\varphi) = \varphi + \theta$ (rotation by θ)

• Character :

$$\chi[(\operatorname{rot}_{\theta},\alpha)] = \int \prod_{n \in \mathbb{Z}^*} dq_n \,\delta(q_n - e^{in\theta}q_n) \, e^{i p_0 \alpha^0}$$

- ► The integral "localizes" to a point !
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$$[\operatorname{rot}_{\theta} \cdot q]_n = e^{in\theta}q_n$$

Massive BMS3 particle

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Unitary reps of BMS₃

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Take $f(\varphi) = \varphi + \theta$ (rotation by θ)

• Character :

$$\chi[(\operatorname{rot}_{\theta}, \alpha)] = \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in\theta}|^2} e^{i p_0 \alpha^0}$$

- ► The integral "localizes" to a point !
- Fourier modes = coordinates on \mathcal{O}_p

•
$$[\operatorname{rot}_{\theta} \cdot q]_n = e^{in\theta}q_n$$

$$\chi \ [(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \ \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in \ \theta}|^2}$$



$$\chi \quad [(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \quad \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{i n(\theta + i\epsilon)}|^2}$$



$$\chi_{p_0}[(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$



$$\chi_{p_0}[(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \quad \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{i n(\theta + i\epsilon)}|^2}$$

 $\chi_{\rm vac}[({\rm rot}_{\theta}, \alpha)]$



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$$\chi_{p_0}[(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

$$\chi_{\rm vac}[({\rm rot}_{\theta}, \alpha)] = e^{-i\alpha^0 c/24}$$



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$$\chi_{p_0}[(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \quad \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{-i\alpha^0 c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

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CHARACTERS & PARTITION FUNCTIONS

$$\chi_{p_0}[(\operatorname{rot}_{\theta}, \alpha)] = e^{i p_0 \alpha^0} \quad \frac{1}{\prod_{n=1}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

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Unitary	reps of semi-direct products	
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Unitary reps of BMS₃

Relation to gravity

Conclusion 00

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CHARACTERS & PARTITION FUNCTIONS

$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{-i\alpha^0 c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

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Unitary reps of semi-direct products Unit	itary reps of BMS ₃ I	Relation to gravity	Conclusion
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$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{\beta c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$

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Unitary reps of semi-direct products Unit	tary reps of BMS ₃	Relation to gravity	Conclusion
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$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{\beta c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$
[BO 2015]

 One-loop partition fct of gravitons on thermal flat space ! [Barnich, González, Maloney, BO 2015]

Unitary reps of semi-direct products Unitary	ary reps of BMS ₃	Relation to gravity	Conclusion
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$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{\beta c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$
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Unitary reps of semi-direct products	Unitary reps of BMS ₃	Relation to gravity	Conclusion
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$$\chi_{\text{vac}}[(\text{rot}_{\theta}, \alpha)] = e^{\beta c/24} \frac{1}{\prod_{n=2}^{+\infty} |1 - e^{in(\theta + i\epsilon)}|^2}$$
[BO 2015]

- One-loop partition fct of gravitons on thermal flat space ! [Barnich, González, Maloney, BO 2015]
- Generalization to higher spins
 [Campoleoni, González, BO, Riegler 2015]
- ► Flat analogue of AdS₃/CFT₂ results

[Giombi, Maloney, Yin 2008] [Gaberdiel, Gopakumar 2010]

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Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion ●0

$BMS_3 = Superrotations \ltimes Supertranslations$

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion •0

$BMS_3 = Superrotations \ltimes Supertranslations$

"Supermomenta"

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion ●○

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CONCLUSION

 $BMS_3 = Superrotations \ltimes Supertranslations$

- "Supermomenta"
- ► UIRREPs classified by supermomentum orbits

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion •0

 $BMS_3 = Superrotations \ltimes Supertranslations$

- "Supermomenta"
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- ► Relation BMS₃/Gravity

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion • o

 $BMS_3 = Superrotations \ltimes Supertranslations$

- "Supermomenta"
- ► UIRREPs classified by supermomentum orbits
- ► Relation BMS₃/Gravity :

Supermomentum \leftrightarrow Bondi mass aspect

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion ●0

 $BMS_3 = Superrotations \ltimes Supertranslations$

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 $\begin{array}{rcl} \text{Supermomentum} & \leftrightarrow & \text{Bondi mass aspect} \\ & \text{BMS}_3 \text{ particle} & \leftrightarrow & \text{Particle dressed with soft gravitons} \end{array}$

Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion ●0

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Unitary reps of semi-direct products 00000	Unitary reps of BMS ₃ 0000000	Relation to gravity 0000	Conclusion 0•



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